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NON-LINEAR TIME HETERONYMOUS DAMPING IN NON-LINEAR PARAMETRIC PLANETARY SYSTEMS

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Abstract: *The analysis of dynamics of time heteronymous weakly and strongly non-linear planetary transmission systems with kinematic couplings – gears constitutes a study of complicated function of many parameters including the gearing material damping properties and damping viscous properties of lubricating oil film in gear mesh. The structures with lightening holes in cog wheel discs are characterized then by time variable damping in gear meshes. The following contribution deals with this time heteronymous non-linear damping.*

Keywords: ***Non-linear dynamics, time variable damping, planetary systems.***

1. Introduction

The presenting study concurs the works (Hortel & Škuderová, 2010; 2011) that deals with matters of linear and non-linear damping of motions in gear mesh of kinematic pairs with constant damping coefficients and with time variable one.

The world development of transmission systems especially in mobile machines leads to high-power high-speed constructions with minimum dimensions and masses. Deep basic research in the area of internal dynamics of weakly and strongly non-linear time heteronymous deterministic systems is focused to the area of complicated planetary, i.e. differential, and pseudoplanetary systems with split power flows. The research here is motivated primarily by whole series of pending issues in the area of dynamic phenomena that influence and closely relate to the safety and dependability of such systems. The issue and existence of these phenomena in many cases of such complicated systems is not still wholly known in term of both theoretical and experimental.

By light high-speed drive systems occur often in discs of the cog wheels the lightening holes for example by the speed reducers of turbines onto propellers, which cause the speed heteronomy in the damping properties of cog wheel discs. The lightening disc holes as well as the forms of discs can have generally according to the functional requirements the different forms. Consequently the *radial* stiffnesses – masses and the damping characteristics that result of these properties, can have the complicated functional relations that depend on the revolutions of appropriate wheels.

2. Mathematical – physical model and analysis of influence of time variable damping by lightening holes in cog wheels

The time variance of disc stiffness – mass depends on the size, number and location of lightening holes along the wheel circumference. It follows from above mentioned, that a real damping in a gear mesh is a complicated time variable i.e. heteronymous function which can influence not only by its quantitatively but mainly by qualitatively properties for example the phase shifts of the amplitude of relative motion in the gear mesh towards the amplitude of the modified resulting stiffness function of gearing and can influence that way the dynamics i.e. the dynamic forces in these systems.

Such time damping function of a disc $k_d(t) = k_d(t+T)$ of a cog wheel with the period $T = 2\pi/\omega$ in the interval $\langle 0;T \rangle$, which fulfils the conditions of an uniqueness and finiteness, with a finite number

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of maximums and minimums and discretenesses (Dirichlet's conditions) can be expressed into a convergent series of the form

$$k_d(t) = k_d(t+T) = \frac{1}{2T}\int_0^{2T} k_d(t)dt + \sum_{m=1}^{\infty}\left\{\left[\frac{2}{T}\int_0^T k_d(t)\cos p\, m\omega t\, dt\right]\cos p\, m\omega t\, dt\right.$$

$$\left. + \left[\frac{2}{T}\int_0^T k_d(t)\sin p\, m\omega t\, dt\right]\sin p\, m\omega t\, dt\right\}, \qquad (1)$$

where $\omega = \pi n/30$ is the angular velocity of cog wheel, n ... the number of the cog wheel revolutions, p ... *the number of the lightening holes* in the gear disc or generally the number of the periods of the courses of the damping who are caused by the inhomogeneity of the wheel disc

By reason that are not yet experimentally developed the functional damping relation will be applied in the following studies, in the "zero" approximation, instead of (2) the simplified function

$$k_d(t) = k_{d0} + k_d \sin p\omega\, t, \qquad (2)$$

where $p = 1,2,3,\ldots$ and k_{d0}, k_d are the constant damping coefficients.

If we introduce the time heteronymous function (2) into the system of the motion equations of the substitutive mathematical – physical model of one branch of power flow of the pseudoplanetary system from Fig.2 (Hortel & Škuderová, 2010) i.e. the sun gear 2 and one satellite 3, we obtain the system of the ordinary differential equations with time variable coefficients in the form (Hortel & Škuderová, 2010)

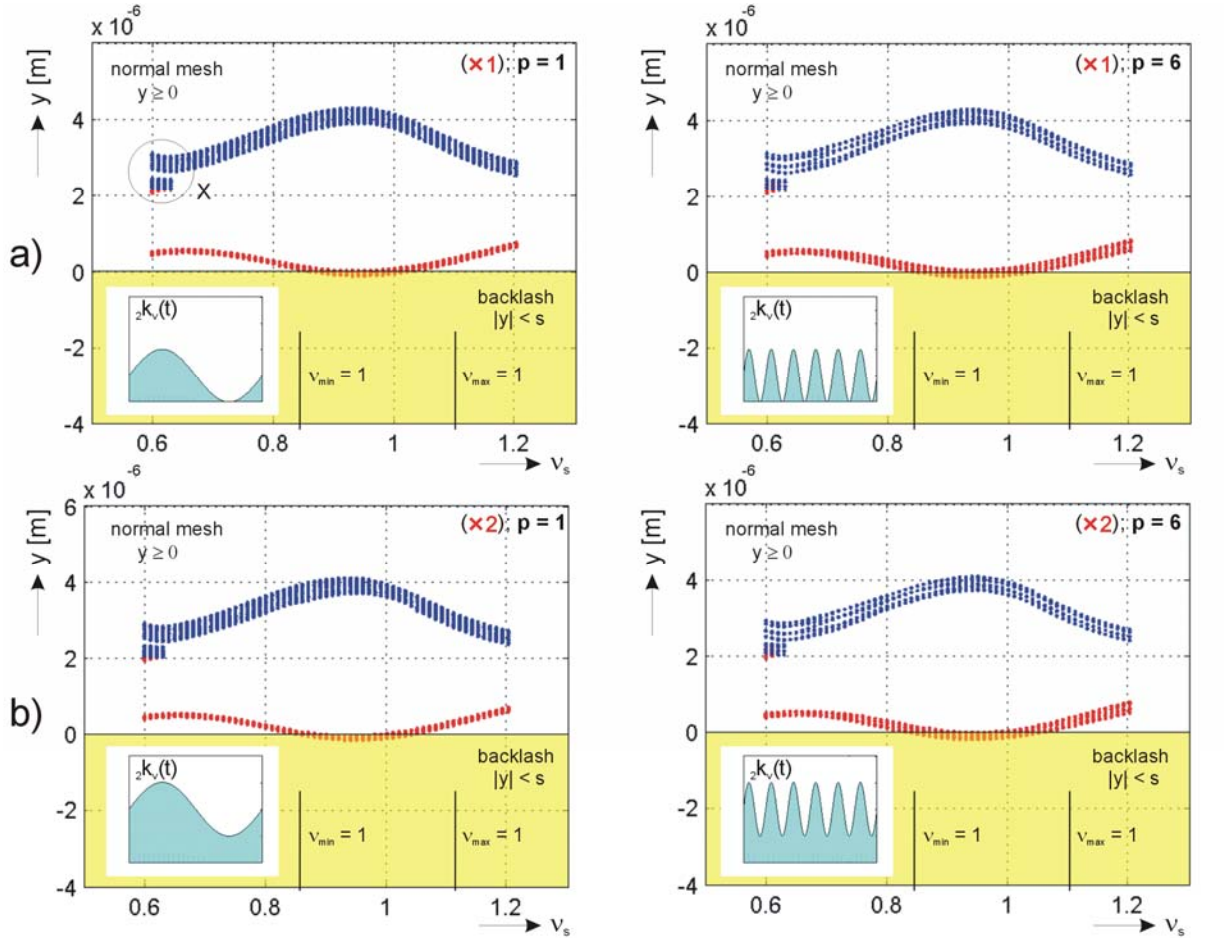
$$\boldsymbol{M}\boldsymbol{v}'' + \left[{}_1\boldsymbol{K}(\beta,\delta_i,H) + {}_1\boldsymbol{k}_d(t)\right]\boldsymbol{v}' + \sum_{K_1>1}\left[{}_{K_1}\boldsymbol{K}(D,D_i,H) + {}_{K_1}\boldsymbol{k}_d(t)\right]\left|\boldsymbol{w}'(\boldsymbol{v}')\right|^{K_1}\operatorname{sgn}(\boldsymbol{w}'(\boldsymbol{v}'))$$

$$+ {}_1\boldsymbol{C}(\varepsilon,\kappa,Y_n,U_n,V_n,H,t)\boldsymbol{v} + \sum_{K>1}{}_K\boldsymbol{C}(\varepsilon,\kappa,I_n,H,t)\boldsymbol{w}^K(\boldsymbol{v}) = \boldsymbol{F}(a_n,b_n,\overline{\varphi},H,t), \qquad (3)$$

where $\boldsymbol{v}$ means here generally the 6-dimensional vector of displacement of system vibration, $\boldsymbol{w}^K(\boldsymbol{v})$ is the K-th power of vector $\boldsymbol{v}$, which is defined by expression $\boldsymbol{w}^K(\boldsymbol{v}) = \boldsymbol{D}(\boldsymbol{w}(\boldsymbol{v})\boldsymbol{w}^{K-1}(\boldsymbol{v}))$. $\boldsymbol{D}(\boldsymbol{w}(\boldsymbol{v}))$ means the diagonal matrix, whose elements at the main diagonal are comprised by elements of vector $\boldsymbol{w}(\boldsymbol{v}) \equiv \boldsymbol{v}$. Furthermore $\boldsymbol{M}$ is the matrix of mass and inertia forces, ${}_1\boldsymbol{K}, {}_1\boldsymbol{k}_d(t)$ and ${}_{K_1}\boldsymbol{K}, {}_{K_1}\boldsymbol{k}_d(t)$ are the matrices of linear and non-linear constant and time variable damping forces, ${}_1\boldsymbol{C}$ and ${}_K\boldsymbol{C}$ are the matrices of linear and non-linear reversible forces and $\boldsymbol{F}(t)$ is the vector of non-potential external excitation with components a_n, b_n and the phase angle $\overline{\varphi}$. H is the Heaviside´s function, which allows to describe the motions – contact bounces – due to strongly non-analytical non-linearities, for example due to technological tooth backlash $s(t)$. Corresponding *constant* linear and non-linear coefficients of damping are denoted by β, δ_i or D, D_i, whereas $\beta(k_1, k_{1m}, H)$, where k_1, k_{1m} are the linear coefficients of material of viscous damping in gear mesh*, $D(k_{2,3}, k_{2,3m}, H)$, where $k_{2,3}, k_{2,3m}$ are the non-linear (2 – quadratic, 3 - cubic) coefficients of the material or viscous damping in the gear mesh. The linear parametric stiffness functions are denoted by the symbols Y_n, U_n, V_n and non-linear parametric functions, so-called parametric non-linearities, by the symbol I_n. ε and κ are the coefficients of the mesh duration and the amplitude modulation of the stiffness function ${}_1\boldsymbol{C}(t)$, t is the time.

The resonance characteristics of linear as well as non-linear systems with constant coefficients with the typical overhangs of the solidifying or softening resonance characteristics are markedly elaborated. This can not be fully declared about the resonance characteristics of linear or non-linear parametric systems with impact effects, where all the phenomena of linear or non-linear systems including the influence of the time variable damping ${}_1k_d(t)$, ${}_{K_1}k_d(t)$ and the changes of the stiffness level – the parametric functions – and the phase shifts of the amplitude of relative motions $y(t)$ in gear mesh in

*) The subscript m is for the tooth backlash.



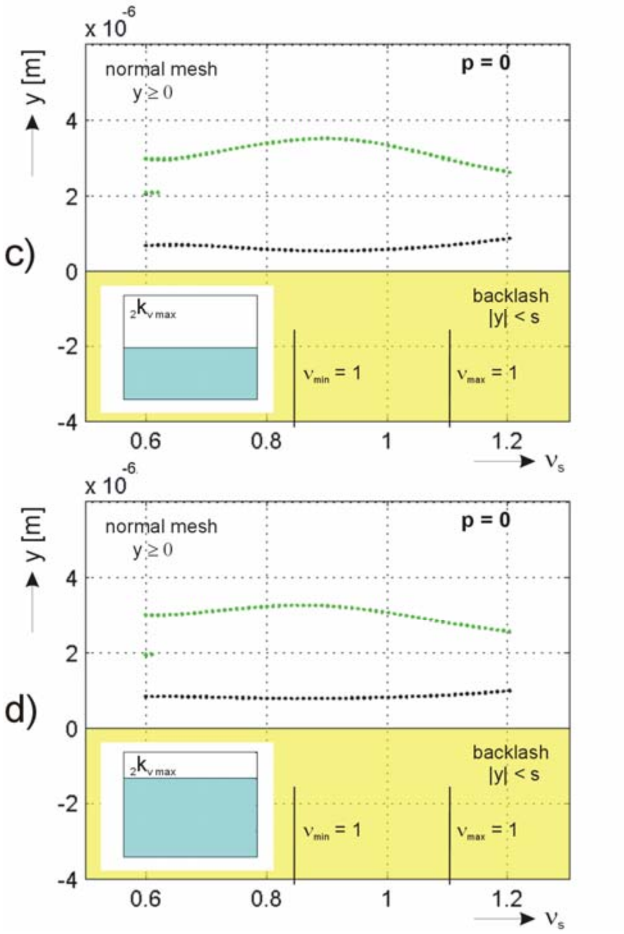


Fig. 1:

The resonance bifurcation characteristics of the internal dynamics of gear pair of non-linear heteronymous system in the resonance range $\nu_s \in \langle 0.6;1.2 \rangle$ *for the constant and with time variable quadratic damping in the gear mesh by the full – unlightening discs (p = 0) and by the lightening wheels with one (p = 1) and six holes (p = 6) for the variable damping* $_2k_v(t)$ *according to eq. (4) with the damping coefficient variations (×1):* $_2k = _2k_d = _2k_m \neq 0\,;\, _2k_{d0} = 0$ *- Fig. a) ; (×2):* $_2k = _2k_{d0} = _2k_d = _2k_m \neq 0$ *- Fig. b). The characteristics in Fig. c) and d) are given for constant quadratic damping with the coefficient of damping* $_2k_{v\max}$ *for variations (×1) and (×2). The bifurcation deviation in the narrow frequency range of* ν_s *in detail "X"- Fig. a) is caused by disperion of closed internal loop-shaped courses in the phase planes {y';y}.*

consequence of action of damping forces towards the parametric excited function $C(t)$.

With regard to limited paper size we illustrate further in brief only the influence of two variations of non-linear – quadratic with time variable damping on internal dynamics of solid system (3). For the whole field of gear mesh, i.e. normal, phase of contact bounces – impact of tooth faces and inverse mesh then results from (2) and (3) the general term for damping

$$ {}_2k_v(t) = \left[{}_2k + {}_2k_{d0} + {}_2k_d \sin p\omega t\right](H1 + H2) + {}_2k_m\left[1 - (H1 + H2)\right], \qquad (4) $$

where $H1 = H(y)$ is the Heaviside's function for the normal gear mesh $y \geq 0$, $H2 = H(-y - s)$ is the Heaviside's function for inverse mesh $|y| > s$.

The values for variation quadratic damping are

$$ (\times 1): {}_2k = {}_2k_d = {}_2k_m \neq 0;\ {}_2k_{d0} = 0, (\times 2): {}_2k = {}_2k_{d0} = {}_2k_d = {}_2k_m \neq 0. $$

These variations ($\times 1$), ($\times 2$) will be theoretically compared for the number of lightening holes of wheel discs $p = 1$ and $p = 6$. The quantitative parameter values of analysed system with six degrese of freedom are presented in (Hortel & Škuderová, 2010). By reason that the non-conservative system with constant damping leads for all the given damping variations on the steady or quasi-steady vibration already at the fifth revolution of gear set, for a comparison the all amplitude-frequency characteristics of system with time variable damping will be displayed for these fifth revolution.

The purpose of this study is to educe the methodology that make possible to analyse the influence of all parameter variations in the area of linear, non-linear, (see (3)) with constant and time variable damping on internal dynamics of here studied complicated system.

In Fig. 1 are given resonance bifurcation characteristics in coordinates $\{\nu_s; y\}$ for the solution variations a) ($\times 1$), b) ($\times 2$) for the number of lightening holes in cog wheels $p = 1$ and $p = 6$ with time variable quadratic damping ${}_2k_v(t)$, where ν_s is frequency relative to the mean value of the resulting stiffness function $C(t)$ in gear mesh. The figures c), d) represent the resonance bifurcation characteristics for the full – unlightening $(p = 0)$ discs of cog wheels with constant quadratic damping ${}_2k_v(t) = {}_2k_{v\max} = konst$, where ${}_2k_{v\max}$ are the maximum values of corresponding damping courses of variationes a), b). The time courses of damping are marked in the individual figures

3. Conclusion

The influences of all possible variations and combinations of linear and non-linear constant and the time variable damping forces will be subjected to deeper qualitative and quantitative analysis in the next studies so that the summary damping characteristics obtained by experimental methods could be simulated and approximated by means of theoretically modified damping characteristics of solved systems.

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