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EEE – METHOD BASED ON FRACTAL DIMENSION FOR ANALYSIS OF TIME SERIES

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Abstract: *In our research we have developed a new analysis of topological one-dimensional objects (especially time series or dividing lines):* ***E****valuation of length changes with* ***E****limination of insignificant* ***E****xtremes. The method, useful for complex data, stems from an estimation of the fractal dimension, so it measures changes of lengths in sequential steps. The EEE method does not use a "ruler" for measurement, but the line is defined by local extremes (maxima and minima). The extremes are eliminated and the length of the function, which is linear by parts, is measured. The lengths are plotted in relation to the number of all steps and the plot is evaluated. Mathematically generated functions (e.g. based on the Hurst coefficient), time series from real production processes and dividing lines (surface profiles and surface roughness) were used for the first experiment. The results show good potential for applications in measurement in off-line evaluations of data sets and on-line monitoring and control.*

Keywords: ***Fractal dimension, time series, dividing line, Hurst coefficient.***

1. Introduction

Evaluation of signals (time series) from experiments, monitoring or production control are standard part of analyses. The choice of the analyses used for the monitoring should correspond with the character of the data obtained. The fractal dimension (describes in books e. g. Mandelbrot, 1982 and Peitgen et al., 1992) and a combination of statistical tools were used experimentally and are interesting and powerful tools for complex data quantification, for poor quality troubleshooting, production optimalization and non-stability of systems troubleshooting in laboratories and in industrial applications (e.g. paper A. Hotař et al., 2009, V. Hotař, 2008).

Our research team developed a new method for describing complexity with a modified technique of an estimated fractal dimension: **E**valuation of length changes with **E**limination of insignificant **E**xtremes (EEE). The method brinks benefits in describing data complexity and results are 2 parameters, one estimate complexity and the second is connected with statistics.

2. Methodology of EEE

The method is based on length evaluation of a curve (a time series). The curve is defined by measured values and they are isolated points - local extremes (maxima and minima). On the curve, unnecessary extremes are classified with a defined rule and a new simplified function is defined by the remaining points. The new function is used for the next classification. An example of a function defined by points and connected into the linear by parts function f is in Fig. 1. In the first step the function f is purged of points which are not local extremes (the local extremes are black points in the Fig. 1). First and last points are added to the extremes. The simplified function g is generated from such received points, Fig. 2.

A relative length of the function g, is computed and the result is saved. The relative length is evaluated from the absolute length of the function from point to point and divided by the length of its projection onto the x axis. The procedure for the elimination of insignificant extremes is applied to the simplified

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function g. The rule of the procedure uses functions formed from maxima and minima of the function g, Fig. 3, the functions g_{max} and g_{min} extended with the first and last points of the function g. The function g_{max} is generated from the maxima of the function g and in this function local maxima are found (black dots in Fig. 3). The function g_{min} is generated from minima and the local minima are found.

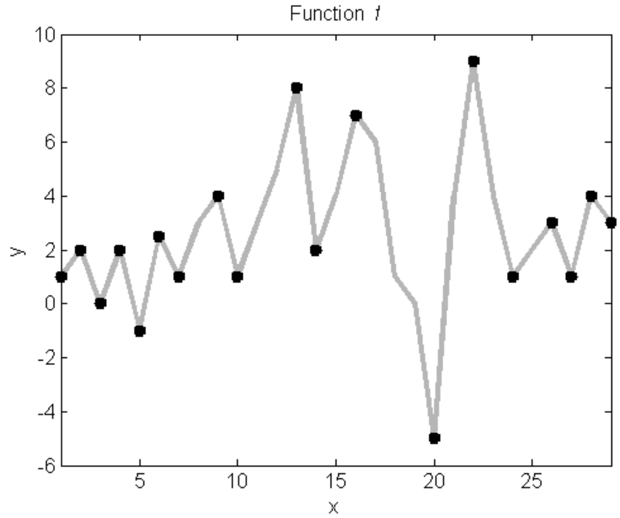


Fig. 1: Function **f** *and its local extremes.*

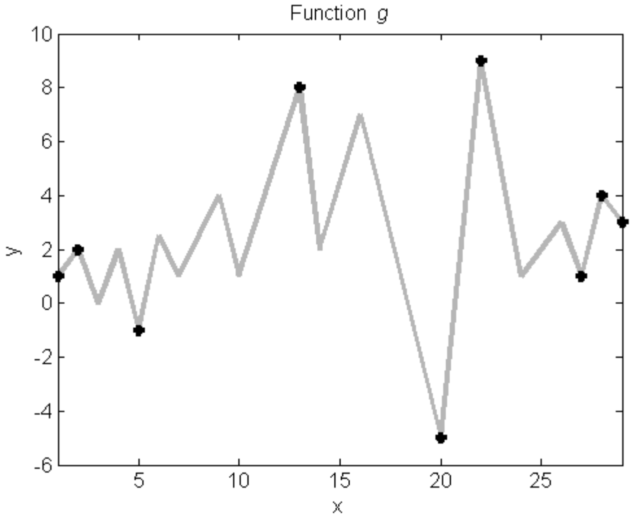


Fig. 2: Simplified function **g** *and its local extremes.*

The local maxima and minima of the functions g_{max} and g_{min} are used for the generation of the function g_{red} , Fig. 4. In this function again local maxima and minima are defined (Fig. 4, black dots). These final local extremes of the function g (Fig. 2, black dots) and the first and last points from the function h, Fig. 5. The relative length of function h is computed and the result is saved.

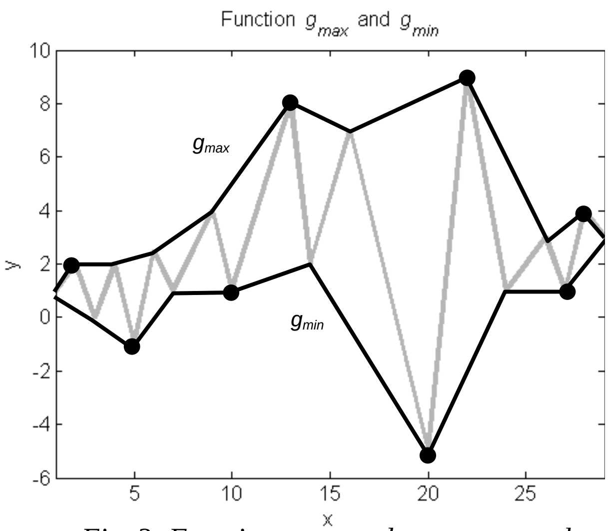


Fig. 3: Functions $\mathbf{g_{max}}$ *and* $\mathbf{g_{min}}$ *generated from local extremes of function* **g**.

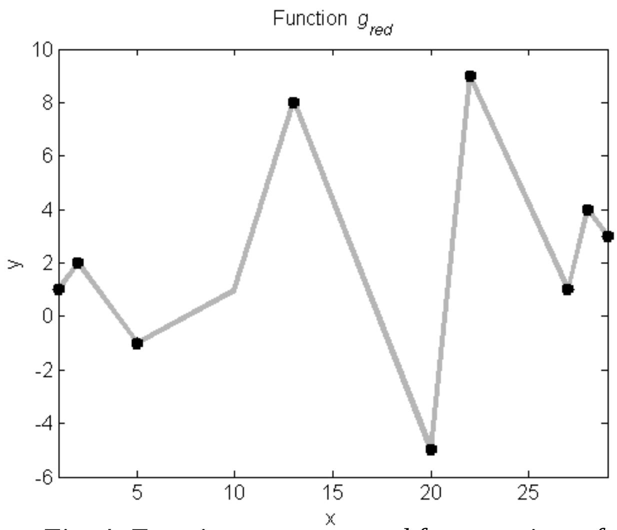
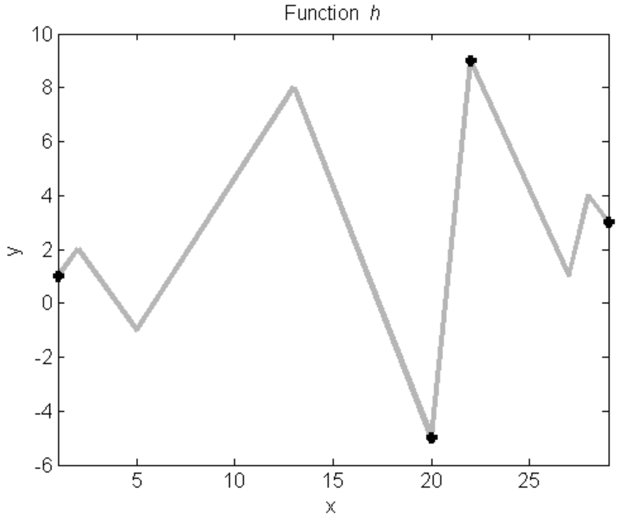


Fig. 4: Function $\mathbf{g_{red}}$ *generated from maxima of function* $\mathbf{g_{max}}$ *and from minima of function* $\mathbf{g_{min}}$.

The same procedure is used for the new simplified function h and its global extremes and the first and last point (Fig. 5, black dots) define the function k. The function is formed from the global maximum and minimum of all functions (f, g, h), therefore the analysis is stopped. All functions are depicted in Fig. 6.

The steps of the analysis are plotted versus the computed relative lengths of functions g, h, k, Fig. 7. The relation between the relative lengths and the steps of elimination are evaluated by a sufficient regression function that can be: a regression line (Fig. 7), a quadratic function or a hyperbolical function. The parameters of the regression functions are used for the evaluation of the function f.



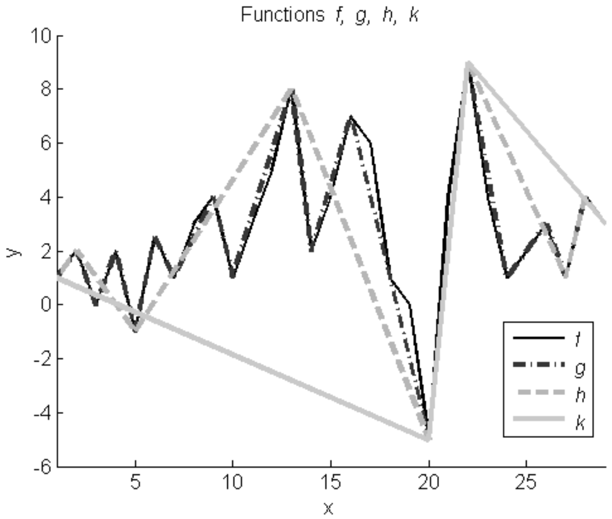
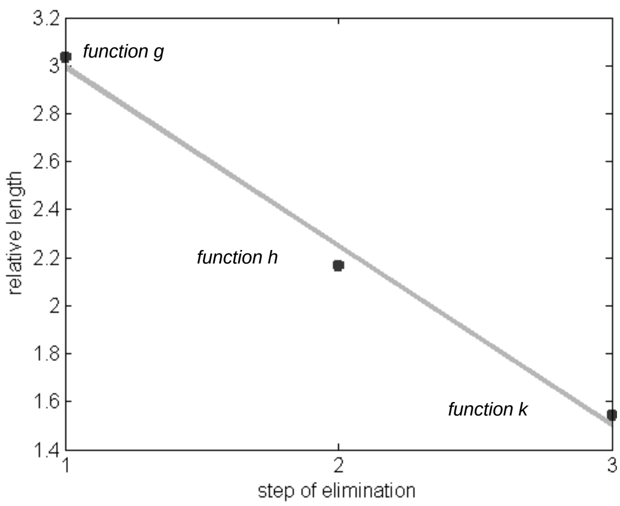


Fig. 5: Simplified function **h** *and its local extremes.*

Fig. 6: Function f and simplified functions **g, h, k**.

3. Experimentation with mathematically generated functions

Time series obtained from simulation of fractional Brownian motion using Cholesky-Factorization of the related covariance matrix (FBM) were used to test of the developed method. An example of testing time series is in Fig. 8 and is generated using the input Hurst coefficient H = 0.4. The coefficient represents the character of time series and can be between value 0 and 1 (higher coefficients generate smoother functions, more information can be found in Evertsz 1996, Peitgen et al., 1992). The dependence between the relative lengths and the steps of elimination is in Fig. 9.



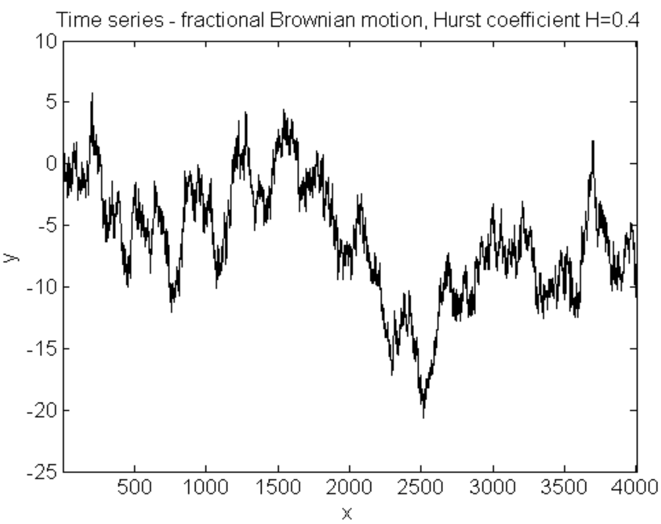


Fig. 7: Plot of the relation between steps of analysis and relative length, regression line.

Fig. 8: Simulation of time series using fractional Brownian motion.

The relation between the steps and the lengths is hyperbolic:

$$y = \frac{d}{x+a} + b \quad . \tag{1}$$

It can be evaluated by parameters d and a. The parameter b is always b = 1. The parameter a has to be computed numerically using an error function.

900 simulated time series from FBM were used for an evaluation of the method EEE with a Hurst coefficient between 0.1 and 0.95. Fig. 10 shows the dependence between Hurst coefficients and the average value of parameters a and d.

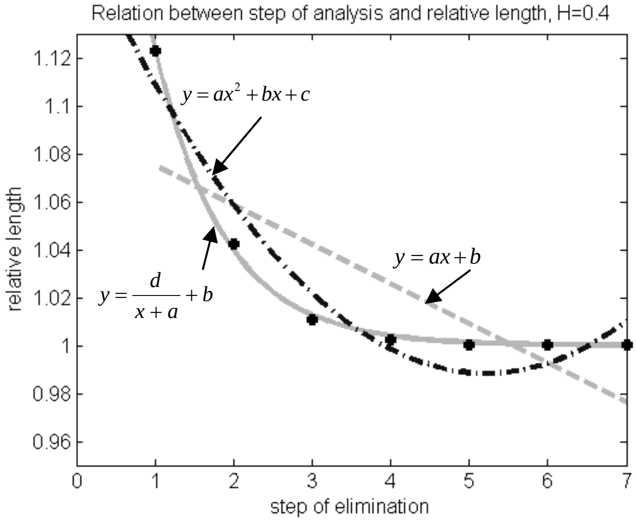


Fig. 9: Relation between step of analysis and relative length for time series FBM, H = 0.4.

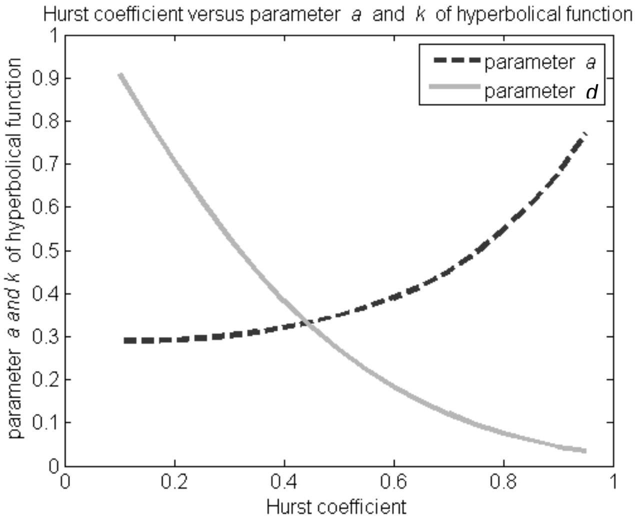


Fig. 10: Hurst coefficient versus average value of parameters **a** *and* **d** *of hyperbolic function.*

4. Conclusion

The method EEE describes the character of curves and can be used to classify their complexity. Currently, tests with real topological one-dimensional curves are being conducted. Time series from measurement, real production processes and the dividing line between two materials are being used. The results of the tests will be compared with parameters of statistical tools, fractal dimensions and spectral analysis. Although some specific problems in higher frequencies of record characteristic have to be solving, the first results have been promising.

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