17th International Conference
ENGINEERING MECHANICS 2011
Svratka, Czech Republic, 9 – 12 May 2011

PROBABILITY OF DESTRUCTIONS OF CERAMICS USING WEIBULL'S THEORY

V. Fuis[*], M. Málek[**], P. Janíček[**]

Abstract: *The probability calculation using two variants of Weibull weakest-link theory is presented in this article. The first varian assumed only one tensile stress (the first principal stress σ_I) and the second variant assumed all three principal stresses in the ceramic component.*

Keywords: ***Computational modeling, ceramic head, in vivo destructions, hip joint endoprosthesis, Weibull theory.***

1. Introduction

Fracturing of metal materials in a fragile condition is assessed by fragile fracture based on the condition of fragile strength. It states that the limit condition of fragile strength occurs when reduced stress, corresponding to the condition at hand, equals the fragile strength limit. Such concept can be perceived as the first level of assessment of its fragile fracture for the ceramics. Higher levels of assessment involve taking into account non-homogeneity of ceramic structure in the sense that it contains numerous micro-failures (pores, cavities and cracks) that increase susceptibility to fragile fracturing. For assessment of fracturing of ceramics cohesion we must use probability approach respecting stochastic division of micro-fractures in the ceramics body volume. This approach describes origination of the limit condition of fragile fracture the so-called „probability of destruction“. This is a statistic approach to destruction of ceramics. There are various statistic reliability approaches. One is the Weibull's weakest-link theory (Weibull, 1939), based on a very simplified assumption that a reliability-assessed ceramics body is perceived as a system consisting of many elements (Andreasen, 1994), (Bush, 1993). If in any elements of the body there originates a stress that causes, under existing physical properties of the ceramics, uncontrolled spreading of a fragile fracture in the element, it usually results in a fragile destruction to the entire body. The issue of ceramics body reliability in view of destruction of its integrity was thus converted to the determination of probability of destruction P_f regarding individual elements of the body. Due to the accidental distribution of defects in the volume of bodies made of ceramic materials, physical properties of individual micro-volumes of these materials differ and thus various micro-volumes have different real fragile strength. It is being proved that statistical distribution of the probability of ceramic materials destructions is of Weibull's type. More micro-volumes in the body increase probability that the body contains a „weak link“, in which the fragile destruction is initiated. Probability of failure of ceramic bodies in the form of fragile destruction is the function of all micro-volumes in the body with various stresses. From the aforementioned facts, W. Weibull (Weibull, 1939) deduced mathematical formulae governing the probability of destruction in ceramic bodies for various „stress levels“ originating in bodies as a result of external load. In the simpler model, he considered only 1-axis tensile stress, on the higher level he worked with real 3-axis stress.

[*] Ing. Vladimír Fuis, Ph.D.: Centre of Mechatronics – Institute of Thermomechanics and Faculty of Mechanical Engineering, Brno University of Technology, Technicka 2, 616 69, Brno, CZ, e-mail: fuis@fme.vutbr.cz

[**] Ing. Michal Málek and prof. Ing. Přemysl Janíček, DrSc.: Institute of Solid Mechanics, Mechatronics and Biomechanics, Faculty of Mechanical Engineering, Brno University of Technology, Technicka 2, 616 69 Brno, CZ, e-mail: ymalek04@stud.fme.vutbr.cz, janicek@fme.vutbr.cz

2. Weibull's model of destruction probability for a 1-axis tensile stress

This model has been elaborated for two modifications of Weibull's theory: three-parametric (contains three ceramics material parameters) and two-parametric (with two parameters). The original Weibull's formulae for destruction probability P_f were deduced in integral shape (Bush, 1993). Since stress, one of the input quantities in the Weibull's theory, is determined using the finite elements method, Weibull's formulae are quoted in the differential shape here. Three-parametric Weibull's formula is as follows:

$$P_f = 1 - e^{-\sum_{i=1}^{n}\left(\frac{\sigma_i - \sigma_u}{\sigma_0}\right)^m \Delta V_i}, \sigma_i > \sigma_u \quad (1)$$

where n number of elements in the body, σ_I first principal stress in the i-th body element, ΔV_i volume of the i-th body element, σ_u is stress [MPa], under which material is not disrupted, σ_0 is normalized material strength [MPa.m$^{3/m}$] of the material volume unit, m Weibull's modulus.

Quantities σ_u, σ_0, m can be considered material properties of ceramics. For two-parametric Weibull's approach, $\sigma_u = 0$, which means that material destruction is possible for any tensile (positive) values of the first principal stress (Bush, 1993). Two-parametric analysis is used more frequently, because it provides more conservative results than the three-parametric analysis. Note to the Weibull's modulus m: The modulus expresses the measure of dispersion of the ceramic material strength. It is defined as a slope of the regression line in Fig. 2 for real 3-point bending experiments (h = b = 2.5 mm, L_1 = 10 mm, L_2 = 0 mm) (Fuis, 2007):

$$\ln(\ln\frac{1}{1-P_f}) \quad \text{versus} \quad \ln(MOR), \quad \text{where} \quad MOR = \frac{3(L_1 - L_2)F}{2h^2 b}. \quad (2)$$

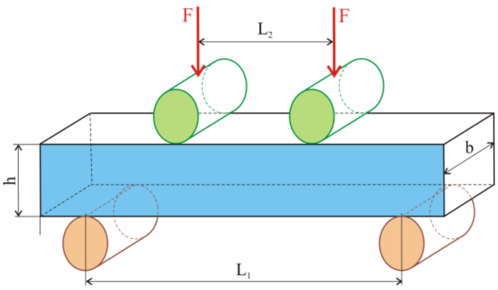

Fig. 1: Modulus of rupture test dimensions.

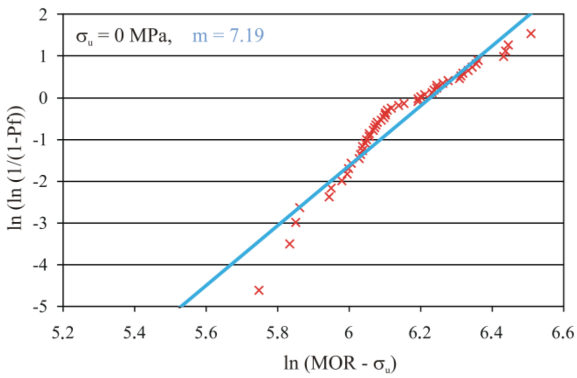

Fig. 2: Weibull plot of the normalized MOR data.

The MOR (Modulus of Rupture) quantity is the maximum flexural stress under which destruction occurs in four-point bending (Fig. 1). Measuring to determine m is realized on many samples (at least 35). Method of P_f probability determination is shown in (Bush, 1993).

3. Weibull's model of destruction probability for a 3-axis general stress

Apart from the aforementioned two and three-parametric analysis of ceramics destruction probability for 1-axis stress, Weibull also designed a modification of destruction probability valid for general spatial (three-axis) stress. He proceeded from the hypothesis that in spherical pores in ceramics, maximum stress around the pore is independent of its size (Andreasen, 1994). He also assumed that destruction of ceramics is caused by a combination of normal stress σ_n (impacting fractures perpendicularly and resulting in the 1st fracture mode) and maximum shear stress τ (acting in the fracture plane and resulting in 2nd fracture mode). Both these stresses are the functions of principal stresses σ_1, σ_2, σ_3. This is standard approach for many physical conditions.

In the final version proposed by Weibull to analyze ceramics destruction probability, he neglected the impact of shear stress on the destruction (Weibull, 1939). Yet the aforementioned method is used frequently. He proposed the following formula to respect the impact of general spatial stress on the probability of destruction:

$$P_f = 1 - e^{-\int_V \left\{ \frac{2m+1}{2\pi\sigma_0^m} \int_0^{2\pi} \int_0^{\pi/2} \left[\cos^2\Phi(\sigma_1\cos^2\Psi + \sigma_2\sin^2\Psi) + \sigma_3\sin^2\Phi \right]^m \cos\Phi \, d\Phi \, d\Psi \right\} dV} \quad (3)$$

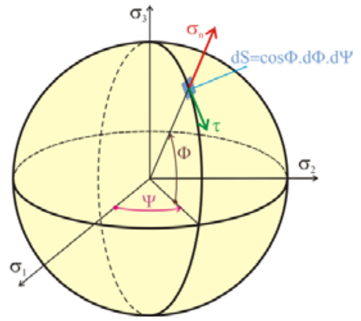

Fig. 3: Unit radius sphere representing all possible flaw/crack orientations.

4. Comparative analyses of Weibull's model for 1-axis and 3-axis stress

Comparative analyses were carried out for the following material characteristics: Weibull's modulus m = 7.19 (Fig. 2), normalized material strength of material volume unit σ_0 = 473,8 [MPa.m$^{3/7.19}$] (Fuis, 2007). Determination of destruction probability according to Weibull's formulae (1) or (3) always requires integration via a corresponding body. For Weibull's model for 3-axis stress, it is integration in the relation (3), which is time-consuming with the current algorithms. To simplify the testing we chose a 10×10×10 mm cube-shaped model body (Figs. 4 – 6 – (Málek, 2010)).

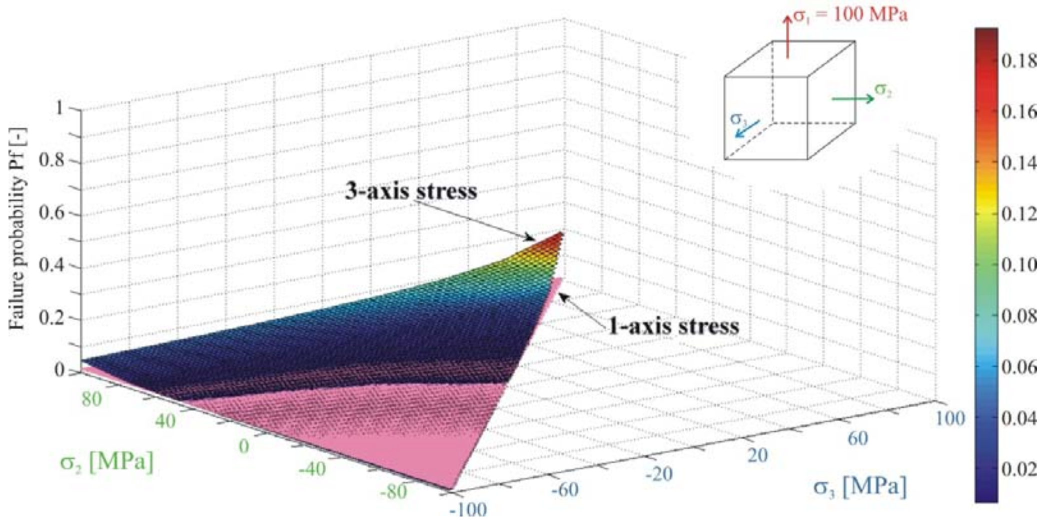

Fig. 4: Surfaces of failure probability in Weibull's model for 3-axis stress (σ_1 = 100 MPa).

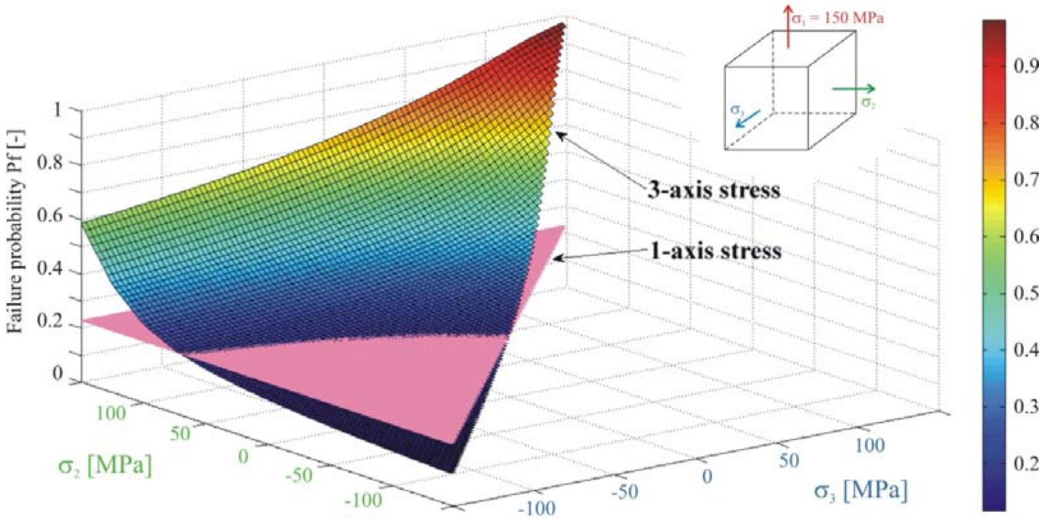

Fig. 5: Surfaces of failure probability in Weibull's model for 3-axis stress (σ_1 = 150 MPa).

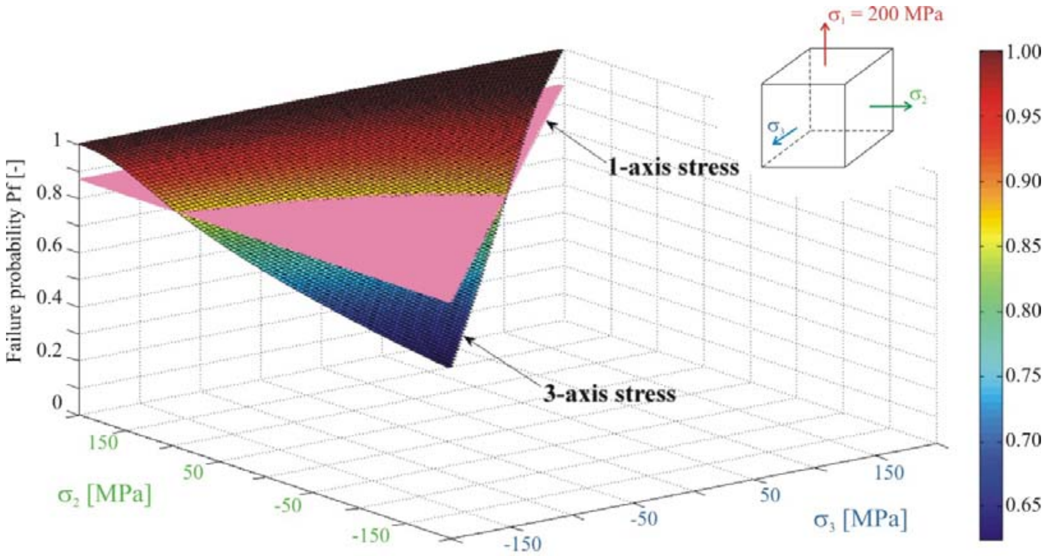

Fig. 6: Surfaces of failure probability in Weibull's model for 3-axis stress (σ_1 = 200 MPa).

We analyzed the following three stress variants (with different first principal stresses (Málek, 2010)):

- the first principal stress σ_1 = 100 MPa, another principal stresses σ_2 and σ_3 are taken the value (– 100 MPa to + 100 MPa) – Fig. 4,
- the first principal stress σ_1 = 150 MPa, another principal stresses σ_2 and σ_3 are taken the value (– 150 MPa to + 150 MPa) – Fig. 5,
- the first principal stress σ_1 = 200 MPa, another principal stresses σ_2 and σ_3 are taken the value (– 200 MPa to + 200 MPa) – Fig. 6.

This covered all types of stresses: 3-axis general ($\sigma_1 \neq \sigma_2 \neq \sigma_3 \neq 0$), 3-axis semi-even ($\sigma_1 = \sigma_2 \neq \sigma_3 \neq 0$), 3-axis even tensile ($\sigma_1 = \sigma_2 = \sigma_3 \neq 0$), 2-axis general ($\sigma_1 \neq \sigma_2 \neq 0$, $\sigma_3 = 0$), 2-axis even tensile ($\sigma_1 = \sigma_2 \neq 0$, $\sigma_3 = 0$), shear ($\sigma_1 \neq 0$, $\sigma_3 = -\sigma_1$, $\sigma_2 = 0$) that were homogenous in the entire cube.

Owing to identical volumes and homogenous stress in all cube elements it sufficed to determine destruction probability in a single element and its multiplication by the total number of cube elements gave us the destruction probability for the entire cube.

Dependences of destruction probability of the model cube on stresses σ_1, σ_2, σ_3 for 1st to 3rd version (i.e. for Weibull's 3-axis stress model) are shown in Figs. 4 - 6. These graphs also show dependence of model cube destruction probability for the two-parameter model for 1-axis stress, i.e. for the first principal tensile stress (Eq. (1) with $\sigma_u = 0$). Figs. 4 - 6 reveal that the second and third stresses substantially affect destruction probability in three-axis stress when all three principal stresses are tensile (positive).

Acknowledgement

This paper was prepared with the support of the project AV0Z20760514.

References

Andreasen, J.H. (1994) Reliability-based design of ceramics. Materials and Design. Vol. 15, Nr. 1, p. 3-13.

Bush, D.R. (1993) Designing ceramic components for structural applications. Journal of Materials Engineering and Performance. Vol. 2 (6), 1993, p. 851-862.

Fuis V, Navrat T, Hlavon P, et al. (2007) Reliability of the Ceramic Head of the Total Hip Joint Endoprosthesis Using Weibull's Weakest-link Theory. IFMBE Proc. Vol. 14, Congress on Medical Physics and Biomedical Engineering, Seoul, South Korea, 2006, pp 2941-2944.

Málek, M. (2010) Reliability analysis of the hip point endoprosthesis ceramic head. Diploma work, Brno University of Technology, Faculty of Mechanical Engineering, 2010, 75 pages, (in Czech).

Weibull, W. (1939) A statistical theory of the strength of material. R. Swedish Acad. Eng. Sci. Proc., No. 151, p. 1-45.