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# INFLUENCE OF FINITE ELEMENT ORDER ON SCF PRECISION FOR U-SHAPED NOTCHES IN FLAT BARS UNDER TENSION

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**Abstract:** *The paper presents results of research on precision of stress concentration factor for flat bars with opposite U-shaped notches under tension. The calculations were performed with the use of the finite element method with utilization of elements of various approximating polynomials orders. Each type of element underwent tests for different characteristic size of element. Precision of numerical calculations for stress concentration factor $K_t$ was compared to errors of analytical calculations performed with employment of various approximate methods. The results gained in the work allow to indicate optimal mesh size and type of finite element providing precise $K_t$ value with minimal DOF number.*

**Keywords:** ***Notch, local approach, stress concentration factor, finite element method.***

## 1. Introduction

Determination of fatigue live of elements with notch still employs a local approach. This kind of solution assumes that the magnitude of fatigue damage of an element is determined by the stress at notch root *σ*. Its value is determined according to nominal net stress *S* and stress concentration factor $K_t$, as well as material constants *E*, *n'*, *K'*. The calculation models, representing the local approach, among others (Stephens et al., 2001) include the Neuber's hypothesis (1) and Glinka-Molski strain energy density method (2):

$$\frac{\sigma^2}{E} + \sigma\left(\frac{\sigma}{K'}\right)^{1/n'} = \frac{(K_t S)^2}{E} \tag{1}$$

$$\frac{\sigma^2}{E} + 2\frac{1}{1+n'}\sigma\left(\frac{\sigma}{K'}\right)^{1/n'} = \frac{(K_t S)^2}{E} . \tag{2}$$

The tests results present conservatism of fatigue life calculations performed with the use of the Neuber's hypothesis (Boroński, 2007) and underestimation of the values for stresses determined with the use of the strain energy density method (Łagoda & Macha, 1998). In order to reduce the above mentioned errors, various types of notation modifications (1) and (2) are proposed. One of them is implementation of power density of stresses parameter (Łagoda & Macha, 1998) to the strain energy density method, which leads to the following notation (3):

$$\frac{\sigma^2}{E} + 2\frac{1-n'}{1+n'}\sigma\left(\frac{\sigma}{K'}\right)^{1/n'} = \frac{(K_t S)^2}{E} . \tag{3}$$

Analysis of dependencies (1) ÷ (3) indicate a considerable influence of stress concentration factor on precision of *σ* calculations with respect to presence of the factor $K_t$ in the second power. For purposes of fatigue live engineering calculations, the values of $K_t$ factor for various notch types and sizes were presented in diagrams or described by the simplified dependence (Pilkey & Pilkey, 2008).

The paper presents results of research on precision of stress concentration factor for flat bars with opposite U-shaped notches under tension. The estimation were performed with the use of the finite element method with utilization of elements of various approximating polynomials orders. Each type of element underwent tests for different characteristic size of element. Error of numerical calculations

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stress concentration $K_t$ factor was compared to errors of analytical calculations performed with employment of various approximate methods.

## 2. Analytical methods

One of the most commonly used analytical approximate dependency for determination of the values of stress concentration factor is the Neuber's trigonometric formula (Pilkey & Pilkey, 2008). It is a combination of $K_{tE}$ and $K_{tH}$ values as presented below (4).

$$K_t = \frac{(K_{tE}-1)(K_{tH}-1)}{\sqrt{(K_{tE}-1)^2+(K_{tH}-1)^2}}+1 \tag{4}$$

The $K_{tE}$ describes the stress concentration factor for shallow elliptical notch in semi-infinitely wide member. The $K_{tH}$ describes the stress concentration factor for deep hyperbolic notch in infinitely wide member. The Neuber's formula provides $K_t$ values determined with 10% precision with respect to the exact solution. Similar precision is characteristic of the method proposed by Pilkey (Pilkey & Pilkey, 2008). It allows direct reading of $K_t$ value from the diagram or to calculate the value from analytical dependence (5). In the dependence the $t$ refers to notch depth in a specimen with $W$ width, and $C1 \div C4$ values depend on the $t$ depth and notch radius $\rho$.

$$K_{tn} = C_1 + C_2\left(\frac{2t}{W}\right) + C_3\left(\frac{2t}{W}\right)^2 + C_4\left(\frac{2t}{W}\right)^3 \tag{5}$$

An exact solution to the issue of flat bar with opposite U-shaped notches was performed by Nisitani with the use of body force method (Nisitani & Noda, 1986). Due to considerable complexity of such solution Nisitani provided the table of results appointed to notches of selected geometrical dimensions. In order to achieve precise approaches for notches of any geometry Noda (Noda et al., 1995) proposed a wide range of notations based on Neuber dependence modification. Such formulated dependencies allow to determine, in a wide range of uses, $K_t$ values with 1% precision with respect to the exact solution.

## 3. Numerical solution

### 3.1. Calculation conditions

The analyses were performed in plane stress state for tension of flat bars with opposite U-shaped notches specimens (Fig. 1) with geometry according to the work (Fatemi et al., 2004). Two notch radius R were tested. The bigger one generating stress concentration factors in tension $K_t = 1.757$ (Noda et al., 1995) was marked as shallow notch. The smaller one generating stress concentration factors in tension $K_t = 2.715$ was marked as sharp notch. Geometrical dimensions of specimens shown in Fig. 1 are presented in Tab. 1.

*Tab. 1: Nominal dimensions of analysed specimens.*

| | H | R |
|---|---|---|
| shallow notch, $\boldsymbol{K_t = 1.757}$ | 41.12 mm | 9.128 mm |
| sharp notch, $\boldsymbol{K_t = 2.715}$ | 35.56 mm | 2.778 mm |

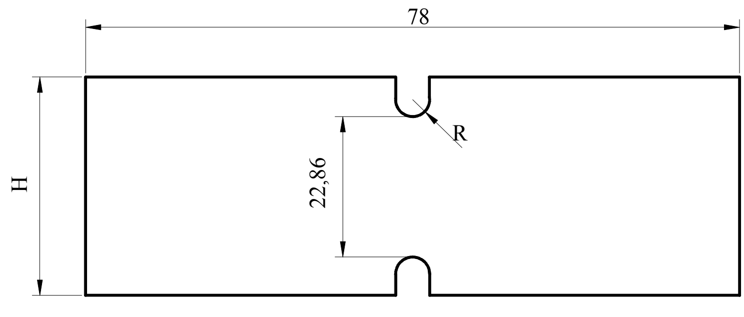


*Fig. 1: Specimens for analyses.*

Linear FEM analyses performed in ANSYS were of two-dimensional character. For the purpose of the calculations the free meshing was used (Cichański, 2010). The analyses used four-node finite elements with approximating polynomials of the order from the first to sixth. Due to dual symmetry, in geometric shape and boundary conditions, the analyses employed a quarter of specimen. Rejection of specimen parts located on other sides of symmetry plane was considered via adequate defining of symmetrical boundary conditions on edges of division.

### 3.2. Analyses results

For selected orders of finite elements approximating polynomials, a numbers of calculations with various characteristic dimension of mesh were every time performed. As the mesh size decrease, the number of finite elements essential for division of specimen Fig. 1 increase. Along with the number of elements, the number of degrees of freedom for the analysed issue increase as well. Based on such prepared mesh a stress distribution was determined. According to the stress values at the notch root the value of stress concentration factor was appointed. The $K_t$ values determined for the sharp notch with respect to the number of degrees of freedom for meshes of various sizes are presented in Fig. 2a. The analyses results for shallow notches are similarly presented on Fig. 2b. Every line presented on the Fig. 2 was determined with the use of elements of defined approximating polynomial order.

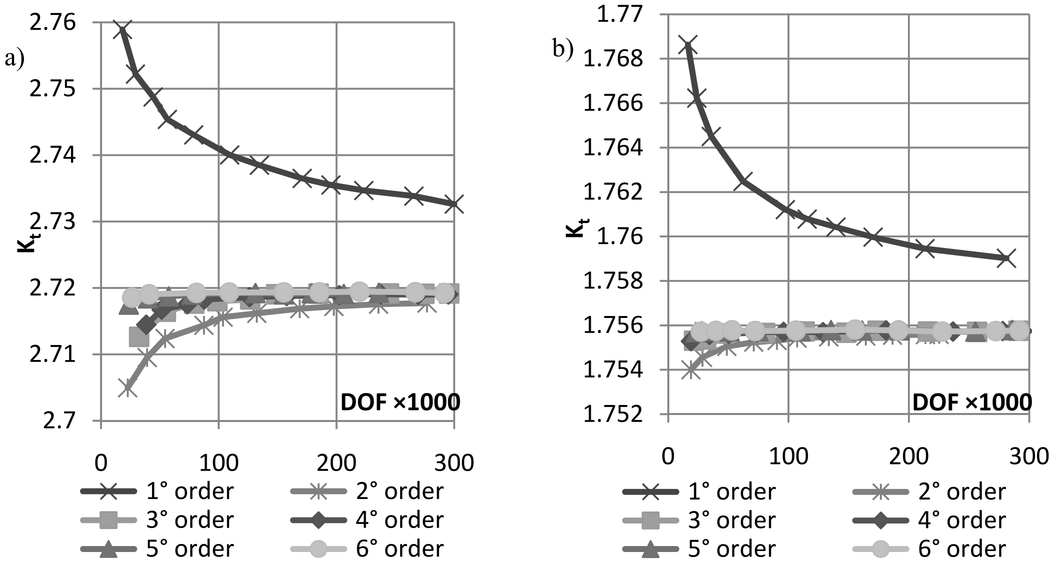


*Fig. 2: Analyses results: sharp notches a) shallow notches b).*

Charts presented in Fig. 2 indicate that for both types of notches the precision of calculations rises with the increase of finite element order. Additional charts processing were performed in order to determine what size of a model can provide $K_t$ with constant precision for different approximating polynomial order. As a measure of the analysis accuracy was taken percent error $\delta_{ord}$ with the reference value $K_t$ determined with the use of sixth order elements. The DOF numbers with respect to the order of approximating polynomial for percent error $\delta_{ord}$ less than 0.02% for all elements orders are presented in Tab. 2 and for elements orders 2° to 6° in Fig. 3.

*Tab. 2: Problem size and $\delta_{ord}$ for sharp and shallow notch.*

| | | approximating polynomial order | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1° | 2° | 3° | 4° | 5° | 6° |
| sharp notch | element size | 0.036 mm | 0.063 mm | 0.18 mm | 0.22 mm | 0.4 mm | 0.9 mm |
| | DOF | 955 170 | 817 488 | 172 200 | 188 540 | 90 645 | 45 124 |
| | $\delta_{ord}$ | 0.319% | 0.012% | -0.001% | -0.01% | -0.002% | 0% |
| shallow notch | element size | 0.04 mm | 0.2 mm | 0.6 mm | 0.7 mm | 1.5 mm | 3 mm |
| | DOF | 957 716 | 107 854 | 21 289 | 25 556 | 8 702 | 3 098 |
| | $\delta_{ord}$ | 0.126% | -0.004% | 0.004% | -0.002% | 0% | 0% |

In the next step calculations were performed to compare the numerical and selected analytical methods. The $K_t$ value determined with the use of Noda method was taken as a reference value for $\delta_{met}$

percent errors (Noda et al., 1995). The values of errors for Neuber method were determined using the dependency (4) and values of errors for Pilkey method were determined using the dependency (5). The values of errors for Finite Element Method were determined using the sixth order of approximating polynomial. Results of methods comparison are presented in Fig. 4.

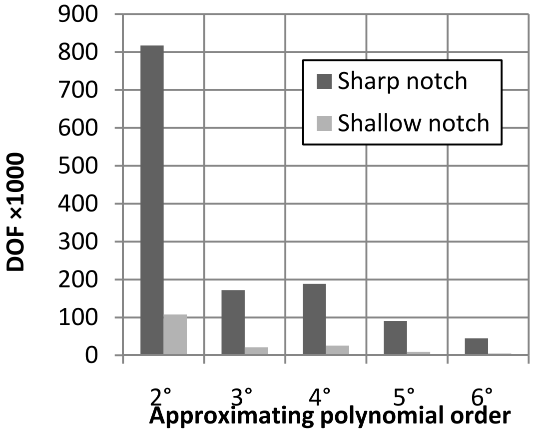


*Fig. 3: DOF number for $\delta_{ord} < 0.02\%$.*

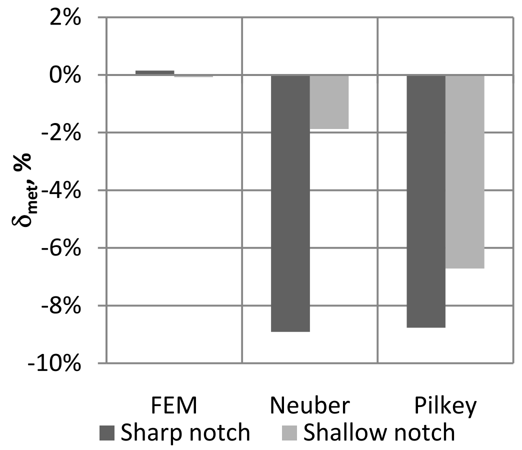


*Fig. 4: Comparison of calculation performed with different methods.*

First order elements analysis even with size above 1 mln DOF calculates $K_t$ with error $\delta_{ord}$ much more bigger than elements with second and higher orders of approximating polynomial. The mesh composed of second order finite elements allows to determine stress concentration factor precisely with five times more degrees of freedom in contrast to elements of third order Fig. 3. For elements with orders higher than third DOF number continuously decrease. The FEM analyses of elements with notches allow to increase precision of calculation for stress concentration factor by ten times if compared to approximate analytical methods Fig. 4. Additional advantage of numerical method is the ability to determine stress and strain distributions around the notch.

## 4. Conclusions

The work results allow to indicate optimal size of the mesh and the type of the finite element which shall allow obtaining high precision of calculations for stress concentration factor with controlled number of the problem DOF. The mesh composed of sixth order elements allows the most precision FEM determination stress concentration factor. The FEM analysis allow significantly decrease of $K_t$ calculations error if compared to approximate analytical methods.

In order to perform further reduction in the number of DOF for first order elements without the loss of precision in calculation of stress concentration factor, one shall indicate a method for refine the mesh around the notch root.

## References

Boroński, D. (2007) Metody badań odkształceń i naprężeń w zmęczeniu materiałów i konstrukcji. Wydawnictwo Instytutu Technologii Eksploatacji, Radom.

Cichański, A. (2010) Influence of mesh morphology near the notch on precision of SCF determination. Journal of Polish CIMAC, 5, 2, Gdańsk, pp.65-71.

Fatemi, A., Zeng, Z. & Plaseied, A. (2004) Fatigue behaviour and life predictions of notched specimens made of QT and forged microalloyed steels. International Journal of Fatigue, 26, pp.663-672.

Łagoda. T. & Macha, E. (1998) Energy solution to fatigue under combined cyclic bending with torsion of smooth and notched specimens. Materials Science, 34, 5, pp.630-639.

Nisitani, H. & Noda, N. (1986) Stress concentration of a strip with double edge notches under tension or in-plane bending. Engineering Fracture Mechanics, 23, 6, pp.1051-1065.

Noda, N.A., Sera, M. & Takase, Y. (1995) Stress concentration factors for round and flat specimens with notches. International Journal of Fatigue, 17, 3, pp.163-178.

Pilkey, W.D. & Pilkey D.F. (2008) Peterson's stress concentration factors. John Wiley & Sons, Inc.

Stephens, R.I., Fatemi, A., Stephens, R.R. & Fuchs H.O. (2001) Metal Fatigue in Engineering, 2nd Edition. John Wiley&Sons, Inc.