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ACOUSTIC CHARACTERISTICS OF PLANE MULTILAYERED SANDWICH INFINITE-INFINITE STRUCTURES

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Abstract: *An exact local model for computing the coincidence frequencies of multilayered sandwich panels and numerical results predicted by the model are presented in the paper. The model is derived within the local theory of linear elastodynamics without any simplifications concerning the structure. The coincidence frequencies for homogeneous panels and for five-layer and seven-layer sandwich panels are obtained and compared with corresponding results predicted by the models existing in the literature. Both flexural and breathing waves propagating in the sandwich structures are considered.*

Keywords: ***Local model, flexural waves, breathing waves, coincidence frequencies, coincidence curve.***

1. Introduction

The plane continuous structures can be divided into three groups i.e., (1) infinite in two perpendicular directions, shortly infinite-infinite (I-I), panels, (2) finite-infinite (F-I) duct covers and (3) finite-finite F-F) plates. Transmission loss (TL) is the basic characteristics of all the structures. In this paper however some other characteristics such as coincidence frequency and critical coincidence frequency of plane, multilayered sandwich panels are considered. The characteristics called here as 'coincidence curve' is defined as the ratio $\omega_c/\omega_{cr} \equiv f_c/f_{cr}$, where ω_c denotes the coincidence frequency for an assumed incident angle θ and ω_{cr} is the critical coincidence frequency, $\omega_{cr} = \omega_c(\theta = 90^0)$.

All the coincidence characteristics are important for many reasons. In particular, the critical coincidence frequency $\omega_{cr} \equiv 2\pi f_{cr}$, is of special importance because of the following reasons (Bhattacharya et al., 1971). First, the critical coincidence frequency is the only frequency at which the transmission loss (TL) of a finite-finite (F-F) purely elastic plate equals to zero. Second, the critical coincidence frequency of the F-F plate is independent of the incident angle of the acoustic wave. Third, the critical coincidence frequency of the F-F simply supported plate equals to the critical coincidence frequency of the infinite-infinite (I-I) panel with the same cross-sectional parameters as the F-F plate. The third property is noted here as follows, $(\omega_{cr})_{I\text{-}I} = (\omega_{cr})_{F\text{-}F}$. Due to the third property the critical coincidence frequency can be computed either within the models (theories) for the F-F plates or within the corresponding models for the I-I panels. Obviously, the latter method is easier.

It is noted that more facts about importance of the critical coincidence frequency, when two-layer plasterboards and the sandwich structures are considered, one can find e.g. in the papers: (Matsumoto et al., 2006; Renji et al., 1997; Renji, 2005; Wang et al., 2005 and Zhou & Crocker, 2010). The main fact is that the acoustic transmission characteristics of the structures are highly dependent on whether the excitation frequency is below or above the critical coincidence frequency.

2. Statement of the problem

The structure considered in this paper is composed of p isotropic layers. It is infinite in directions x, y and its thickness, being the sum of thicknesses of the layers, extends in direction z.

Statement of the acoustic problem for the multilayered structure contains the following Eqs: the kinematic assumptions (1) - satisfied individually for each layer, the wave Eqs (2) - satisfied individually for each layer, the Saint-Venant compatibility Eqs - satisfied individually for each layer,

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the calibration condition (3) - satisfied individually for each layer, the boundary conditions (4) - for the (outside) surfaces of the structure, the compatibility Eqs (5) between adjoining layers and the constitutive Eq (the Hooke law) - satisfied individually for each layer.

$$\underline{u} = grad\phi + rot\underline{\psi} \quad \equiv \quad u_k = \phi_{,k} + e_{klm}\psi_{m,l} \ , \qquad \underline{u} = \{u_1\ u_2\ u_3\} \ , \qquad k,l,m = 1,2,3 \ . \tag{1}$$

$$[(\lambda + 2\mu)/\rho]\nabla^2\phi - \ddot{\phi} = 0 \ , \qquad (\mu/\rho)\nabla^2\psi_m - \ddot{\psi}_m = 0 \ , \quad m = 1,2,3 \ . \tag{2}$$

$$\psi_{k,k} \equiv \psi_{1,1} + \psi_{2,2} + \psi_{3,3} = 0 \ . \tag{3}$$

$$\begin{aligned} \sigma_{33}(x,y,z=z_r) \equiv \sigma_{zz}(x,y,z=z_r) = q_r \ , \quad & \sigma_{31}(x,y,z=z_r) \equiv \sigma_{zx}(x,y,z=z_r) = 0 \ , \\ & \sigma_{32}(x,y,z=z_r) \equiv \sigma_{yz}(x,y,z=z_r) = 0. \end{aligned} \tag{4}$$

$$\begin{aligned} \sigma_{33(j)} = \sigma_{33(j+1)} \ , \quad \sigma_{31(j)} = \sigma_{31(j+1)} \ , \quad \sigma_{23(j)} = \sigma_{23(j+1)} \ , \\ u_{1(j)} = u_{1(j+1)} \ , \qquad u_{2(j)} = u_{2(j+1)} \ , \qquad u_{3(j)} = u_{3(j+1)} \quad . \end{aligned} \tag{5}$$

Symbols, u_k, λ and μ, ρ, σ_{lm}, appearing in (1)-(5), denote, displacement components, Lame's parameters, material density and stresses, respectively. For a particular j^{th} layer the Lame parameters λ and μ as well as density ρ, appearing in the wave Eqs (2), should be replaced with $\lambda_{(j)}$, $\mu_{(j)}$ and $\rho_{(j)}$, respectively. Symbols q_r denote the normal acoustic loading acting on the surface denoted with r = 1, 2. If we assume that the subscript r equals to 1 ($q_r = q_1$) for the top/left outside surface of the plate then r equals to 2 (q_r=q_2) for the bottom/right outside surface of the structure, respectively. Symbol z_r in (4) denotes coordinates of the outside surfaces of the multilayered panel. It is noted that for the problem considered here the displacement field (1) can be simplified.

The acoustic loadings q_r are defined as follows (Renji, 2005; Wang et al., 2005),

$$q_1 = Z_{air}\dot{u}_z(z = -h/2), \qquad q_2 = Z_{air}\dot{u}_z(z = +h/2) \ , \qquad Z_{air} = \rho_{air}c \ . \tag{6}$$

Symbols h, ρ_{air}, c denote thickness of the structure, air impedance and sound velocity in the air, respectively. The solution resulting from the above statement is called in the further text as the exact local model.

3. Numerical results

In order to show a broad applicability of the exact local model outlined in the paper the coincidence frequencies were computed for a homogeneous (one-layer) I-I panel, for a five-layer sandwich I-I panel and for a seven-layer sandwich I-I panel. To test the model and the computing program the results for the homogeneous structure are compared with the counterparts predicted by the Kirchhoff and Mindlin models. All the numerical results are presented in the Figs. 1 - 2 and in the Tables 1 - 3.

In Fig. 1 the coincidence curves and in Table 1 the coincidence frequencies for the flexural waves propagating in a homogeneous I-I panel of the following parameters, $h_{(1)}$ = 25 mm, $E_{(1)} = 0.1\cdot10^{10}$ Pa, $\nu_{(1)}$=0.16, $\rho_{(1)}$=1200 kg/m^3, are given. These input data are approximately parameters of the plasterboard. The (shear modulus)/density ≡ $\mu_{(1)}/\rho_{(1}$ ratio in the case equals to $3.59\cdot10^5$. Results denoted with the subscript SK are predicted by the local exact model, the counterparts denoted with subscripts K, M are predicted by the Kirchhoff and Mindlin model, respectively. It is seen that the coincidence curve resulting from the Kirchhoff theory is much below the curves predicted by the local and Mindlin models, within the range of the incident angle eta ≡ θ between 42 and 72 degrees.

Tab. 1: Flexural coincidence frequencies (Hz) for the plasterboard.

Θ	42	48	54	60	66	72	78	84	90
$(f_c)_{SK}$	19465	10496	7450.4	5897.4	4987.9	4426.9	4084.7	3898.8	3839.7
$(f_c)_K$	6266.1	5080.1	4286.5	3740.7	3361.7	3101.7	2932.3	2836.5	2805.6
$(f_c)_M$	17801	9437.9	6771.8	5416.3	4617.7	4121.6	3817.5	3651.6	3598.8

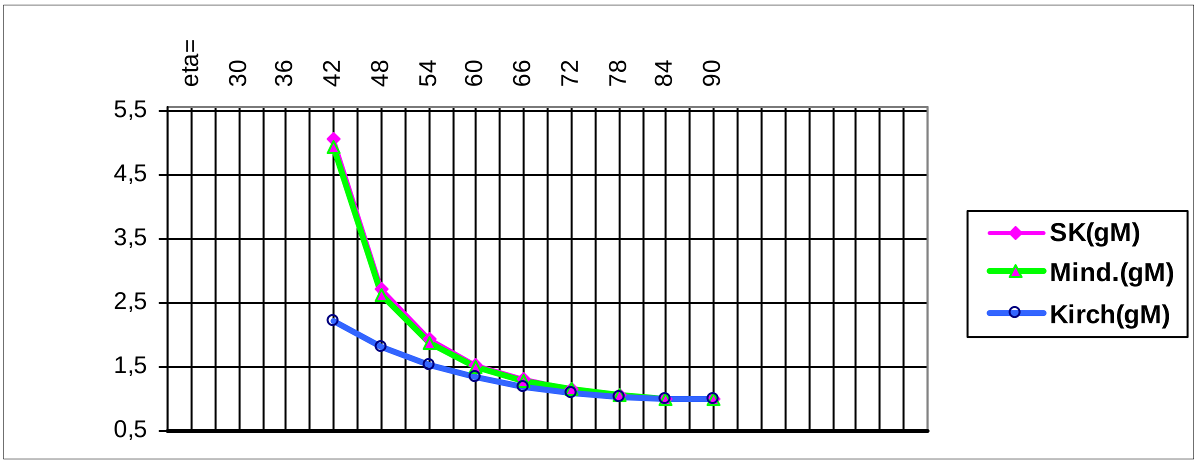

Fig. 1: Coincidence curves for the plasterboard.

In Fig. 2 the coincidence curves and in Tables 2 - 3 coincidence frequencies for a five-layer sandwich I-I panel and a seven-layer sandwich I-I panel are presented. Parameters of the five-layer structure are as follows: $h_{(1)}=1$ mm, $E_{(1)}=0.68910^{11}$ Pa, $\nu_{(1)}=0.276$, $\rho_{(1)}=2680$ kg/m^3, $h_{(2)}=0.5$ mm, $E_{(2)}=0.39\cdot10^{10}$ Pa, $\nu_{(2)}=0.08$, $\rho_{(2)}=1175$ kg/m^3, $h_{(3)}=62$ mm, $E_{(3)}=0.3059\cdot10^{9}$ Pa, $\nu_{(3)}=0.85$, $\rho_{(3)}=32.8$ kg/m^3, $h_{(2)}=h_{(4)}$, $E_{(2)}=E_{(4)}$, $\nu_{(2)}=\nu_{(4)}$, $\rho_{(2)}=\rho_{(4)}$, $h_{(1)}=h_{(5)}$, $E_{(1)}=E_{(5)}$, $\nu_{(1)}=\nu_{(5)}$, $\rho_{(1)}=\rho_{(5)}$. The layers 2, 4 may be for instance of the glue necessary to connect the outer aluminum layers and the middle honeycomb core. Parameters of the seven-layer structure are as follows: $h_{(1)}=1$ mm, $E_{(1)}=0.68910^{11}$ Pa, $\nu_{(1)}=0.276$, $\rho_{(1)}=2680$ kg/m^3, $h_{(2)}=0.5$ mm, $E_{(2)}=0.39\cdot10^{10}$ Pa, $\nu_{(2)}=0.08$, $\rho_{(2)}=1175$ kg/m^3, $h_{(3)}=30$ mm, $E_{(3)}=0.3059\cdot10^{9}$ Pa, $\nu_{(3)}=0.85$, $\rho_{(3)}=32.8$ kg/m^3, $h_{(4)}=2$ mm, $E_{(4)}=0.925\cdot10^{7}$ Pa, $\nu_{(4)}=0.25$, $\rho_{(4)}=92.5$ kg/m^3, $h_{(2)}=h_{(5)}$, $E_{(2)}=E_{(5)}$, $\nu_{(2)}=\nu_{(5)}$, $\rho_{(2)}=\rho_{(5)}$, $h_{(1)}=h_{(7)}$, $E_{(1)}=E_{(7)}$, $\nu_{(1)}=\nu_{(7)}$, $\rho_{(1)}=\rho_{(7)}$. The layer 4 is of a cork conglomerate material. The results in Fig. 2 and in Tab. 2 refer to the flexural waves of the sandwich structures. In Tab. 3 the coincidence frequencies for the breathing waves are given.

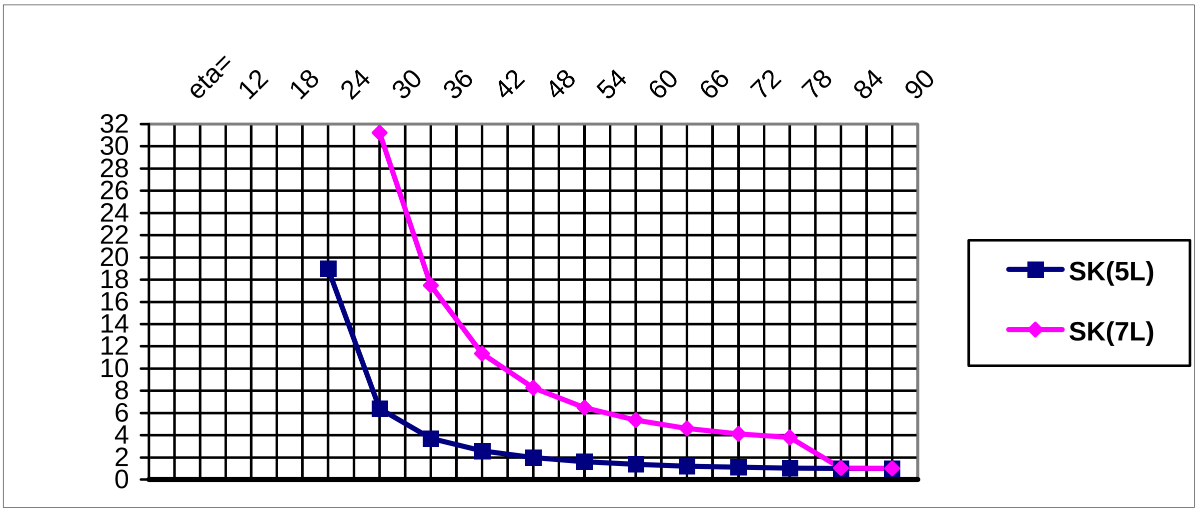

Fig. 2: Coincidence curves for the five-layer(5L) and seven-layer (7L) sandwich structures.

Symbol Δ in Tab. 2 denotes the percentage differencies between the coincidence frequencies.

Tab. 2: Flexural coincidence frequencies (Hz) for the five-layer (f_{c5}) and seven-layer (f_{c7}) panel.

Θ	12	18	24	30	36	42	48	54	60	66	72	78	84	90
$(f_{c5})_{SK}$	51813	10400	2709.7	911.1	529.6	369.9	283.7	231.4	197.4	174.8	159.7	150.0	144.6	142.8
$(f_{c7})_{SK}$		87137	63686	4952.8	2770.7	1801.5	1311.4	1027.4	848.5	730.8	652.9	603.2	160.8	158.6
Δ		737.86	2250.3	443.61	423.17	387.02	362.25	343.99	329.84	318.08	308.83	302.13	11.20	11.06

Tab. 3: Breathing coincidence frequencies (Hz) for the five-layer (fc5) panel.

Θ	12	18	24	30	36	42	48	54	60	66	72	78	84	90
$(f_{c5})_{SK}$						27185	22218	19012	16857	15387	14390	13745	13382	13265

It is seen in Fig. 2 that the seven-layer panel seems to be much better for the acoustic purposes than the five-layer panel. The seven-layer structure is obtained by dividing the five-layer panel within the middle plane and inserting the layer of the cork material between the two symmetric parts.

4. Conclusions

Coincidence frequencies and the coincidence curves for the homogeneous plasterboard and for two multilayered sandwich panels are obtained by applying the local exact model outlined in section 2. In order to test the model the characteristics for the homogeneous panel are compared with corresponding results predicted by the Kirchhoff and Mindlin models.

The comparison enables us to say that prediction of the coincidence frequencies by the Kirchhoff model is highly inaccurate, in particular for the lower values of the incident angle of the acoustic wave. The predictions of the Mindlin model and the local exact model are much closer.

Inserting of the compliant thin layer of cork material between two symmetric (about the middle plane) parts of the five-layer panel increases significantly the coincidence frequencies. It is also shown that the coincidence frequencies of the five-layer panel associated with the breathing acoustic waves propagating in the structure occur within the range of hearing by the human ear.

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