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# SIMULATION OF FATIGUE CRACK PROPAGATION UNDER CONTACT LOADING CONDITIONS

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**Abstract:** *The article is a contribution to numerical simulations of pitting arise phenomena on the gear teeth. The basic assumption of the presented simulations was that pitting (pits) is a result of fatigue crack propagation under (rolling) contact loading conditions. The solution approach consisted of numerical simulations of fatigue cracks growth in the FEM framework and a comparison of numerical and experimental results. An acceptable agreement of the numerical and experimental results confirms (not proves) that fatigue crack propagation can present a true damage mechanism of the pitting wear rise. A theoretical basis of the presented simulations is an approximation of fatigue crack growth description by the so called Paris law in conjunction with FEA of crack tip loading conditions and fracture criteria evaluation. This allows, at the given crack geometry and loading amplitude in each simulated loading cycle, to estimate the crack growth rate and extension increment direction, and to model its increment by which the crack was lengthened in the following simulation. The simulations were performed under the ABAQUS CAE FEM programme which enables to create in-house codes using the Python scripting language and which is the basis of all FEA including the simulation of the crack growth.*

**Keywords:** ***Gears, pitting, FEM simulation, crack propagation.***

## 1. Introduction

Pitting phenomena belong to a contact fatigue problem area, which rise under rolling contact conditions especially. The published approaches to the numerical simulation of pitting damage rise process can be divided into two main domains. The basic assumption of the first group is that a pressured fluid lubricant penetrates into cracks and influences its growth. This approach can be represented e.g. by Fajdiga et al. (2004), who simulate fatigue crack growth from a surface initial crack and contact loading approximated by pressure distribution corresponding to the EHD lubrication theory. In the second group of approaches other possible damage mechanisms of pitting rise are assumed. E.g. Ding et al. (2003) simulate pitting rise from subsurface initial cracks.

The presented article belongs to the first group of the above mentioned approaches. The fatigue crack growth is simulated from the surface initial crack. The contact conditions between real gear teeth near the pitting crack mouth were computed and the pressured fluid lubricant penetration into pitting crack was assumed.

## 2. Applied phenomenological crack growth theories

Validity of the small scale yielding conditions is the basic assumption of the presented crack growth simulations. The phenomenological theory of fatigue crack propagation – the Paris law – applied in this work supposes that the initial cracks are so long that all parts could be modelled as an isotropic continuum and a dimension of crack tip plastic zone is negligible compared to the crack length.

The crack growth predictions were based on the computation and evaluation of fracture mechanics J-integral criterion. J-integral was used for both calculating fatigue crack growth rate according to the Paris law (1), where $C$ and $m$ are material parameters and $\Delta J$ is a J-integral amplitude, and evaluating the fatigue crack growth direction, which corresponds to the direction in which the maximum of the J-

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integral value was calculated. This criterion is equivalent to the maximal tangential stress criterion in terms of linear elastic fracture mechanics conditions.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta J)^m \qquad (1)$$

## 3. Experimental works

Experimental works were performed at two levels. Firstly, simple so called CT test specimens were employed to provide data for validation of the crack growth prediction models. Secondly, a setup of gearing testing machine was debugged and fatigue tests of the real gearing were carried out.

### 3.1. CT specimens testing

The main goal of the CT specimens testing was both the Paris law parameters identification used for the crack growth rate prediction and verification of the implemented approach to crack growth direction prediction (maximum of J-integral) according to observing the crack growth under mixed mode loading conditions (Španiel et al., 2008). The CT specimens were manufactured from the 18CrNiMo7-6 material, which is normally used for gears and shafts. After heat finishing, the following characteristics of the material can be mentioned: Young's modulus 210 000 MPa, yield stress 1100 MPa, strength 1250 MPa. The estimated Paris law parameters are the following: $C = 1.42\text{e-}5$ mm/[cycle.(N.mm)$^{1.3}$] and $m = 1.3$.

### 3.2. Gearing testing

The main goal of the gearing testing was to obtain experimental data for subsequent verification of the numerical model of the fatigue crack growth under contact loading conditions. Verification criteria of numerical models were both the geometrical parameters of pits and the assessment of the number of the loading cycles, which are necessary for the pits rise. Parameters of tested gearing are listed in Tab. 1.

*Tab. 1: Basic geometrical parameters of the tested gearing.*

| ***Geometrical parameters of tested gearing*** | | |
|---|---|---|
| Gearing ratio | $i_{mp}$ | 3.2 |
| Modulus | m | 5 mm |
| Axis distance | $a_w$ | 160 mm |
| | Pinion | Gear |
| Teeth number | 15 | 48 |
| Gear thickness | 17 mm | 12 mm |
| Loading time/loading cycles | 68 hours/5.9e6 cycles | |
| Torque moment | 400 Nm | |

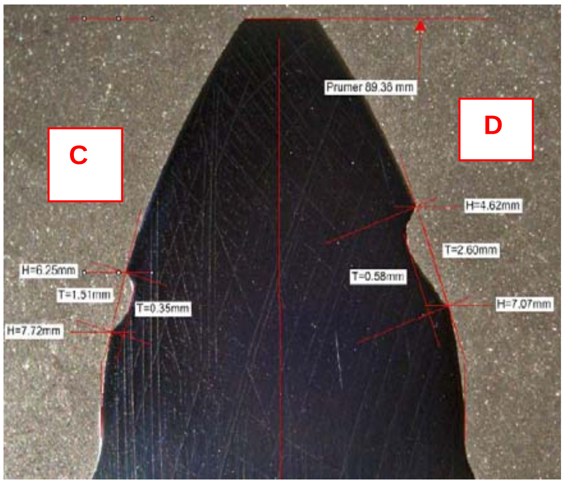


*Fig. 1: Pits location (pitting).*

In order to perform the gearing tests in real time, a short testing mode was chosen. The experimental test stand was designed as the so called Niemann's close torque chain, which normally consists of two connected gear boxes (experimental and technological). The required test toque is induced by a leverage mechanism.

The pits geometry and pitting cracks in various propagation stages were analyzed using suitable material cuts and metallographic samples of the most damaged gear teeth, Fig. 1. It can be derived from the metallographic samples that the fatigue cracks are initiated along the whole contact surface. In the tooth heel domain the cracks are initiated in the direction of the tooth head and in the tooth head domain in the direction of the tooth heel. The results of the experimental tests show that pitting cracks propagate especially in the so called one tooth contact condition area, thus the pitting damage is concentrated in this area.

## 4. Numerical simulations

The pitting rise investigation mentioned in this article is based on the simulations of the gearing contact conditions, which induce boundary conditions for subsequent fatigue crack growth computational predictions. Basic mechanical quantities defining contact conditions are: contact pressure, shear stress, relative slip range and rate. Actually these quantities could be affected by both properties of fluid lubricant used and contact surface roughness. In the FEA models the tribological relations are approximated by friction coefficient *f*, whose value can be in the range of 0.05 - 0.15. Simple linear isotropic material model was assumed according to the material properties.

According to both published works and own experiences the penetration of pressured fluid lubricant into pitting cracks should be considered in simulations, because the pressured lubricant can cause crack opening during contact rolling around the cracked region. A simple cavity model was used to include pressured fluid lubricant penetration in the FEA models. The pressure of the lubricant closed in the crack is assumed to be equal to the actual contact pressure near the crack mouth.

### 4.1. Numerical models

Simulation programs were created using developmental interface of the commercial ABAQUS CAE FEM programme, which provides both a function for stress intensity factors, T-stress, J-integral evaluation and a Python language interface for in-house programme codes submit. FE analyses are quasistatic and planar considering plane strain conditions. The schedule of complex pitting crack growth simulation is shown in Fig. 2.

- At first the simulation of contact conditions around the future crack mouth location (pinion without initial crack) is performed to estimate the relevant contact pressure distribution.

- Subsequently the simulations of pitting crack growth are carried out. The initial crack is included in the first computational step. In the next computational steps the crack is incrementally extended by a relatively small length increment in the given direction. Each computational step consists of a contact rolling simulation around the crack region (2.1, Fig. 2), J-integral amplitude ($\Delta J$), crack growth direction, the number of loading cycles calculation (2.2, Fig. 2) and the pitting crack geometry modification (2.3, Fig. 2).

Influence of the pressured fluid lubricant closed inside the crack is included in the FE models using special elements, which allow simulating charging resp. discharging of deformable cavities by uncompressible fluid.

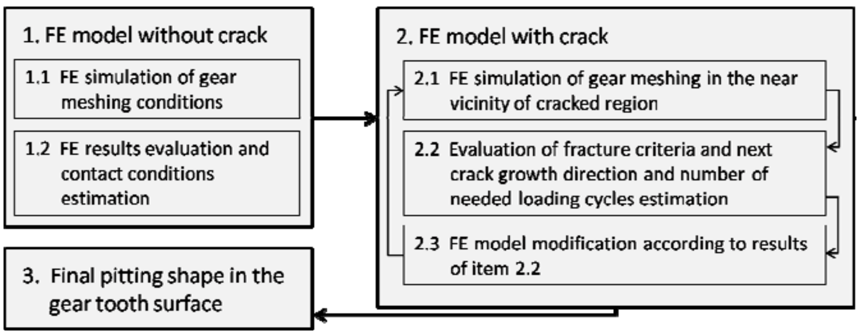


*Fig. 2: Pitting crack growth simulation schedule.*

### 4.2. Results

The main goal of the above mentioned simulations was the numerical model verification. A sensitivity study of the influence of the model parameters (friction coefficient, initial crack inclination to the contact surface, length of the crack increment etc.) on the pitting crack growth behaviour was performed. The FEA model of pinion with the final crack (predicted pit shape) after the crack growth simulation is shown in Fig. 3. Estimation of the number of loading cycles was based on the J-integral amplitude calculation in each simulation step. The crack length increment $\Delta a$ was every time equal to 0.015mm. The predicted number of loading cycles required for the pit rise according to Fig. 3 is approximately equal to 740 000. Neither the first nor the last simulation step, resp. crack extension, was included in the final number of the loading cycles. The J-integral amplitudes in these steps correspond to the initiation, resp. fracture stage of crack behavior.

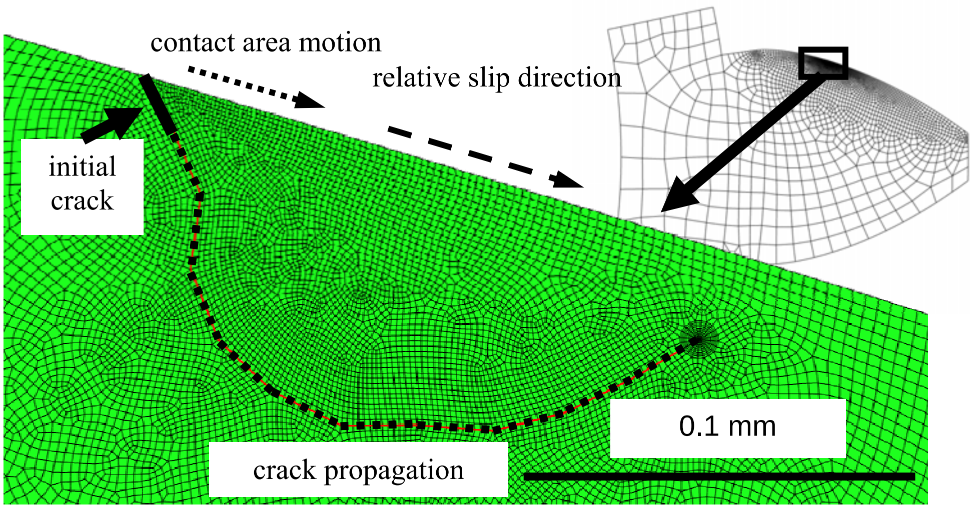


*Fig. 3: Final crack resp. pit shape after simulation – pitting.*

*Tab. 2: Pitting rise prediction according to Fig. 3.*

| ***Crack length [mm]*** | ***J-integral value [Nmm]*** | ***Number of loading cycles [-]*** | ***Crack length [mm]*** | ***J-integral value [Nmm]*** | ***Number of loading cycles [-]*** |
|---|---|---|---|---|---|
| 0.015 ini. | - | - | 0.135 | 0.08093 | 3.71E+04 |
| 0.035 | 0.004179 | 1.75E+06 | 0.155 | 0.082998 | 3.59E+04 |
| 0.055 | 0.026429 | 1.59E+05 | 0.175 | 0.105771 | 2.62E+04 |
| 0.075 | 0.019907 | 2.30E+05 | 0.195 | 0.085879 | 3.43E+04 |
| 0.095 | 0.042368 | 8.60E+04 | 0.215 | 0.064731 | 4.96E+04 |
| 0.115 | 0.044134 | 8.16E+04 | fracture | 0.113319 | |

## 5. Conclusion

The main result of the above mentioned simulation is confirmation that the Paris law can be a relevant mathematical model to describe the pitting crack growth under rolling contact conditions. The complex parametrical FE model of real gearing was created using the Python programming language. This FE model simulates both the contact conditions of real gears including pressured fluid lubricant penetration into crack during contact rolling and the incremental crack propagation. Both the dimensions of the final pit (Fig. 3) and the predicted number of loading cycles are approximately 10x smaller than the experimental results. These differences could be caused by a simple cavity model, in which is supposed, that the fluid lubricant can flow towards and outwards the crack tip according to the actual contact pressure in the vicinity of the crack mouth. This simplified approximation will be replaced by a more sophisticated one within the future work. New cavity model will be based on the numerical simulation of the lubricant flow between teeth during gearing meshing and which will include so called lubricant closure inside the crack.

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