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AEROELASTIC INSTABILITIES ON THE HEIGHT RISE BUILDINGS

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Abstract: *The wind action on structures may be represented by peak velocity pressures, in conditions of strong wind, or by the basic stochastic descriptors of the velocity fluctuation, i.e. turbulence intensity, power spectral density and correlation between velocities at different points of the structures and probability distribution. The energy of turbulent fluctuations is distributed over a range of frequencies and the turbulence intensity is changing with height and terrain category. The resonance response due to gust wind takes into account also motion of the structures itself. Application examples will be given for the multi storey concrete and steel buildings.*

Keywords: ***Wind velocity, intensity of turbulence, power spectral density function,*** **stochastic** ***response.***

1. Introduction

Wind load is the fundamental type of action especially on slender structures. The mean part of the wind pressure – static type is associated with mean wind speed. The fluctuating part is weakly stationary random process with respect time t and homogeneous random process with z. The wind action is represented as random field in time and place. It is assumed linear elastic behavior of structures, the response can be obtained by superposition of the response due to mean wind and than due to fluctuating part $v(z,t)$. The intensity of turbulence is defined as the variance of fluctuating part to the mean wind velocity and depends on height z and orography factor and roughness length, it should be between 5 - 30 %. The energy of turbulence (gustiness) is distributed over a range of frequencies. The effect of the wind on the structures as well as the response of the slender structures depends on the size, shape and dynamic properties of the structures.

2. Wind forces

The wind velocity and the velocity pressure and force are composed of a mean and a fluctuating component. The aerodynamic wind force for the area A at height z is:

$$F(z,t) = c_{\mathrm{D}} \frac{1}{2} \rho \left(v_m(z) + v(z,t)\right)^2 \cdot A \cong c_{\mathrm{D}} \left[\frac{1}{2} \rho \cdot v_m(z)^2 + \rho \cdot v_m(z) \cdot v(z,t)\right] \cdot A \tag{1}$$

The fluctuating part of wind pressure $c_{\mathrm{d}} \cdot \rho \cdot v_m(z) \cdot v(z,t)$ is distributed non-simultaneously on the on building; therefore we can use for autocorrelation function expression (2).

$$R_{\mathrm{FF}}(\tau) = \lim_{\mathrm{T}\to\infty} \int_{-\mathrm{T}/2}^{\mathrm{T}/2} F(z,t) \cdot F(z,t+\tau) dt = c_{\mathrm{D}}^{\ 2} \cdot \rho^2 \cdot v_m(z)^2 \cdot A^2 \cdot \overline{v(z,t) \cdot v(z,t+\tau)} \tag{2}$$

Wind distribution over frequencies can be expressed by the power spectral density function of along wind turbulence. For the natural wind at height z =10 m over the ground we can use according (Pirner et al, 2003) expression (3) or by (Davenport, 1962) expression (4):

$$S_v(n) = \frac{1}{3} \frac{v_{m,10}^2}{n} \cdot \frac{\left(1800n / v_{m,10}\right)^2}{\left[1 + \left(1800n / v_{m,10}\right)^2\right]^{4/3}} \tag{3}$$

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$$S_v(n) = 4K\frac{v^2_{m,10}}{n}\cdot\frac{\left(1200n/v_{m,10}\right)^2}{\left[1+\left(1200n/v_{m,10}\right)^2\right]^{4/3}} \tag{4}$$

Where K is terrain roughness factor(open country $K = 0.005$; city $K = 0.025$) and $v_{m,10}$ is the reference wind velocity at height 10 m. The similar distribution over frequencies (see EN 1991-1-4, 2005) is expressed by the non-dimensional power spectral density function $S_L(z,n)$:

$$S_L(z,n) = \frac{n\cdot S_v(z,n)}{\sigma_v^2} \tag{5}$$

The along wind spectral density for horizontal excitation for symmetrical geometrical shape and stiffness or mass distribution:

$$S_{y,direction}(n,z) = \sum_{i=1}^{\infty}\left|H_i(z,n)\right|^2\cdot\iint_A \phi_{i,y}(z_1)\cdot\phi_{i,y}(z_2)\cdot S_{p1p2}(n,z_1,z_2)\cdot dA_1 dA_2 \tag{6}$$

where $\left|H_i(z,n)\right|$ frequency characteristic of the structure according to harmonic impulse $Dir(z-z_k)\cdot e^{i2\pi nt}$ at the point k

$S_{p1p2}(n,z_1,z_2)$ is the power spectral density of the wind pressure at different points.

$\phi_{i,y}(z)$ is the natural frequency of along wind vibration of the mode "*i*"

According to experimental measurements is possible to express relationship between spectral density of wind pressure and power spectral density of wind velocity.

$$S_p(n) = 4\left(\frac{q_{m,10}}{v_{m,10}}\right)^2\cdot S_v(n) \tag{7}$$

Using aerodynamic admittance coefficient we can obtain for horizontal excitation at the top of structure expression:

$$S_{y,direction}(n) = \left|H(n)\right|^2\cdot\frac{4\cdot q_m^2(z=h)}{v_m^2(z=h)}S_v(n)\cdot A^2\chi^2 \tag{8}$$

The coefficient of aerodynamic admittance χ^2 depends on roughness and dimensions of building.

The standard deviation of the horizontal displacement for the symmetrical structures:

$$\sigma_v^2 = \frac{4\cdot q_m^2(z=h)}{v_m^2(z=h)}\int_0^{\infty}\left|H(n)\right|^2\ \chi^2\ S_v(n)\cdot A^2 dn \tag{9}$$

where
$$H(n) = \frac{1}{M_1\cdot\left(n_{1,y}^2 - n^2 + 2i\cdot n_b\cdot n\right)4\pi^2} \tag{10}$$

3. Numerical Analysis

3.1. Response of height steel building

We can find the maximal dynamic displacement for the 150 m tall steel building, system of 36 steel columns and cross-beams forming a space frame with stiff joints on rectangular plan 35 x 35 m (see Fig. 1) in terrain category III. The equivalent mass per unit $m(z) = 245.10^3$ kg/m.

Equivalent vibrating mass:

$$M_1 = \int_0^{150}\left(\frac{z}{h}\right)^2\cdot m(z)dz = 1.225\cdot 10^7\ \text{kg},$$

$$C = \omega^2 M_1 = 3.79\cdot 10^7\ \text{Nm}^{-1},\quad A = 150\ \text{x}\ 35 = 5250\ \text{m}^2$$

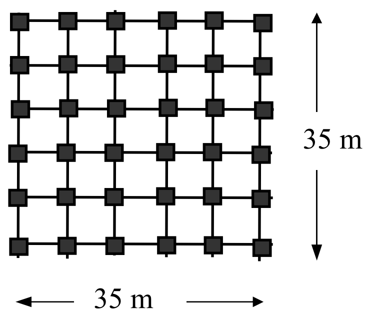

Fig. 1: Rectangular plan.

The fundamental flexural frequencies $n = n_{1,y} = 0.28$ Hz. Mean velocity at the top of building:

$$v_m(z=h) = c_r(z)\cdot c_o(z)\cdot v_{m,10} = 1\cdot 0.19\left(\frac{z_o}{z_{o,II}}\right)^{0,07} ln\left(\frac{z}{z_o}\right)\cdot v_{m,10}$$

Mean wind pressure:

$$q_m(z) = \frac{1}{2}\cdot \rho\cdot c_D\cdot v_m(z)^2\cdot b = \frac{1}{2}1.25\cdot 1.5\cdot 35\cdot v_m(z)^2 = 32.81\cdot v_m(z)^2$$

The logarithmic decrement of structural damping for steel building (EN 1991-1-4, 2005):

$$\delta_y \cong 2\pi\cdot\xi = 0.05$$

For fundamental flexural mode $\phi_1(z) = (z/h)^{\xi}$ and $n = n_{1,y}$ we can obtain standard deviation of the horizontal displacement for the symmetrical structures: according to (9) and (10):

$$\sigma_v^2 = \frac{4\cdot q_m^2(z=h)}{v_m^2(z=h)}\chi^2\, S_v(n_{1,y})\cdot A^2\,\frac{\pi^2\cdot n_{1,y}}{C^2\cdot\delta^2} \tag{11}$$

Aerodynamic admittance functions for the 1st flexural mode: $\chi^2 = 0.0516$

The power spectral density function for $n = n_{1,y}$ is: $S_v(n) = 135.33$

Response according to random vibration approach:

According to (11) we can obtain standard deviation of the horizontal displacement for the symmetrical structures:

$$\sigma^2_{y,direction} = 4.712.10^{-3} \quad \sigma_{y,direction} = 0.0686 \text{ m}$$

For the maximal non harmonic horizontal displacement due to mean and fluctuating part of wind we can write:

$$y_{\max}(z=150) = y_{\mathrm{m}}(z=150) + \sigma_{y,direction} = 0.077675 + 0.0686 = 0.1463 \text{ m}$$

Deflection due to mean wind velocity for slender steel building is:

$$y_{\mathrm{m}}(z) = \frac{\int_0^h q_{\mathrm{m}}(z)\cdot\phi_1(z)dz}{4\pi^2 n_{1,y}^2\cdot M_1}\phi_1(z) = \frac{1/2\cdot\rho\cdot c_{\mathrm{D}}\cdot b\int_0^h\left(0{,}19\left(\frac{z_{\mathrm{o}}}{z_{\mathrm{o,II}}}\right)^{0,07} ln\left(\frac{z}{z_{\mathrm{o}}}\right)\cdot v_{\mathrm{r}}\right)^2\phi_1(z)dz}{4\pi^2 n_{1,y}^2\cdot M_1}\phi_1(z) = 0.07765 \text{ m}$$

Conventional mixing model due to EN 1991-14, (Hubová et al., 2000) and (Hubová, 2007) with detail method of structural factor:

The drag wind force F_w in wind direction is defined by:

$$F_w = c_s c_d\cdot c_f\cdot q_p(z_e)\cdot A_{ref} \tag{12}$$

$$c_s c_d = \frac{1+2\cdot k_p\cdot I_v(z_e)\cdot\sqrt{B^2+R^2}}{1+7\cdot I_v(z_e)} \tag{13}$$

$$q_p(z) = [1+7I_v(z)]\cdot 1/2\cdot\rho\cdot v_m^2(z) \text{ is peak velocity pressure} \tag{14}$$

The resonance response factor:

$$R^2 = \frac{\pi^2}{2\cdot\delta}\cdot S_L(z_e, n_{1,x})\cdot R_h(\eta_h)\cdot R_b(\eta_b) \tag{15}$$

The aerodynamic admittance functions for fundamental mode:

$$\begin{aligned} R_h(\eta_h) &= \frac{1}{\eta_h} - \frac{1}{2\eta_h^2}(1-e^{-2\eta_h}) \qquad \eta_h = \frac{4{,}6\cdot h}{L(z_e)}\cdot f_L(z_e, n_{1,y}) \\ R_b(\eta_b) &= \frac{1}{\eta_b} - \frac{1}{2\eta_b^2}(1-e^{-2\eta_b}) \qquad \eta_b = \frac{4{,}6\cdot b}{L(z_e)}\cdot f_L(z_e, n_{1,y}) \end{aligned} \tag{16}$$

Response according to the (EN 1991-1-4, 2005) is obtained from Eq. (12) – (16):

$$y_{\max}(z=150) = 0.1624 \text{ m}$$

3.2. Response of height concrete building

We can find the maximal dynamic displacement for the concrete building (see Hubová, 2010) in terrain category III using equations (4) and (6) - (11).

$$M_1 = \int_0^{76.42} \left(\frac{z}{h}\right)^2 \cdot \frac{m_b}{h} dz = 1.2056 \cdot 10^7 \text{ kg}, \; C = \omega^2 M_1 = 2.2 \cdot 10^8, \; A = 3530.6 \text{ m}^2$$

$$v_m(z = h) = 33.41 \text{ m/s}, \; v_{m,10} = 28 \text{ m/s}, \quad n = n_{1,y} = 0.68 \text{ Hz}, \quad n_{2,y} = 3.311 \text{ Hz}$$

$\xi_y = 0.01$, $\delta_y \cong 2\pi \cdot \xi = 0.0628$ is logarithmic decrement of structural damping

$$q_m(z) = \frac{1}{2} 1.25 \cdot 1.5 \cdot 46.2 \cdot v_m(z)^2 = 43.31 \cdot v_m(z)^2, \; \chi^2 = 0.0151$$

$$S_v(n) = \frac{1}{3} \frac{v_{m,10}^2}{n} \cdot \frac{\left(1800n / v_{m,10}\right)^2}{\left[1 + \left(1800n / v_{m,10}\right)^2\right]^{4/3}} = 30.969$$

$$\sigma_{y,direction}^2 = 3.55.10^{-5}$$

Horizontal response due to mean wind load and contribution of the fluctuating component is:

$$y_{max}(z = 76.42) = y_m + y_f = 0.01005 + 0.013685 = 0.0237 \text{ m.}$$

Maximal deflections for concrete building with rectangular plan and with regular distribution of mass 473 279 kg/m according to the EN 1991-1-4 is:

$$y_{max}(z = 76.42) = 0.048 \text{ m.}$$

4. Conclusions

Displacements on the top of buildings according two different calculations give us different values. The simplified method according to EN is conservative and based on peak pressures. The separation of turbulence intensity can get better understand of the wind effect on structures. The random vibration procedure presents full dynamical evaluation of response of sensitive height rise structures and it is significant for engineer. Results and conclusion in this height buildings show that is necessary to take into account stochastic and resonance effect of turbulence intensity. Fluctuating part of wind load can be obtained using random process simulation with different power spectral density function.

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References

Background document of ENV 1991-2-4, (1996) Wind Action, Draft.

Davenport, A.G. (1962) The response of slender, line like structures to a gusty wind, Proc. Inst. Civil. Eng., 6610, p.23/III.

EN 1991-1-4: Euro code 1: Action on structures – Part 1-4: General actions-Wind actions, 2005.

Hubová, O. & Prekop, Ľ. (2000) Wind excited response of a line slender structures, in: Proc. Int. Conf. Dynamics of Civil Engineering and transport Structures and Wind Engineering, Vyhne, pp. 41-44.

Hubová, O. (2007) The effect of the wind on the structure, Slovak Journal of Civil Engineering 2007/3, Volume XV, ISSN 1210-3896.

Hubová, O. (2010) Simplified approach for assessing building wind response. Proc. 10 th Int. Scientific conference VSU' 2010, Sofia 3-4 June 2010, ISBN 1314-071X.

Chmielewski, T & Górski, P. (1998) Along Wind Response of the TV Tower, in: Proc of 2nd East European Conference EECWE'98, Praque, pp.105- 112.

Koloušek, V. & Pirner, M. & Fischer, O. &, Náprstek, J.(1983) Wind effects on Civil Engineering Structures, Academia Praha, Elsevier, koed. London 1984.

Petersen, Ch. (1996) Dynamik der Baukonstruktionen, Braunschweig/Wiesbaden. Friedrich Vieweg & Sohn Verlag-gesellschaft mbH, ISBN 3-528-08123-6.

Pirner, M. & Fischer, O. (2003) Zatížení staveb vetrem, ČKAIT Praha.