17th International Conference
ENGINEERING MECHANICS 2011
Svratka, Czech Republic, 9 – 12 May 2011

IDENTIFICATION OF CONSTITUTIVE MODEL BLOOD VESSEL WALL FROM WATER HAMMER EXPERIMENT

D. Hromádka*, R. Žitný*, L. Horný*, H. Chlup*, J. Veselý*, J. Kronek*, E. Gultová*

Abstract: *This paper presents a water hammer experiment performed on an physical model of blood vessel. Physical model of blood vessel was manufactured from latex tubes textile rubber bands and elastic matrix. Latex tube outer surface was covered by elastic matrix. Textile rubber bands were wound on elastic matrix and they imitated reinforcing effect of collagen fibers. Mathematical model describes pulsation of pressure after instantaneous closing of valve at outlet of pipeline a part of which is a short elastic tested tube. Constitutive viscoelastic model was used to describe the behavior of blood vessel wall.*

Keywords: ***Water hammer, constitutive model, blood vessel, pressure pulsation, oscillations.***

1. Introduction

The water hammer phenomenon was studied for more than one hundred years. This phenomenon is usually explained by considering an ideal reservoir – pipe – valve system in which a steady flow of fluid is almost instantaneously stopped by a valve closure. Water hammer theory is based upon Joukovsky's, momentum and continuity equations (Ghidaoui et al., 2005) which must be solved numerically. Hybrid models are usually solved by method of characteristics for the water hammer equation and by FEM for the solid (compressible) structure. Kochupillai (Kochupillai et al., 2004) presented finite element formulation based on fluid velocity of flow. Other methods for solving water hammer equation are presented in (Ghidaoui 2005). Bessems (Bessems et al., 2008) in his paper focused on modeling the linear viscoelastic behavior of blood vessel wall during dynamic loading.

In this paper we are focused on water hammer experiment on physical model of blood vessel. Mathematical model formulation is simplified by the fact that only a small part of system boundary is flexible (tested section of elastic tube is very short comparing with a long rigid piping in experimental setup). Thus the effect of a moving pulse wave can be neglected and the whole system can be approximated by the 'windkessel' model that is by a system of ordinary differential equations. One specimen of elastic tubes was tested: a simple tube having a composite structure as a human blood vessel. And this is the primary aim: development and manufacturing of reproducible and uniform physical model of blood vessel having a structure of intima, media and adventitia layers, layers reinforced by oriented collagen fibers with a limited extensibility (tested bands have the limiting extensibility 2). The connection between layers was performed by using a silicone matrix. It is believed that the water hammer tests could be useful in further development of these physical model of blood vessels.

2. Methods

Experiments were carried out using the experimental setup shown schematically in Fig. 1. Water flows from a reservoir through a vertical pipe and elbow to the tested section closed by a valve (stop cock). Pressure at the water level in the reservoir and at the valve outlet is atmospheric, therefore the only driving force is gravity (height H). The tested elastic pipe has approximately the same initial inner radius (R=10 mm) as the connected piping and the valve, the length of tested elastic section is about

* Ing. David Hromádka, prof. Ing. Rudolf Žitný, CSc., Ing. Lukáš Horný, Ing. Hynek Chlup, Ing. Jan Veselý, Ing. Jakub Kronek and Ing. Eva Glutová: CTU in Prague; Technická 4; 160 07, Prague; CZ, e-mails: david.hromadka@fs.cvut.cz, rudolf.zitny@fs.cvut.cz, lukas.horny@fs.cvut.cz, hynek.chlup@fs.cvut.cz, jan.veselý@fs.cvut.cz, jakub.kronek@fs.cvut.cz, david.hromadka@fs.cvut.cz

10% of the connected rigid pipes (of the length *L*). Parameters of water hammer experiment are presented in Tab. 1.

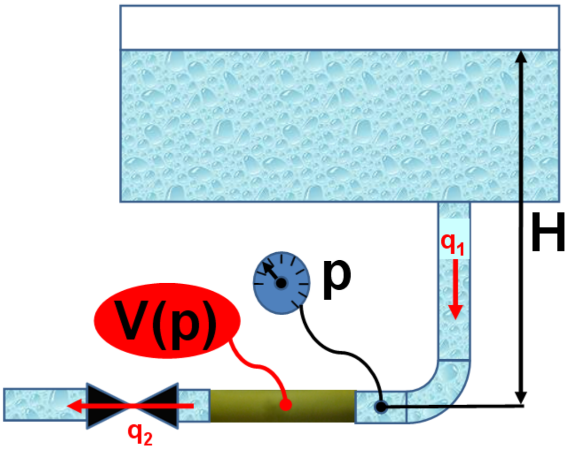

Fig. 1: Scheme of experimental setup with reservoir.

Tab. 1: Water hammer experiment parameters.

Specimen	***Value***
Inner diameter	*0.0174 mm*
Wall thickness	*0.0024 mm*
Length	*0.1450 mm*
Setup	***Value***
Inner pipe diameter	*0.02 m*
Wall thickenss of pipe	*0.0028m*
Water level	*1 m*

2.1. Model

Mathematical model is reduced only to 3 ordinary differential equations for pressure $p(t)$ in the viscoelastic pipe and the flow rate $q_1(t)$ in the rigid piping (and at inlet to the elastic section) and the flow rate $q_2(t)$ through valve. However this is not enough, the simplified model assumes either uniform or parabolic velocity profile

The final form of differential equations is continuity equation (1), balance momentum equation (2) where λ is friction factor calculated as 64/Re in laminar and as $0.316/Re^{0.25}$ in the turbulent flow regime. Coefficient ζ determines local pressure losses in elbow and at inlet from reservoir. Function f(t) in Eq. (2) is a function decreasing from 1 to 0 and describes the time course of gradual closing the valve at outlet.

$$\frac{dV}{dt} = q_1 - q_2 \tag{1}$$

$$q_2 = q_1 f(t) \tag{2}$$

$$L\rho\frac{dq_1}{dt} = \pi R^2(\rho gH + p_a - p) - \frac{\rho}{\pi R^2}\left(\frac{\lambda L}{4R} + \frac{\xi}{2}\right)q|q| \tag{3}$$

$$\frac{dp}{dt} = \frac{\partial G}{\partial V}(q_1 - q_2) + \frac{\partial G}{\partial \dot{V}}\left(\frac{dq_1}{dt} - \frac{dq_2}{dt}\right) \tag{4}$$

$$p = G(V,\dot{V}) = \frac{h_0 \eta_3}{\left(\frac{V}{V_0}\right)^{0.5} r_0 2V} \frac{dV}{dt} - \frac{V_0 h_0}{V r_0} \sum_{i=1}^{3} \mu_i \left(\left(\frac{V}{V_0}\right)^{-0.5\alpha_i} - \left(\frac{V}{V_0}\right)^{0.5\alpha_i} \right) \tag{5}$$

The Eqs. (4, 5) describe relationship between pressure, volume and rate of volume change of the viscoelastic section. The elastic part is determined experimentally from inflation tests. Ogden's constitutive equation (hyperelastic spring) combined with parallel dashpot are used for description of inflated axisymmetric membrane (artificial blood vessel). Initial conditions for system (1) (2) (3) and (4) are zero pressure (in fact overpressure), volumetric flow rate and the volume corresponding to the steady state and fully opened valve. Fully implicit numerical solution was implemented in a simple Fortran program.

2.1.1. Elastic response

The inflation test of the physical model of blood vessel revealed nonlinear pressure-volume relationship, see Fig. 3. Thus hyperelastic model has been adopted describing pure elastic contribution. Ogden's model of the strain energy density W, proposed originally for elastic response of polymer materials, fit experimental data successfully. Its mathematical form is given by equation (6). Here μ_k denote stress-like material parameters and α_k are dimensionless. Experimentally identified values from static tests are listed in Table 2.

$$W = \sum_{k=1}^{3} \frac{\mu_k}{\alpha_k} \left(\lambda_T^{\alpha_k} + \lambda_Z^{\alpha_k} + \lambda_R^{\alpha_k} - 3 \right) \tag{6}$$

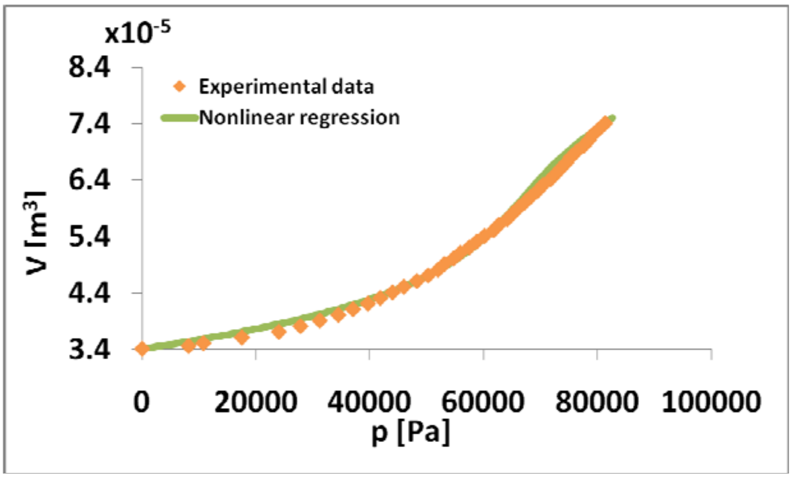

Fig. 3: Nonlinear pressure-volume relationship fitted by Ogden model.

Tab. 2: Estimated material parameters for elastic contribution.

Parameter	***Value***
μ_1	*1.219 MPa*
μ_2	*-3.355 MPa*
μ_3	*0.3449 MPa*
α_1	*-4.211*
α_2	*-1.153*
α_3	*6.028*

2.1.2. Dynamic response

Results from experiment and simulation are shown in Fig. 4 (points represent recorded pressure from water hammer experiment and the green line is numerical prediction for parameters in Tab. 2. In simulation was used constant viscosity η_3 = 3 Pa.s

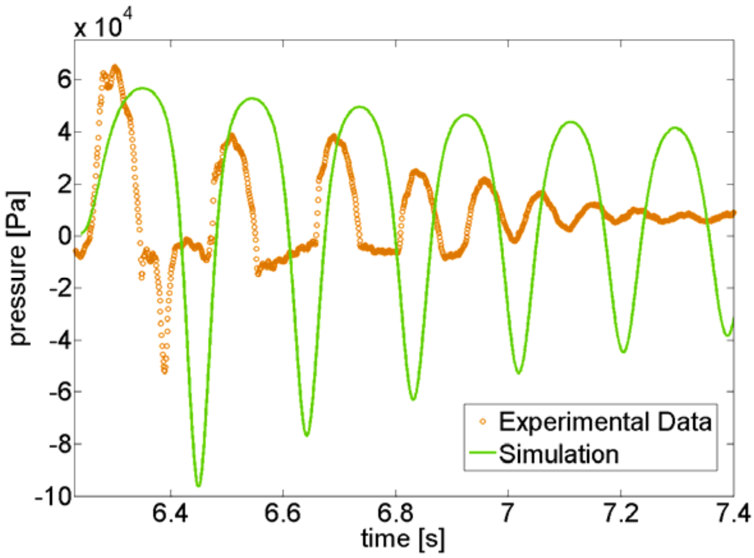

Fig. 4: Pressure responses after almost instantaneously closed valve.

3. Conclusions

Calculated natural frequency is little bit lower than the measured. Discrepancy of the response shape is caused first of all by the fact that the inflation test was performed only with inner overpressure (behavior of elastic section at negative pressure obviously cannot be extrapolated from Fig. 3). The second problem concerns the damping constant – this value was not optimized, and it seems that the single linear dashpot model is not suitable. Experiments did not confirmed the hypothesis that the initial frequency of oscillations will be higher due to increased stiffness at large deformation (effect of limited extensibility) and that the frequency will be gradually decreasing at an attenuated response tail.

Acknowledgement

This work has been supported by Institutional research plan MSM6840770012-Transdisciplinary Research in the Field of Biomedical Engineering II and by Project GA106/08/0557 - Material properties of veins and their remodelling.

References

Bessems, D. 2008 Experimental validation of a time-domain-based wave propagation model of blood flow in viscoelastic vessels. Journal of Biomechanics, 41, pp. 284–291.

Ghidaoui, S. M. 2005. A review a water hammer theory and practice. Applied mechanics reviews, Jan. 2005, vol. 58, no 1-6, p. 49-75.

Kochupillai, J. 2004 A new finite element formulation based on the velocity of flow for water hammer problems, International Journal of Pressure Vessels and Piping, 2005, vol. 82, p. 1-14.