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MODELING OF FATIGUE CRACK GROWTH IN CERAMICS UNDER COMPRESSIVE CYCLING

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Abstract: *This contribution presents a simple numerical model for crack growth under mode-I compressive fatigue loading. Residual tensile stress at the crack tip, which is responsible for crack advance, is simulated by the plastic-like behavior in compression. Tensile part of the constitutive law accommodates a phenomenological hysteresis behavior. This constitutive law is implemented into an implicit discrete 2D numerical model. The model is used to simulate the compression-compression fatigue fracturing of ceramics, particularly polycrystalline alumina. It is shown that the model is capable of capturing trends observed in experimentally obtained crack growth behavior.*

Keywords: ***Compression-compression fatigue, discrete model, hysteresis, crack arrest.***

1. Introduction

Fatigue crack propagation, which determines the durability of structural elements, is of importance in most civil and mechanical engineering applications. The fatigue crack can propagate under periodic loads that are much lower than monotonic strengths. A simple and useful predictive tool was given in Paris & Erdogan (1963), who proposed a power law equation relating the crack growth rate ($\mathrm{d}a/\mathrm{d}N$) to the stress intensity factor difference (ΔK). So called Paris-Erdogan law, $\mathrm{d}a/\mathrm{d}N = C\Delta K^m$, has two parameters, C and m, which need to be found by matching experimental results.

Numerical modeling is often used in regimes beyond validity of the analytical formula (e.g. variable ΔK or short cracks). Key feature of fatigue numerical model is hysteresis, which has been mostly prescribed phenomenologically, i.e. directly in constitutive law (for instance Desmorat et al., 2006; Nguyen et al., 2001; Roe & Siegmund, 2003). Recently, the phenomenology of hysteresis has been overcome by meso-level model – no hysteresis at the level of constitutive models resulted in combination with a material mesostructure to hysteretic loops in overall specimen response (Grassl & Rempling, 2008). Though such simulations provide useful insights into mechanisms leading to loading-unloading hysteresis, they are computationally extremely demanding.

This study focuses on one particular case, where Paris-Erdogan law is not directly applicable, i.e. the compression-compression mode-I fatigue. Traditionally, an attention has been paid to metallic materials (e.g. Fleck et al., 1985; Reid et al., 1979) but many experiments were done also on ceramics (Ewart & Suresh, 1986; Subhash et al., 1999). The crack propagates, because residual tensile stresses develop at the notch tip at the end of each loading cycle. The local and global stress intensity factors differ from each other. What is important for the crack propagation is the local one. This local stress intensity factor evolves during crack propagation and it might be hard to estimate it and use it with the Paris-Erdogan law. Instead of making analytical attempts, we developed a simple numerical model, which accommodates both (i) tensile damage-like hysteretic behavior to describe damage accumulation due to cycling according to Nguyen et al. (2001) and (ii) compressive plastic law, which produces residual tensile stresses. The model is applied to simulation of crack propagation in polycrystalline alumina in a specimen with stress concentrator. These results are compared with experimental observations published in Ewart & Suresh (1986).

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2. Kinematics and statics overview

Instead of usual finite elements, material is replaced by a 2D assembly of rigid bodies (cells). These are interconnected by springs (normal, shear and rotational, Fig. 1a). Every cell has three degrees of freedom. Displacement jumps determining spring extensions are calculated through the rigid body motion (Bolander & Saito, 1998). Geometry of the cells is irregular, given by Voronoi tessellation on pseudorandom set of points with a restricted minimal mutual distance (Bolander et al., 2000). The tessellation procedure enables to increase mesh density around the notch tip and along the crack path – see Fig. 1c. So far, we focused our effort to develop only 1D constitutive behavior described further. For this reason, we eliminated shear as much as possible and thus a straight crack path is predefined in the model – see Fig. 1b. The discrete model is chosen here for the sake of simplicity, the usual finite elements could be used as well. However, the discrete model can also reflect material inhomogeneity, because the size of the discrete cells was set approximately to the real size of the grains.

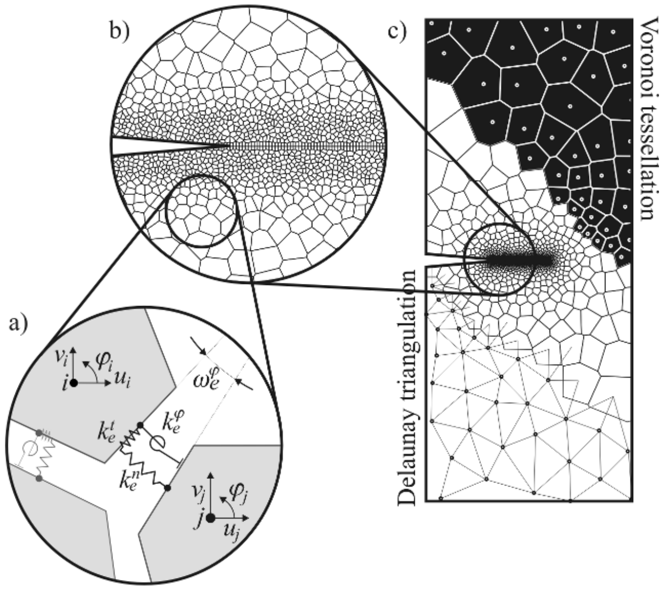

Fig. 1: Schematic explanation of the discrete model: a) mesh of irregular geometry given by Voronoi tessellation; b) predefined crack path; c) spring contact between rigid bodies.

3. Constitutive model

We lump the entire nonlinear phenomenon at the crack tip into one layer of spring contacts, where the crack may occur. The nonlinear behavior of these spring sets is represented by damage variable d (determined solely by normal spring) and plastic strains $\boldsymbol{\varepsilon}^p$ (nonzero only in the normal direction): $\boldsymbol{\sigma} = [(1\text{-}hd)\boldsymbol{D}_n+(1\text{-}d)\boldsymbol{D}_{sr}(\boldsymbol{\varepsilon}\text{-}\boldsymbol{\varepsilon}^p)$. $\boldsymbol{D}$ is matrix of material elastic constants (n for the normal spring and sr for the shear and the rotational spring), $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}^p$ is the total and the plastic strain vector and h is a parameter controlling a portion of the stiffness recovered in compression similarly to Desmorat et al. (2006). In tension, h equals 1. In compression, h might lie between 0 and 1. In this work, we assume h equals 0.999. Damage variable, d, is calculated from normal stress, σ, and normal strain, ε, (stress and strain in normal spring) $d = (1\text{-}\sigma/E(\varepsilon\text{-}\varepsilon^p))/h$; E is elastic modulus for plain stress.

What remains is to determine functions returning stress σ for arbitrary value of strain ε in normal spring. The model combines damage-like approach with plasticity. Change of normal spring extension, $\Delta\delta$, in a spring of length t results into change of strain $\Delta\varepsilon=\Delta\delta/t$. The strain change is decomposed into three components: elastic $\Delta\varepsilon^e$, plastic $\Delta\varepsilon^p$ and fracture/opening $\Delta\varepsilon^o$. In compression, the fracture component equals zero and the plastic part changes (increases) only when the descending linear softening branch is reached – see Fig. 2b.

On the contrary, the tensile part does not have any plastic increment. The strain increment is divided only into the elastic and the fracture component, so that both return the same stress: elastic part from 1D elasticity and fracture part from the traction-separation law sketched in Fig. 2a. Traction ($T = \sigma A$) is related to separation ($\delta^o=\varepsilon^o t$) through following differential equations (Nguyen et al., 2001).

$$\dot{T} = \begin{cases} K^{+}\dot{\delta}^{o} & \text{if } \dot{\delta}^{o} > 0 \\ K^{-}\dot{\delta}^{o} & \text{if } \dot{\delta}^{o} < 0 \end{cases} \tag{1}$$

$$\dot{K}^{+} = \begin{cases} -K^{+}\dot{\delta}^{o} / \delta_f & \text{if } \dot{\delta}^{o} > 0 \\ \left(K^{+} - K^{-}\right)\dot{\delta}^{o} / \delta_f & \text{if } \dot{\delta}^{o} < 0 \end{cases} \tag{2}$$

Parameter δ_f governs a cumulation of the damage due to cycling. State variables K^{+} and K^{-} are tangent stiffnesses in T-δ^{o} relation – see Fig. 2a; K^{+} is used for positive increments, whereas K^{-} for negative increments of the opening. Unloading always takes direction to the origin. Traction T cannot exceed the linear softening boundary – Fig. 2a; when this happens, the stress is dictated by the standard crack band model (Bažant and Oh, 1983) and both K^{+} and K^{-} are set to point to the origin. Though we simulate cycle by cycle, it is not possible to calculate experimentally performed 300000 cycles. Thus, the simulation of one "effective" cycle is understood to correspond with many experimental cycles.

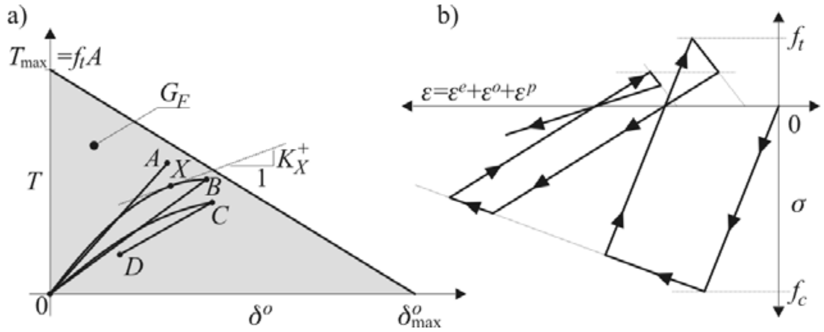

Fig. 2: a) Hysteresis according to Nguyen et al. (2001); b) coupling plasticity and damage.

4. Matching experimental data

The authors attempted to match experiments performed on polycrystalline alumina in Ewart & Suresh (1986). The loading was performed as sketched in Fig. 3a, the maximal stress was -29.8 MPa and the minimal one -298 MPa (11% of compressive strength). Specimen geometry is shown in Fig. 2b (D = 15.9 mm, a_0 = 6.3 mm, l = 37 mm, thickness 9.4 mm).

The minimal distance between points of discrete model was set to 13 μm. This produces an average cell size about 18 μm, which is approximately the real grain size in polycrystalline alumina. Following material parameters were used in simulation: tensile strength 260 MPa, compressive strength 2620 MPa, Elastic modulus 372 GPa, fracture energy in tension 35 J/m^2, fracture energy in compression 10000 J/m^2 (almost horizontal softening line compared to the rest). Parameter δ_f was determined by "trial and error" method to be approximately 1E-6 m. Stiffness recovery in compression was suppressed (h = 0.999), because in the experiment the crack was cleaned ultrasonically every 5000 cycles.

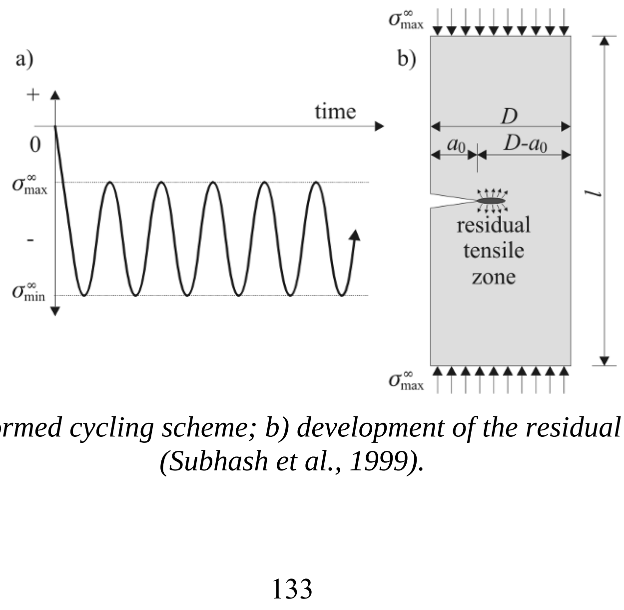

Fig. 3: a) Performed cycling scheme; b) development of the residual tensile zone (Subhash et al., 1999).

5. Results and Conclusions

Fig. 4 shows the simulated and the experimentally observed crack growth rate. An overall trend in crack propagation with cycles is well captured. However, large differences still remain. Mainly, the crack arrest occurs too early and it is too sudden. Moreover, the initial crack growth rate is too large. Both these issues should be suppressed; otherwise a predictive capability of such a model would be useless.

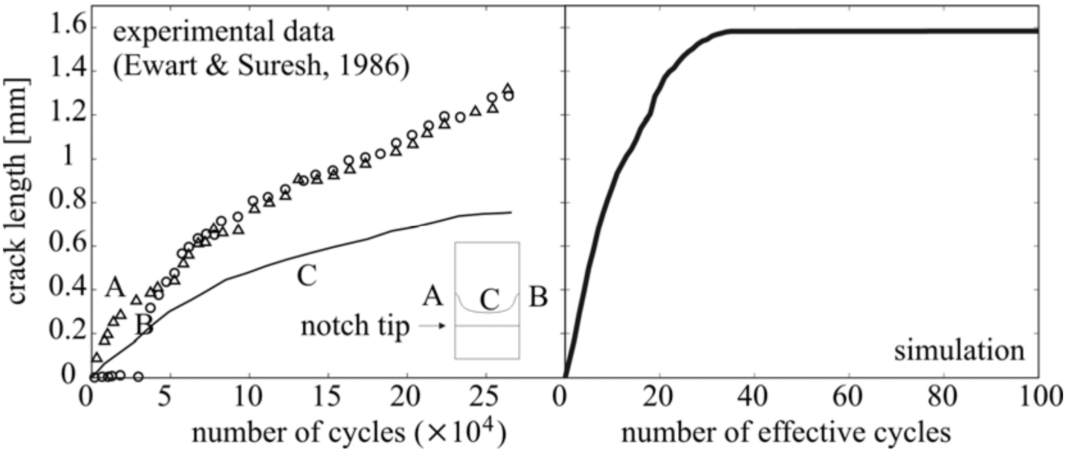

Fig. 4: Comparison of model results to experimental observation.

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