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ASSESSMENT OF THE INFLUENCE OF THE ELECTROMECHANICAL INTERACTION ON ROTORDYNAMIC INSTABILITY IN ELECTRIC MACHINES

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Abstract: *The electromechanical interaction in rotating electric machines induces additional forces between the rotor and the stator. A simple electromechanical computational model, which was developed by Holopainen et al., was used for study of this problem. Dependence of the eigenfrequencies of the rotor on shaft stiffness and load level of the motor is results of this work.*

Keywords: ***Electric, machines, rotor, magnetic pull.***

1. Introduction

Rotating electrical machines are composed from two main parts, the stator and the rotor. The magnetic fields in the air gap between the stator and the rotor induce electromagnetic forces, which acting on the machine structure. If the stiffness of the air gap is constant along whole circumference, that means the rotor and the stator have an ideal cylindrical shape and they are ideally concentric, the magnitude of the electromagnetic forces is zero. If the stiffness of the air gap is not constant, the magnitude of the electromagnetic forces is nonzero and is called as unbalanced magnetic pull (UMP). UMP induces two eccentricity harmonics of the magnetic fields in the air gap, which affect dynamic behavior of the rotor.

2. Methods

Holopainen et al. (2002) created a simple eletromechanical computational model for assessment of the influence of the electromechanical interaction on rotordynamic behavior of the rotating electrical machines. Initial assumptions of this computational model are:

- the rotor and the stator have an ideal cylindrical shape,
- axis of rotation of the rotor makes whirling motion around axis of the stator,
- a whirling radius is constant,
- a damping of the surrounding medium is viscous type.

This model is a combination of the de Laval rotor, which describes mechanical behavior of the rotor and the simple parametric model of the electromagnetic forces, which operates between the stator and the rotor, presented by Arkkio et al. (2000) This model can by written as follow

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} z \\ \dot{z} \\ q_{p-1} \\ q_{p+1} \end{bmatrix}^{\bullet} + \begin{bmatrix} 0 & -1 & 0 & 0 \\ k_e/m & b/m & -1/m & -1/m \\ -k_{p-1} & 0 & -a_{p-1} & 0 \\ -k_{p+1} & 0 & 0 & -a_{p+1} \end{bmatrix} \cdot \begin{bmatrix} z \\ \dot{z} \\ q_{p-1} \\ q_{p+1} \end{bmatrix} = \begin{bmatrix} 0 \\ f(t)/m \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

Where m is the mass of the rotor, b is the mechanical damping coefficient, f is the excitation force, k_e is the effective stiffness of the rotor, which is calculated as $k_e = k - k_0$, k is the shaft stiffness

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coefficient, k_0, $k_{p\pm1}$, $a_{p\pm1}$ are the parameters of the resulting electromagnetic force, the unknown z is a complex number, that describes the position of the centre of gravity of the rotor, $z(t) = x(t) + iy(t)$, x and y are the Cartesian coordinates of the position of the centre of gravity of the rotor and the unknown $q_{p\pm1}$ are variables related to the eccentricity harmonics of the magnetic fields, that are called as quasi-displacements.

Notice:

k_0, $k_{p\pm1}$ are real parameters proportional to the square of the fundamental flux density in the air gap.

$a_{p\pm1}$ are complex parameters, $a_{p\pm1,\mathrm{Im}}$ refer to whirling frequencies at witch the splits of the two eccentricity harmonics become zero, $a_{p\pm1,\mathrm{Re}}$ depend on the dimensions of the machines.

The subscripts of the parameters $p+1$ and $p-1$ refer to the respective eccentricity harmonics of the electromagnetic fields. p denotes number of pole pairs of the machine.

Value of the parameters $k_0, k_{p\pm1}$, $a_{p\pm1}$ are determined by the finite element method. In this approach the effects of core saturation in the model were included.

Equation (1) can be easily written as

$$\begin{aligned} \bar{\boldsymbol{M}}\dot{\bar{\boldsymbol{x}}} + \bar{\boldsymbol{K}}\bar{\boldsymbol{x}} &= \bar{\boldsymbol{f}} \\ \boldsymbol{E}\dot{\bar{\boldsymbol{x}}} + \bar{\boldsymbol{K}}\bar{\boldsymbol{x}} &= \bar{\boldsymbol{f}} \end{aligned} \tag{2}$$

Where $\overline{\mathbf{M}} = \mathbf{E}$ is the augmented matrix of the mass, $\overline{\mathbf{K}}$ is the augmented matrix of the stiffness, $\bar{\mathbf{f}}$ is the augmented vector of the excitation and $\bar{\mathbf{x}}$ is the augmented vector of the unknowns.

The homogenous part of the solution of the equation (2) comprises the eigenvalue problem.

$$\left(\bar{\boldsymbol{K}} + \lambda \boldsymbol{E}\right)\boldsymbol{u} = \boldsymbol{0} \tag{3}$$

3. Results

3.1. Assessment of the influence of the shaft stiffness and load level on the eigenfrequencies of the rotor

The assessment was performed for three load level of the four-pole cage induction motor about performance 15 kW. Essential mechanical properties are: mass of the rotor $m = 30\ kg$, mechanical damping coefficient $b = 2 \cdot 10^3\ \frac{kg}{s}$ and shaft stiffness coefficient $k \in \langle 0{,}5 \div 23 \rangle \cdot 10^6\ \frac{N}{m}$. Parameters of the electromagnetic forces are listed in Tab. 1.

Tab. 1: Parameters of the electromechanical forces.

Parameter	k_o [MN/m]	k_{p-1} [MNrad/ms]	k_{p+1} [MNrad/ms]	a_{p-1} [rad/s]	a_{p+1} [rad/s]
No load motor	4.37	8.37	94.7	(-0.59 + 25i)·2π	(-1.7 + 25i)·2π
50% load	4.89	9.1	102.5	(-0.42 + 25.4i)·2π	(-2.09 + 23.8i)·2π
100% load	6.22	15.7	74.9	(-0.50 + 25.8i)·2π	(-1.68 + 22.6i)·2π

The imaginary parts of the eigenvalues of the matrix $\overline{\mathbf{K}}$ correspond with eigenfrequencies of the rotor. The two higher eigenfrequencies are associated with the forward (FW) and the backward (BW) whirling modes of the rotor. The two lowest eigenfrequencies are associated with the eccentricity harmonics of the electromagnetic fields ($p+1$ and $p-1$). The Fig. 1 shows eigenfrequencies as the function of the load of the motor and shaft stiffness. These dependencies show, as the shaft stiffness is decreased, the whirling modes and electromagnetic modes interact strongly with each other's. The influence of the electromagnetic modes is stronger at full load, therefore further is analyzed this case.

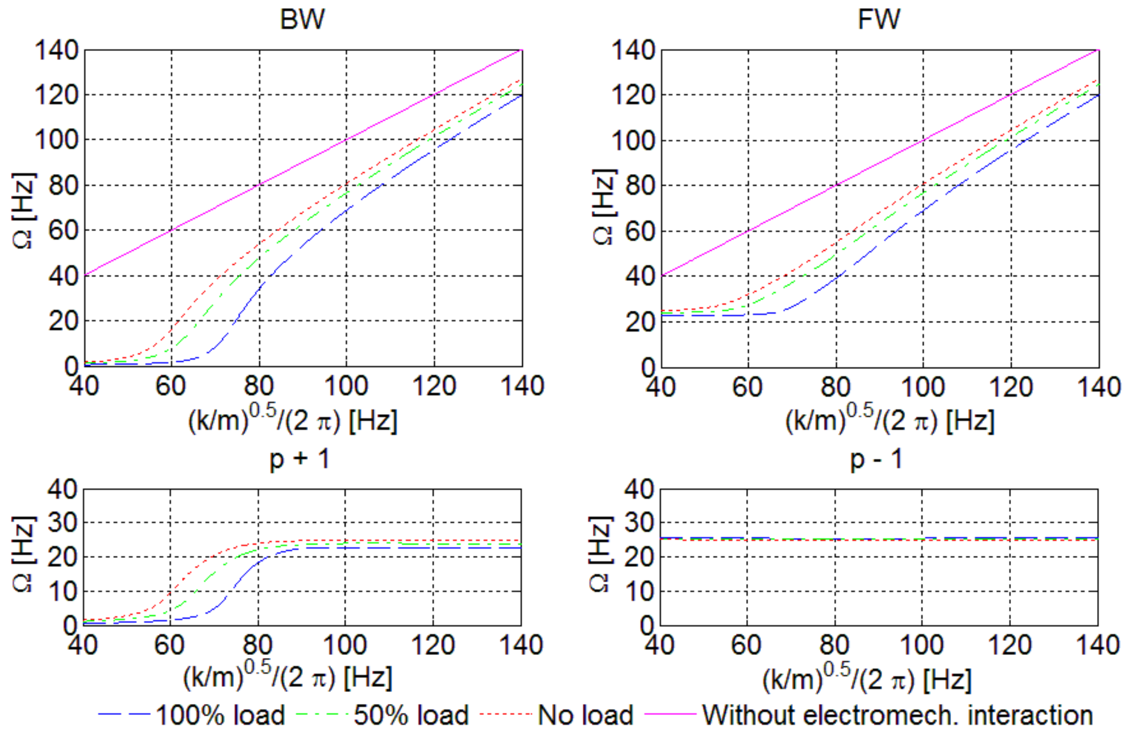

Fig. 1: Eigenfrequencies as function of the shaft stiffness and load level of the motor.

The real parts of the eigenvalues of the matrix $\overline{\mathbf{K}}$ correspond with decay constants. The Fig. 2 shows the decay constants as the function of the shaft stiffness of the full load motor. These dependencies show, that the UMP causes inside of the motor additional damping. Under the certain conditions can be decay constant of the $p+1$ mode positive and thus the motion of the rotor can be instable. Therefore in the next is assessed the influence of the parameters of the electromagnetic force on the rotor dynamics stability.

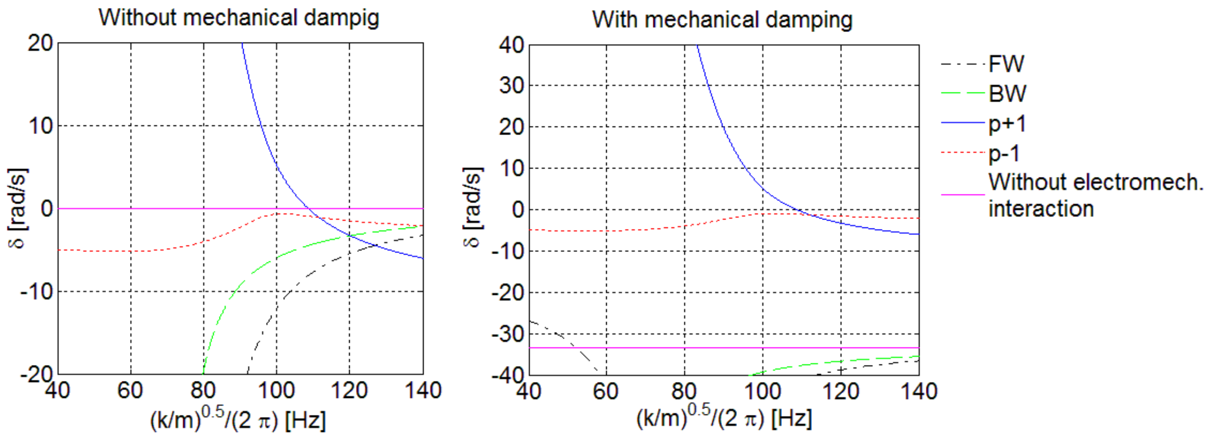

Fig. 2: Decay constants as function of the shaft stiffness.

3.2. Rotor dynamic stability

Figs. 3 - 6 show stability charts. This figures show following: if the value of the effective stiffness increases the stability of the system increases too. In addition an increase of parameters $k_{p\pm1}$ and a decrease of parameters $a_{p\pm1,\mathrm{Re}}$ may destabilize the system.

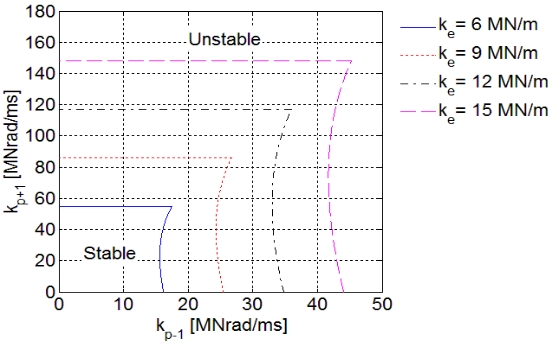

Fig. 3: Stability chart 1.

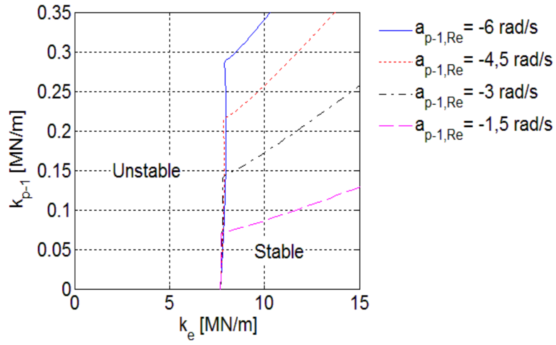

Fig. 5: Stability chart 3.

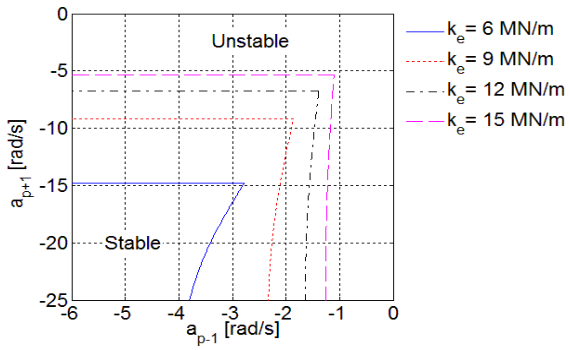

Fig. 4: Stability chart 2.

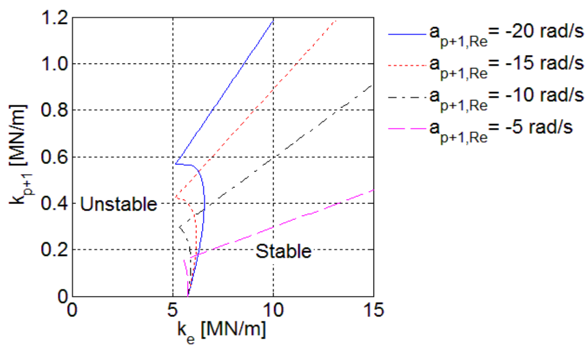

Fig. 6: Stability chart 4.

4. Conclusion

The results showed that electromagnetic interaction in rotating electrical machines leads to decreasing of the critical speed of rotor and to increasing of damping of the rotor.

The goal of this work was verification of the mathematical model described by Holopainen et al. first and next step will be modification of the model for calculation with the non-linear unbalanced magnetic pull and inclusion of the non-linear bearings. This non-linear model should be presented in the conference.

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