17th International Conference
ENGINEERING MECHANICS 2011
Svratka, Czech Republic, 9 – 12 May 2011

RING-CORE RESIDUAL STRESS MEASUREMENT: ANALYSIS OF DEPTH INCREMENT DISTRIBUTION FOR INTEGRAL EQUATION METHOD

A. Civín*, M. Vlk*, P. Navrátil*, M. Matug*

Abstract: *The ring-core method is the semi-destructive experimental method. Correctly determined and properly used calibration or relaxation factors for the residual stress measurement by the ring-core method are essential. Knowledge of their dependence on the geometric changes of the ring-groove and on the disposition of the residual state of stress through the depth of metallic material gives results, correspond of theirs appropriate application. This paper is focused on the evaluation of calibration factors a_{ij} and b_{ij}, necessary for the residual state of stress determination by the integral equation method. The finite element method is used for simulation of the residual state of stress and to calculate relieved strains on the top of the core. Three types of the depth increment distribution are studied, i.e. constant, increasing and optimized depth increment distribution, which is commonly used.*

Keywords: ***Ring-core method, integral method, calibration factors, residual stress, strain gauge.***

1. Introduction

The ring-core method (RCM) is a semi-destructive experimental method used for the evaluation of homogeneous and non-homogeneous residual stresses, acting over the depth of drilled core. Therefore, the specimen is not totally destroyed during measurement and it could be used for further application in many cases.

In this paper, the most suitable mathematical theory to evaluate non-uniform residual stress fields, which is the integral equation method (IEM), is discussed. This method overcomes typical drawbacks of the incremental strain method (ISM), which lead to incorrect results, where a steep gradient of residual state of stress occurs. The incremental strain method assumes that the measured deformations $d\varepsilon_a$, $d\varepsilon_b$, and $d\varepsilon_c$ are functions only of the residual stresses, acting in the current depth z of the drilled groove and they do not depend on the previous increments dz, including another residual stresses. More information about the ISM could be found in papers Civín & Vlk (2010). Anyway, relieved strains do not depend only on the stress acting within the drilled layer and its position, but also on the geometric changes of the ring groove during deepening. These two factors are taken into account by the integral equation method, which has been particularly developed for the practical use by Schajer.

The IEM assumes that strain relaxation, correspond to the particular depth of drilled groove, is superposition of all deformations caused by partial residual stresses, acting within every drilled layer of all depth increments, see Figs. 1 and 2.

Papers made by Ajovalasit et al., (1996); Zuccarello, (1996) generally describe the IEM like a method, with a high sensitivity to the measurement errors due to the numerical ill-conditioning of the equation set. Strain gauges on the top of the core are not enough sensitive to the strains, relieved in the deeper layers of the drilled groove. Therefore, influence of the strain measurement errors on the calculated residual stress depends particularly on the number and magnitude of the depth increment distributions Δz_i and consequently on the maximum depth H of the groove. Influence of the step distribution on the determination of calibration factors a_{ij}, b_{ij} and on the subsequent residual stress state determination has been investigated for three different types of depth increment distribution. Total depth of drilled groove $H = 5\ mm$ has been made by $n = 8$ constant, increasing and optimized (Ajovalasit et al.,

* Ing. Adam Civín, assoc. prof. Ing. Miloš Vlk, CSc., Ing. Petr Navrátil and Ing. Michal Matug: Institute of Solid Mechanics, Mechatronics and Biomechanics, Brno University of Technology, Technická 2896/2; 616 69, Brno; CZ, e-mails: civin.adam@seznam.cz, vlk@fme.vutbr.cz, ynavra26@stud.fme.vutbr.cz, ymatug01@stud.fme.vutbr.cz

(1989); Zuccarello, (1996)) depth increments. Optimum depth increment distribution should minimize error sensitivity of the experimental measurement and considerably improves the numerical conditioning.

This paper describes how application of the ring-core method with theory of the IEM and the finite element method (FEM) could be used for a numerical simulation and determination of uniform or non-uniform residual state of stress. The numerical simulation is used for the measurement of relieved residual strains on the top of the model's core at real positions of the strain gauge rosette's measuring grids. Calibration factors´ matrices ***a*** and ***b***, which are lower triangular, need to be calculated first to describe uniform and non-uniform residual state of stress by the integral equation method.

2. Integral equation method

Like each method, the IEM has its own theoretical background to define certain relations between known and unknown parameters. The integral of the infinitesimal strain relaxation components caused by the residual stresses at all depths, relaxed in the range ($0 \leq z \leq H$), is described by Eq. (1):

$$\varepsilon_{k(H)} = \frac{1}{E}\int_0^H \left\{ \left[A_{(H,z)} \cdot \left(\sigma_{1(z)} + \sigma_{2(z)}\right) + B_{(H,z)} \cdot \left(\sigma_{1(z)} - \sigma_{2(z)}\right) \right] \cdot \cos 2\alpha_{k(z)} \right\} dz \qquad k = a, b, c \tag{1}$$

where $\varepsilon_{a(H)}$, $\varepsilon_{b(H)}$, $\varepsilon_{c(H)}$ are strains, measured by the strain gauge rosette on the top of the core´s surface after milling a groove having depth H, $\sigma_{1(z)}$ and $\sigma_{2(z)}$ are the unknown residual stresses acting at current depth z, $\alpha_{k(z)}$ is the angle between the maximum principal stress $\sigma_{1(z)}$ and the direction of the strain gauge´s measuring grid $k=a, b, c$ and $A_{(H,z)}$, $B_{(H,z)}$ are calibration functions, dependent on the shape and geometry of the ring-groove (Fig. 1). Using a three-grid rosette, Eq. (1) leads to a three linear equation set from which the principal residual stresses and their orientation can be evaluated too.

For $i = 1, ..., n$ finite depth increments Eq. (1) can be written as:

$$\varepsilon_{ki} = \sum_{j=1}^{i} \varepsilon_{kij} \quad k = a, b, c \tag{2}$$

where the strain ε_{kij} depends only on the stress existing in the j^{th} layer by means of Eq. (3).

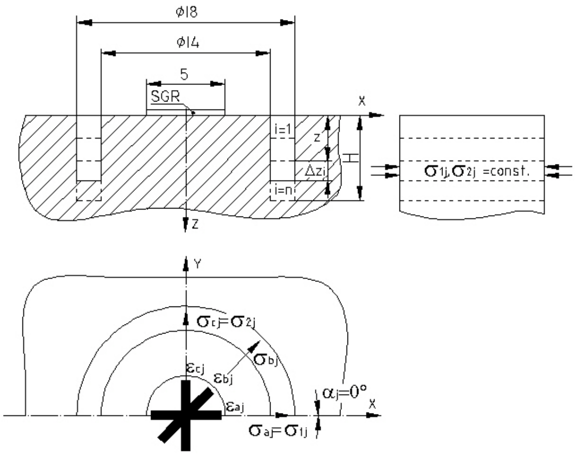

Fig. 1: Ring-core method: geometry and general notation.

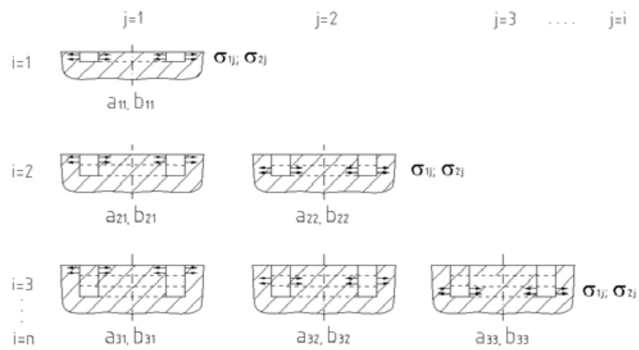

Fig. 2: Loading cases based on theory of the integral equation method.

Consequently, it is necessary to divide the maximum depth H into n intervals with the depth increment of Δz_i and to approximate the function of the principal residual stresses $\sigma_{1(z)}$ and $\sigma_{2(z)}$ in each interval with uniform distribution (Fig. 2). Therefore, considering i finite depth increments, Eq. (1) can be written as:

$$\varepsilon_{kij} = \frac{a_{ij}}{E}\left(\sigma_{1j} + \sigma_{2j}\right) + \frac{b_{ij}}{E}\left(\sigma_{1j} - \sigma_{2j}\right)\cos 2\alpha_{kj} \tag{3}$$

in which ε_{kij} is the strain component, relaxed on the surface solely due to the stress acting in the j^{th} layer, when i^{th} depth increments have been achieved, a_{ij} and b_{ij} are calibration factors and σ_{1j}, σ_{2j} are stresses acting within the j^{th} layer.

Calibration factors a_{ij} and b_{ij} of the lower triangular matrices $\boldsymbol{a}$ and $\boldsymbol{b}$ cannot be determined by calibration coefficients K_1 and K_2 used for the IEM and described in papers Civín & Vlk (2010). They can be possibly obtained only by the finite element simulation.

3. FEM simulation

A prerequisite for correct and accurate measurement of residual strains on the top of the core is to use the finite element simulation. The ANSYS analysis system is used for the subsequent FE-simulation.

FE-analysis is based on a specimen volume with dimensions of a x $a = 50\ mm$ and thickness of $t = 50\ mm$. Due to symmetry, only a quarter of the model has been modelled with centre of the core on the surface as the origin. The shape of the model is simply represented by a block with planar faces with a quarter of the annular groove drilled away (Figs. 3 and 4). The annular groove has been made by $n = 8$ increments with the different step´s size Δz_i (Tab. 1). The maximum depth of drilled groove is $H = 5\ mm$. Dimension of outer diameter is D = $2r_i = 18\ mm$ and groove width is $h = 2\ mm$.

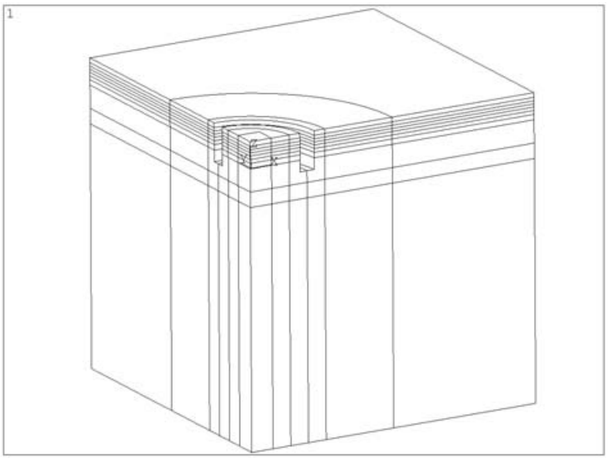

Fig. 3: Quarter of global solid model.

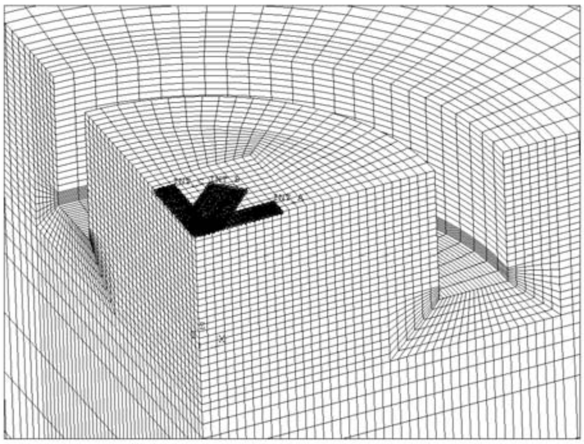

Fig. 4: Detail of core with finite element mesh.

Linear, elastic and isotropic material model is used with material properties of Young's modulus $E = 210\ GPa$ and Poisson's ratio $\mu = 0.3$. Length and width of each measuring grid is $l = 5\ mm$ and $w = 1.9\ mm$ respectively. In case of known directions of principal residual stresses, placing of the three-element strain gauge rosette on the top of the ring-core is shown in Fig. 2. Strain measurement on the top of the core is made by integration across rosettes' measuring grid surface.

Tab. 1: Distribution of the depth n = 8 increments Δz_i for a total depth of H = 5 mm.

Depth distribution:	Depth increment Δz_i [mm]: Δz_1	Δz_2	Δz_3	Δz_4	Δz_5	Δz_6	Δz_7	Δz_8
Constant	0.5	0.5	0.5	0.5	0.5	0.5	1.0	1.0
Increasing	0.1	0.25	0.4	0.55	0.7	0.85	1.0	1.15
Optimized[1)]	0.6	0.45	0.4	0.4	0.45	0.5	0.7	1.5

[1)] proposed in papers by Ajovalasit et al., (1996); Zuccarello, (1996)

4. Calibration factors determination

For correct determination of depth-varying principal residual stresses σ_{1j}, σ_{2j} by the IEM, it is necessary to determine calibration factors a_{ij}, A_{ij} and b_{ij}, B_{ij} for each depth increment distribution.

In order to determine factors a_{ij} and A_{ij} it is necessary to consider for Eq. (3) biaxial state of uniform stress with $\sigma_{1j} = \sigma_{2j} = 1\ MPa$, where $\varepsilon_{aij} = \varepsilon_{bij} = \varepsilon_{cij} = \varepsilon_{ij}$:

$$a_{ij} = \frac{E\varepsilon_{ij}}{2\sigma_{1j}} \ ; \quad A_{ij} = \frac{a_{ij}}{E} = \frac{\varepsilon_{ij}}{2\sigma_{1j}} \qquad (4, 5)$$

To evaluate factors b_{ij} and B_{ij} it is necessary to consider for Eq. (3) a pure shear state of uniform stress with $\sigma_{1j} = -\sigma_{2j} = 1\ MPa$, and for $\alpha_{aj} = 0°$:

$$b_{ij} = \frac{E\varepsilon_{aij}}{2\sigma_{1j}} \; ; \; B_{ij} = \frac{b_{ij}}{E} = \frac{\varepsilon_{aij}}{2\sigma_{1j}} \qquad (6, 7)$$

Finally, particular principal residual stresses σ_{1j}, σ_{2j}, acting in j^{th} layer of drilled groove with $i = 1, .., n$ depth increments, can be determined by using Eq. (5) and Eq. (7):

$$\sigma_{1j} = \frac{1}{4}\left[\frac{\varepsilon_{aij} + \varepsilon_{cij}}{A_{ij}} + \frac{\varepsilon_{aij} - \varepsilon_{cij}}{B_{ij}}\right] \; ; \; \sigma_{2j} = \frac{1}{4}\left[\frac{\varepsilon_{aij} + \varepsilon_{cij}}{A_{ij}} - \frac{\varepsilon_{aij} - \varepsilon_{cij}}{B_{ij}}\right] \qquad (8, 9)$$

For considered depth of ring-groove $H = 5\ mm$ are calibration factors a_{ij} and b_{ij}, determined for each type of depth increment distribution (see Tab. 1), written in Tab. 2 ÷ 4.

Tab. 2a: Constant depth increment.

a_{ij} [1]	j=1	2	3	4	5	6	7	8
i=1	-0,0238							
2	-0,0426	-0,0291						
3	-0,0570	-0,0459	-0,0295					
4	-0,0679	-0,0573	-0,0444	-0,0270				
5	-0,0755	-0,0652	-0,0533	-0,0396	-0,0228			
6	-0,0807	-0,0705	-0,0591	-0,0465	-0,0331	-0,0180		
7	-0,0859	-0,0759	-0,0649	-0,0531	-0,0412	-0,0298	-0,0283	
8	-0,0876	-0,0778	-0,0669	-0,0553	-0,0438	-0,0329	-0,0383	-0,0111

Tab. 2b: Constant depth increment.

b_{ij} [1]	j=1	2	3	4	5	6	7	8
i=1	-0,0219							
2	-0,0424	-0,0296						
3	-0,0580	-0,0488	-0,0326					
4	-0,0700	-0,0621	-0,0509	-0,0329				
5	-0,0793	-0,0721	-0,0626	-0,0500	-0,0315			
6	-0,0864	-0,0795	-0,0709	-0,0602	-0,0469	-0,0290		
7	-0,0955	-0,0891	-0,0812	-0,0719	-0,0614	-0,0500	-0,0606	
8	-0,1004	-0,0942	-0,0866	-0,0776	-0,0678	-0,0577	-0,0857	-0,0451

Tab. 3a: Linearly increasing depth increment.

a_{ij} [1]	j=1	2	3	4	5	6	7	8
i=1	-0,0024							
2	-0,0045	-0,0093						
3	-0,0077	-0,0174	-0,0207					
4	-0,0112	-0,0263	-0,0365	-0,0341				
5	-0,0144	-0,0344	-0,0497	-0,0560	-0,0433			
6	-0,0167	-0,0402	-0,0591	-0,0698	-0,0669	-0,0429		
7	-0,0180	-0,0432	-0,0641	-0,0771	-0,0775	-0,0624	-0,0316	
8	-0,0184	-0,0443	-0,0659	-0,0797	-0,0814	-0,0684	-0,0435	-0,0147

Tab. 3b: Linearly increasing depth increment.

b_{ij} [1]	j=1	2	3	4	5	6	7	8
i=1	-0,0013							
2	-0,0040	-0,0077						
3	-0,0074	-0,0169	-0,0201					
4	-0,0111	-0,0264	-0,0376	-0,0362				
5	-0,0146	-0,0352	-0,0524	-0,0621	-0,0516			
6	-0,0174	-0,0424	-0,0644	-0,0802	-0,0838	-0,0619		
7	-0,0194	-0,0474	-0,0727	-0,0923	-0,1016	-0,0949	-0,0630	
8	-0,0205	-0,0504	-0,0776	-0,0993	-0,1112	-0,1092	-0,0915	-0,0554

Tab. 4a: Optimized depth increment.

a_{ij} [1]	j=1	2	3	4	5	6	7	8
i=1	-0,0318							
2	-0,0519	-0,0257						
3	-0,0659	-0,0384	-0,0222					
4	-0,0770	-0,0474	-0,0329	-0,0208				
5	-0,0864	-0,0548	-0,0403	-0,0312	-0,0213			
6	-0,0937	-0,0605	-0,0458	-0,0376	-0,0317	-0,0200		
7	-0,0998	-0,0652	-0,0503	-0,0426	-0,0384	-0,0310	-0,0223	
8	-0,1042	-0,0688	-0,0537	-0,0463	-0,0431	-0,0372	-0,0352	-0,0261

Tab. 4b: Optimized depth increment.

b_{ij} [1]	j=1	2	3	4	5	6	7	8
i=1	-0,0300							
2	-0,0520	-0,0263						
3	-0,0670	-0,0410	-0,0244					
4	-0,0793	-0,0515	-0,0374	-0,0247				
5	-0,0903	-0,0605	-0,0469	-0,0383	-0,0280			
6	-0,0998	-0,0681	-0,0544	-0,0473	-0,0429	-0,0301		
7	-0,1092	-0,0756	-0,0616	-0,0554	-0,0539	-0,0481	-0,0414	
8	-0,1198	-0,0839	-0,0693	-0,0636	-0,0641	-0,0617	-0,0696	-0,0830

5. Conclusion

This paper provided basic information about the integral equation method, used particularly for the non-uniform residual stress determination by the experimental ring-core method. To determine important calibration factors a_{ij}, b_{ij} of lower triangular matrices $\boldsymbol{a}$ and $\boldsymbol{b}$, appropriate equations and the FE-model have been used. Application of optimized step distribution increases the spatial resolution without a significant increase of the error sensitivity and for a given number of steps it allows to minimize sensitivity of the strain measurement errors.

Acknowledgement

This work has been supported by the specific research FSI-S-11-11/1190.

References

Ajovalasit, A.et al. (1996) Determination of Nonuniform Residual Stresses Using the Ring-Core Method Transactions of the ASME – Journal of Engineering Materials and Technology, vol. 118, no. 2, pp. 224-228.

Civín, A. & Vlk, M. (2010) Assessment of Incremental Strain Method Used for Residual Stress Measurement by Ring-Core Method, in: 48th Int. Conference Experimental Stress Analysis 2010, Velké Losiny, pp. 27-34.

Civín, A. & Vlk, M. (2010) Ring-Core Residual Stress Measurement: Analysis of Calibration Coefficients for Incremental Strain Method. Bulletin of Applied Mechanics, Vol. 6, No. 24, pp. 77-82.

Zuccarello, B. (1996) Optimization of Depth Increment Distribution in the Ring-Core Method, Journal of Strain Analysis for Engineering Design, vol. 31, no. 4, pp. 251-258.