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DEVELOPMENT OF MATHEMATICAL MODEL OF THE HYDRAULIC DAMPER

J. Šklíba*, **R. Svoboda****

Abstract: *The physical model of two cylinder hydraulic damper is composed from several groups of equations, which describe: a) the dynamic equilibrium of the forces loading the valves, b) the dynamic equilibrium of volume flows in the working spaces of the damper, c) equation of the state of the air in the accumulator, d) the dependencies of discharge coefficient on Reynolds number, e) the dependency of hydraulic capacity on the concentration of free air in the working liquid. According to a number of places in which is the pleasure concentrated, we have models with three, five or seven pressures. It is described and commented the development of physical model and its precising.*

Keywords: ***Hydraulic damper, damper valves, free air in working liquid, volume flow, discharge coefficient.***

1. Introduction

At present types of dampers with a two-phase working liquid consisting of oil and air with the possibility of developing steam and gas cavern located under and above the piston are used on a regular basis. These types reflect reality much better. The original assumption of an ideal oil-based working liquid with no other ingredients surely did not correspond with reality as there is always at least minimal amount of air present, and can therefore be released from as well as dissolved in the liquid. Some working liquids create so called steam cavern during the work cycle. This means that in case of drop of pressure to the value of saturated vapor P_n a bar of saturated vapor is created while the pressure remains constant. If afterwards the pressure has a tendency to increase, first the vapor is condensed at pressure $P = P_n$, and only after its condensation will the pressure increase. Other working liquids, such as Czech damper oils, do not develop a steam cavern; however, under certain conditions, they can release dissolved air in form of bubbles of different sizes. This is also the case of working liquid becoming lacteous immediately after the beginning of a working cycle.

As a matter of fact, the concentration of air dissolved in oil changes dramatically (especially upon the flow through valves) during the whole working cycle, which comparatively alters the physical parameters of the working medium. Depending on actual operational conditions, especially pressure and temperature, the air in some areas of the damper dissolves and is released from the oil at the same time. It is not yet possible to describe this dynamic process of gradual dissolving and re-release of air form the oil at full length.

For these reasons, the dynamic process of dissolving and releasing air in oil is at least approximately replaced with the assumption of constant level of air concentration stabilized after certain time of the damper operation. A model of damper specified like this works similarly as the original model with the only difference that instead of ideal one-time liquid as the working medium, the two-phase liquid with invariable ratio of oil and dissolved air is used, which resembles the reality much more. Nonetheless, it was not possible to accept the calculation processes and equations derived from the original models without any change, only with specified physical parameters of the mixture instead of pure oil. It was necessary to modify them with regards to retaining the physical principles valid for the intended working medium; in addition, it was necessary to include the description of dynamics (of development and disappearance) of the steam and air cavern.

* prof. RNDr. Jan Šklíba, CSc.: Faculty of Mechanical Engineering, Technical University of Liberec, Studentská 2; 461 17, Liberec; CZ, e-mail: jan.skliba@tul.cz

** RNDr. Rudolf Svoboda, CSc.: Techlab s.r.o.; Sokolovská 207; 190 00, Praha 9; CZ, e-mail: svobodaru@techlab.cz

2. Two-phase working liquid with constant air concentration in oil

As it has already been stated, the real working liquid (oil) always contains certain amount of air, either in form of bubbles, or air dissolved in liquid. As the amount and form of air contained in oil changes the physical qualities of the working medium fundamentally, it is not possible to neglect this fact during the construction of the dasher model (Šklíba, 2007). In equations of the model of dasher, compressibility plays a crucial role. While the compressibility of mixture depends mainly on the volume of free air contained in oil in form of bubbles, air dissolved in oil (i.e. air bound to the molecules of oil) changes the compressibility only minimally.

Unfortunately, the ratio of free and bound air in oil does not remain stable, but it changes dynamically during the whole working cycle. Depending on actual operational conditions, namely pressure and temperature, the air keeps dissolving as well as releasing from oil continuously in areas of damper. The most significant changes are located in areas with the biggest pressure and temperature gradients, especially when flowing through valves and other constricting elements. That means that the concentration of air is time- and space-dependent within individual areas of the damper, and it is different in each area of the damper.

In case of a mixture with non-zero concentrations of free air, the equilibrium of volume flows cannot be used to deduce the total of volume flow. The mixture of oil and air is compressible, the density depends on the pressure, and the incoming volumetric stream flowing through a constricting element to the area with lower pressure is increasing to outcoming stream (the bubbles contained in oil expand in an area with lower pressure). Therefore, it is necessary to replace the equilibrium of volume flows with the condition of equilibrium of mass flows, which now expresses the principle of matter conservation:

„The sum of all mass flows between areas of the damper is equal to zero“. To be specific, it means that we deduct the mass flow flowing from area y to area k form the total mass of medium contained in area j-X, and we add the same value to the mass of medium i area k. As for the oil without air bubbles, i.e. with zero concentration $K_m = 0$, the condition of equilibrium of mass flows is equivalent to the condition of equilibrium of volume flows, and the equation of equilibrium operating with mass flows of mixture Q_m can be replaced by the original equation of a damper with oil filling.

The present standard dynamic model of hydraulic damper deals with pressures concentrated into three regions (regions above the piston, under the piston and the accumulator). This model consists of systems of differential equations describing equations of motion of valves, dynamic balance of flows in the damper, evolving and dissolving of columns of saturated vapors and algebraic equations describing dependence of discharge coefficients of flows on corresponding Reynolds numbers.

The precise model of hydraulic damper deals with pressures concentrated into seven regions. In this model the flows through valves are modeled more precisely and pressure gradients in supply channels are respected for all four valves.

For both standard and precise models the flows among regions of the damper are supposed to be one-dimensional, the liquid is compressible, in the state of unsaturated solution (it does not contain free air), viscose. The columns of liquid are described by concentric parameters.

3. Dynamic model of the double-tube car damper

System of equations of hydraulic damper model (Šklíba, 2006) with pressures concentrated into three regions represents:

a) Dynamic balance of streams in the regions of damper (for three-pressure model)

Let W_i, i =1, 2, 3 designates volume flows and changes of volume caused by movements of piston, outer cylinder and valve (index 1, 2, 3 corresponds to the regions above the piston, under the piston and in the accumulator). For $W_i > 0$ the stream flows into the region resp. volume of the region decreases, for $W_i < 0$ the stream flows out of the region resp. volume of the region grows.

When no column of saturated vapor exists (the region is full of liquid) and the pressure does not fall under the level of saturated vapor pressure P_{np}, it holds

$$C_i \cdot \dot{P}_i = W_i, \qquad \dot{V}_{pi} = 0, \qquad \text{i = 1, 2,} \qquad P_i \geq P_{np} \text{ .and. } V_{pi} = 0. \tag{1}$$

When the pressure falls down the level of saturated vapor pressure or the column of saturated vapor is developed, the pressure remains constant and the volume of column of saturated vapors changes:

$$\dot{P}_i = 0, \qquad \dot{V}_{pi} = -W_i, \qquad \text{i = 1, 2,} \qquad P_i < P_{np} \text{ .or. } V_{pi} > 0. \tag{2}$$

The pressure P_3 in the accumulator is given by equation

$$(C_3 + V_{V3} / p_3) \cdot \dot{P}_3 = W_3, \tag{3}$$

where V_{B3} is volume of the air in the accumulator. Hydraulic capacities C_{ki} are given by relations

$$C_{k1} = -(V_{10} - (S - S_0).y(t)).(E_k + E_{kP} P_1)^{-1} \tag{4}$$

$$\begin{aligned} C_{k2} &= -(V_{20} - S.y(t)).(E_k + E_{kP} P_2)^{-1} \\ C_{k3} &= \ V_{30}.(E_k + E_{kP} P_3)^{-1} \end{aligned} \tag{5}$$

and resulting volume flows W_i by formulas

$$\begin{aligned} W_1 &= Q_{12} - Q_{21} + Q_{13} \ - (S - S_0).\dot{y}(t) - S_{21}\dot{x}_2 + S_{11}\dot{x}_1, \\ W_2 &= -Q_{12} + Q_{21} - Q_{23} + Q_{32} \ + S.\dot{y}(t) - S_3\dot{x}_3 - S_{11}\dot{x}_1 + S_4\dot{x}_4 + S_{21}\dot{x}_2 \\ W_3 &= Q_{23} - Q_{32} + Q_{13} - S_4\dot{x}_4 + S_3\dot{x}_3 - S_a\dot{x}_6 \end{aligned} \tag{6}$$

when the working liquid is a mixture of and oil and of a free air, we introduce V_{Vi} stands for volumes of the free air in the mixture.

$$\begin{aligned} &\left(C_{ki} + \frac{V_{Vi}}{P_i} \right) \dot{P}_i = W_i, \qquad i = 1, 2 \\ &\left(C_{k3} + \frac{V_{V3} + V_{B3}}{P_3} \right) \dot{P}_3 = W_3 \end{aligned} \tag{7}$$

Quantity $Q_{ij}(i \neq j)$ represents the total stream flowing from ***i***-th to ***j***-th region. The resulting flows Q_{ij} are further branched to the single flows through valves, calibrated orifices and parasitic flows through different leaks, for instance

$K\left(P_i - P_j\right)$ parasitic flow through the piston in the working cylinder,

$\alpha_0 K_1 \sqrt{P_1 - P_2}$ or. $\alpha_0' K_1 \sqrt{P_2 - P_1}$ flow through the calibrated valve orifices,

$\alpha_1 k_1 x_1 \sqrt{P_1 - P_2}$ or $\alpha_2 k_2 x_2 \sqrt{P_2 - P_1}$ flow through the valve gaps, and so on.

For instance the resulting flow $\tilde{Q}_{21}$ flowing through the discharge valve is given by relations

$$\begin{aligned} \tilde{Q}_{21} &= 0 \quad , && P_2 \leq P_1 \\ &= K(P_2 - P_1) + \alpha_0' K_1 \ \sqrt{P_2 - P_1} && P_2 > P_1, \quad x_2 < 0 \\ &= K(P_2 - P_1) + (\alpha_0' K_1 + k_2 \alpha_2 x_2)\sqrt{P_2 - P_1}, && P_2 > P_1, \quad 0 \leq x_2 \leq x_{2\alpha} \\ &= K(P_2 - P_1) + (\alpha_0' K_1 + k_2 \alpha_2 x_{2\alpha})\sqrt{P_2 - P_1}, && P_2 > P_1, \quad x_2 > x_{2\alpha} \end{aligned} \tag{8}$$

Similar equations hold for remaining three valves.

b) State equation of the gas in the accumulator

$$V_b = (V_{0a} - V_{30} - F_a x_b) = \Phi(n, P_3, T) \tag{9}$$

c) Dependence of discharge coefficients of flows (Šklíba, 2005, 2004) through valves α_i (i=1,...,4) and calibrated piston orifices α_5, α_6 on corresponding Reynolds numbers

$$\alpha_i = \alpha_{i00} + \alpha_{i0}(1 - \exp(\alpha_{1i} \operatorname{Re}_i)), \quad \text{where} \quad \operatorname{Re}_i = \frac{2Q_{vi}}{\pi d_{Hi} \nu} \tag{10}$$

and Q_{Vi} represent flows through valves or calibrated orifices with hydraulic diameter b_{Vi}

d) Dynamic balance of forces on valves (see Figs. 1 and 2)

$$m_k(\ddot{x}_k - \ddot{y}(t))\cdot(-1)^{k+1} + \delta_k \dot{x}_k + c_k(x_k + x_{kP}) + \tilde{Z}_k = (F_{k1}P_1 - F_{k2}P_2)\cdot(-1)^{k+1} \tag{11}$$

$$\tilde{Z}_k = \tilde{Z}_k(x_k, \dot{x}_k, \ddot{x}_k, \alpha_k, (P_1 - P_2)\cdot(-1)^{k+1}, L_k)\,, \qquad k = 1,2\,,\ x_k \geq 0 \tag{12}$$

$$\tilde{Z}_k = \delta_{0k}\dot{x}_k + c_k^* x_k\,, \qquad k = 1,2\,,\ x_k < 0 \tag{13}$$

$$m_k \ddot{x}_k + \delta_k \dot{x}_k + c_k(x_k + x_{kP}) + \tilde{Z}_k = (F_{k1}P_3 - F_{k2}P_2)\cdot(-1)^{k+1} \tag{14}$$

$$\tilde{Z}_k = \tilde{Z}_k(x_k, \dot{x}_k, \ddot{x}_k, \alpha_k, (P_3 - P_2)\cdot(-1)^{k+1}, L_k)\,, \qquad k=3,4\,,\ x_k \geq 0 \tag{15}$$

$$\tilde{Z}_k = \delta_{0k}\dot{x}_k + c_k^* x_k\,, \quad m_k = m_{kv} + 0.33 * m_{prk} + \rho S_k L_k\,, \qquad k = 3,4\,,\ x_k < 0 \tag{16}$$

where $\tilde{Z}_i$, $i = 1,\dots,4$ represent the hydrodynamic forces in case of open valve and the resistance valve seat forces in case of closed valve. Starting from the general relation and substituting relation for the space flow through the throttled valve slot we obtain the following expression for the hydrodynamic force (often called the Bernoulli force too) which acts on the plate valve (other types of valves are usually not used for hydraulic dampers - see Šklíba, 2004)

$$\tilde{Z} = \alpha^2 \sum_{i=1}^{2} \sigma_{i0} x^i . \Delta P + \alpha \sum_{i=0}^{1} \gamma_{i0} x^i (\Delta P)^{\frac{1}{2}} . \dot{x} + \rho S L \ddot{x} \tag{17}$$

where α is discharge flow coefficient, x is valve stroke, S sign front area of the valve. The first member in the relation represents the stationary force component acting as a non-linear reverse spring, the second represents a non-stationary component acting as a non-linear damping force and the third member represents inertia forces of the mass of liquid included in so called 'damping space' – this mass is added to the mass of the valve and to the reduced mass of the valve spring.

The standard dynamic model of hydraulic damper consists therefore of four non-linear differential equations of the second order and five non-linear differential equations of the first order, completed with seven algebraic equations. The system is of thirteenth order.

Differential equations for each valve, which describe pressure and flow through the supply channels, are added to the precise model. The state vector of the precise model consists therefore of twenty one components.

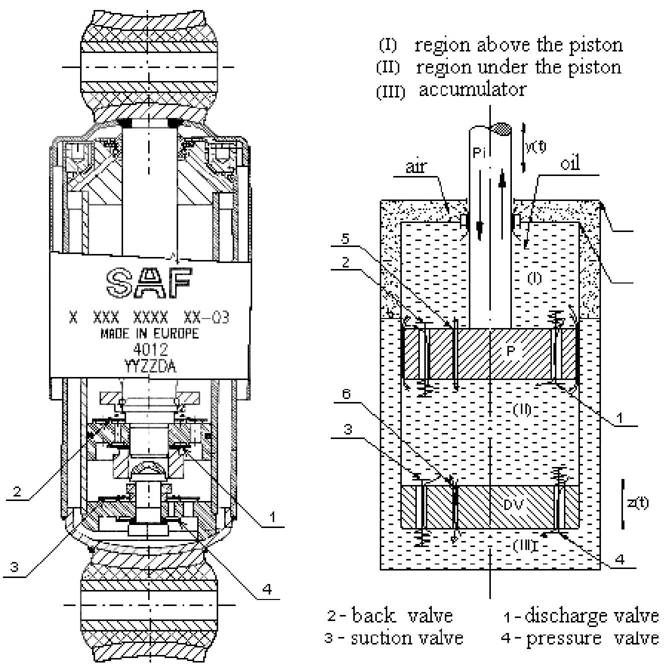

Fig. 1: Scheme of two-cylinder hydraulic damper.

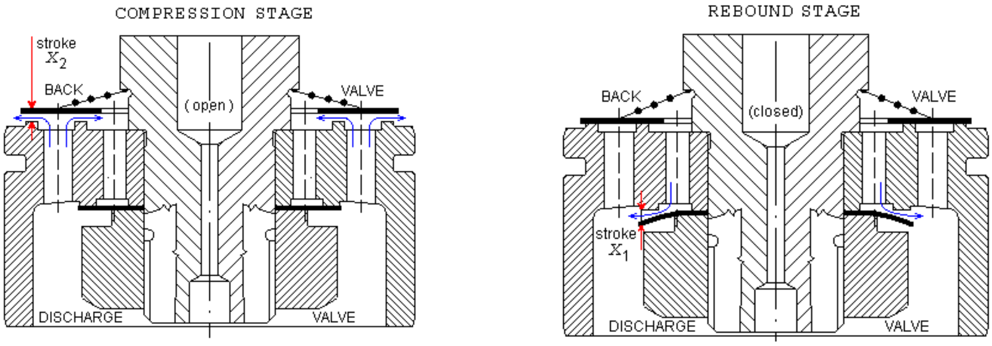

Fig. 2: Function of the valves in the piston – compression (left) and rebound (right).

As it has been stated above, we imagine the mixture in the areas above and under the piston as formally divided in liquid and gas component. The mass concentration of the mixture K_m ,remains the same, so there will be the same mass ratio of oil and air in areas above and under the piston regardless of the current values of pressure and volume of the mixture; the only thing that changes in dependence on pressure is the ratio of oil and air volumes in these areas. The situation in an accumulator is rather more difficult, as there is not only a stable bar of free air (that is considered neither to dissolve in oil nor to pass from other areas of the damper), but also a bar of bound air in the mixture, which has already been in the accumulator, or which has entered through two valves in the bottom of the cylinder.

During the working cycle both the oil volumes and the volumes of air contained in the mixture change dynamically in areas above and under the piston. As for the damper with oil filling, the change of pressures was calculated from the total change of volumes and volume flows and from the hydraulic capacity of oil in a given area. However, we must also take into account the changes of air volumes in the area for the damper with mixture of oil. The pressures in areas above and under the piston will therefore be determined by the equations where the term in the denominator represents a sort of „hydraulic capacity" of the mixture in the given area.

The relief and discharge valve are not considered as separate components as in the quasistatic model; they are rather considered as compound hydraulic components created by the combination of parallel and serial intake canals, calibrated orifices, and a volume of the pre-stressed springs, that is the elementary hydraulic components forming the valve set between two areas of the damper.

4. Issues of stability of the dynamic model of the damper

Fig. 3 shows calculated characteristics of one valve set of a damper with different (initial) volume concentrations of free air in oil. It is evident that with increasing concentration of free air, the rate of the power growth (initially jump) is decreasing significantly in both areas of transitions of the compression and expansion phases. At higher concentrations, however, the characteristics appear to be influenced negatively by the self-excited oscillation.

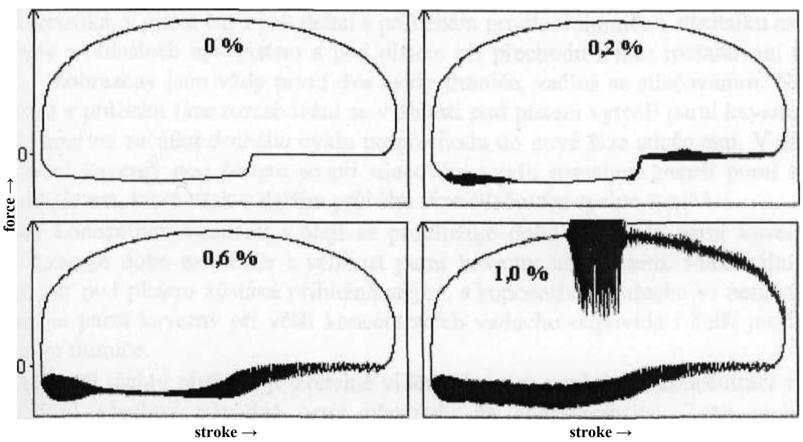

Fig. 3: Characteristics of the damper at different volume concentrations of air in oil.

There can be three kinds of origin of the self-excited oscillation (Šklíba, 2005) on the calculated characteristics:

– It is a fictitious self-excited oscillation caused by faulty numerical integration of the damper equations; then, it is necessary to find a more suitable integration method.
– Self-excited oscillation is a feature of the mathematical model of the damper, but in fact it does not happen; then, it is necessary to adjust/generalize the model of the damper.
– Self-excited oscillation really happens in an existing damper.

Along with numerical integration (and independently on it), complex proper values of linearized system of equations of the damper are calculated (real part of the proper value represents the damping, imaginary part indicates natural frequency).

As it is known from the theory of differential equations, the system is stable if all proper values are damped, i.e. if their real parts are negative. If at least one proper value has a positive real part, the system is unstable, and self-excited oscillation will occur with a frequency corresponding to the frequency of the unstable proper value. This situation agrees with the numerical calculations of self-excited oscillation of the damper exactly. The self-excited oscillation starts to evolve/disappear on the characteristics at the moment when the damping of the proper value changes the sign from negative to positive values, respectively from positive to negative. At that, the natural frequency, deducted from the calculated development of self-excited oscillation, matches the calculated natural frequency – imaginary part of the proper value exactly.

The formulas imply high dependence of stable qualities of the valve subsystem on the modulus of elasticity of the working medium. It is not possible to describe the damper as a whole analytically; however, it is reasonable to assume that a quantity that has a significant impact on the properties of all four valves as basic subsystems of the damper will have a vital influence on the dynamics of the damper as a whole. Exactly the same situation occurs at the calculations of a damper with two-phase liquid with constant concentration of air in oil. Because of the volume of free air in oil, the compressibility of the mixture increases sharply, i.e. its modulus of elasticity E. decreases. The result is a dramatic deterioration of the stability properties, the higher is the concentration of air in oil, the worse are the stability properties of the model. The damping effects of the flows from the valves motion are not sufficient to compensate for the negative impact of significantly higher compressibility of oil with free air added.

Data for "whistling" dampers, in which the self-excited oscillation occurs during operation, could not be gained, so for this reason, there is no comparison with the calculation. So far, all previous experience with calculations of K50 damper (see Figs. 4a and 4b) and dampers of other types suggests that the origin of calculated self-excited oscillation is to be found in the imperfections of the existing dynamic model of damper, especially in the description (model) of the real working fluid mixed with air.

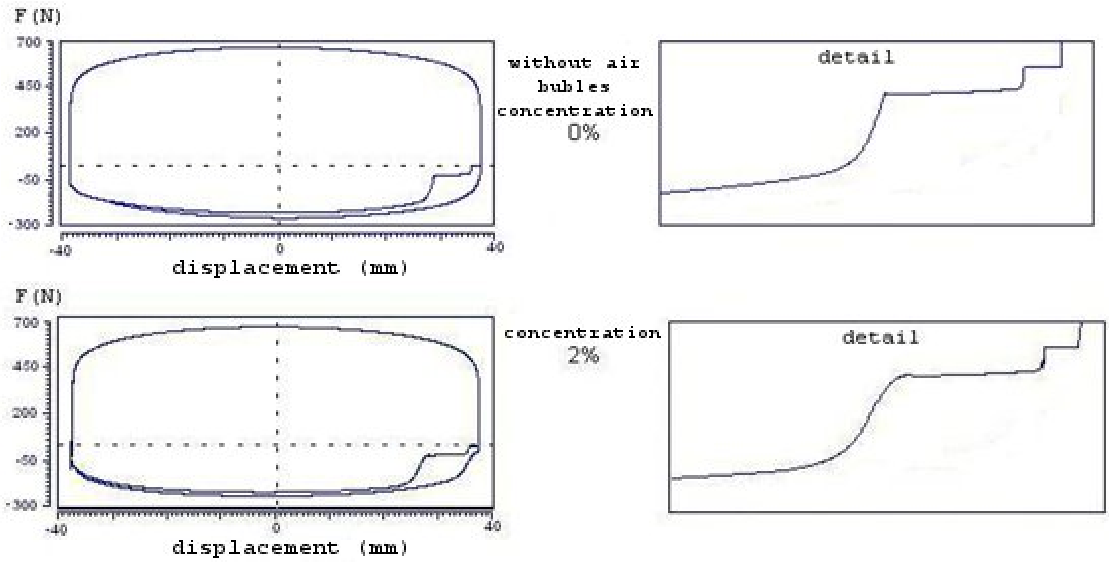

Fig 4a: The influence of the concentration of free air in the working liquid.

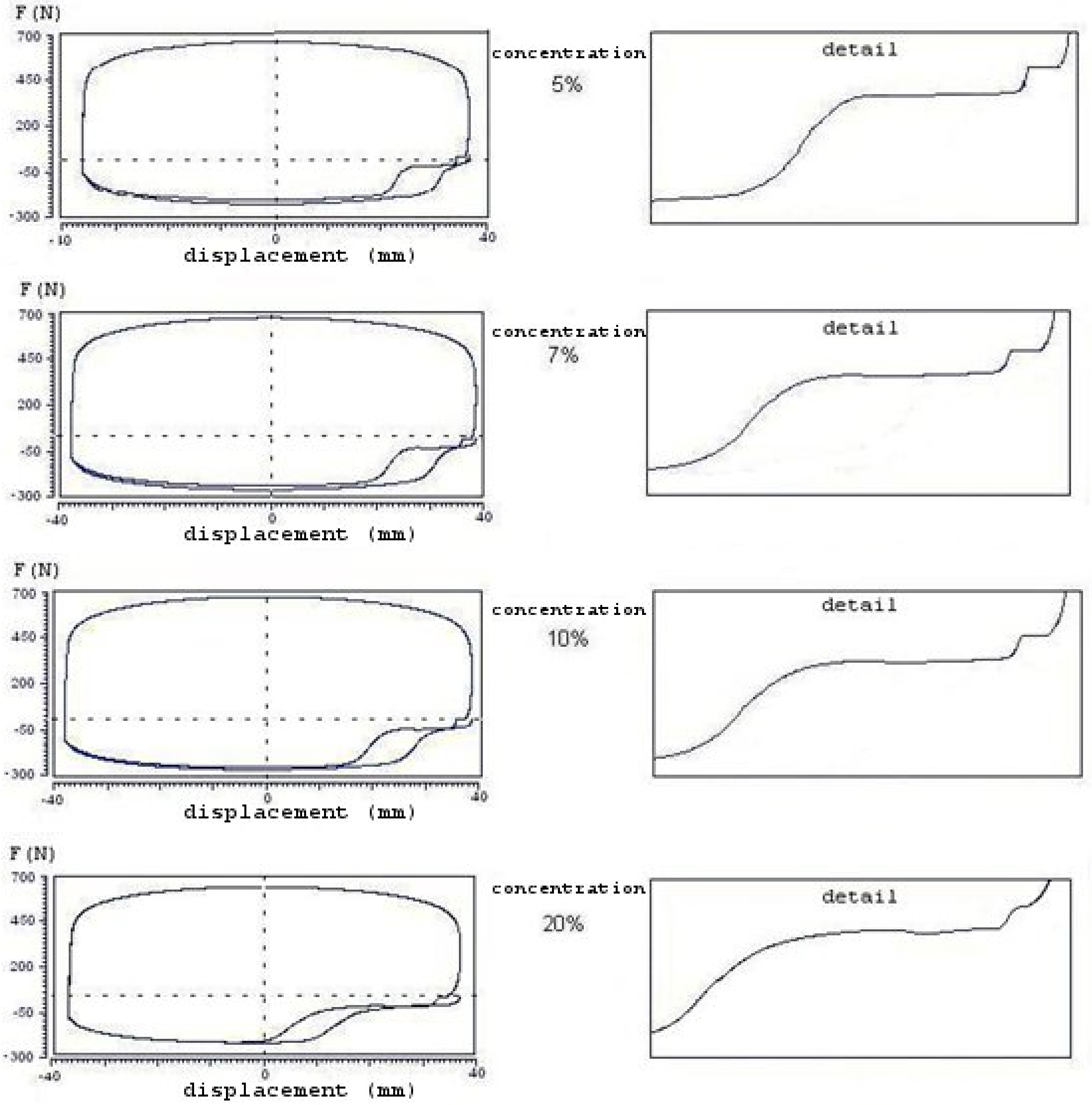

Fig. 4b: The influence of the concentration of free air in the working liquid.

5. Preparation of the model with a variable concentration of free air

The research will cover either the theoretic or the experimental part: the researchers' workplace is equipped with a system capable to detect either cavitation cores or dynamic impacts of cavitation directly. Cavitation is detected by ABS (Acoustic bubble spectrometer), which is an acoustic apparatus permitting by means of two piezoelectric converters to evaluate the quantity and size of cavitation bubbles in the volume between the two converters.

Apertures situated on the low-pressure part also allow to detect cavitation optically. The lighting is realized through ND-Yag continuous laser. The chamber is also adapted to install a PVDF hydrophone, which permits the detection of acoustic manifestations.

The results from measuring the quantity and size of bubbles at various work cycles (the rate of flow, the pressure at the entrance and the setting of the choke element will be modified) will be handed over to the Institute of Thermomechanics of the Academy of Sciences of the Czech Republic.

The expansion and compression velocity are then the guiding parameters capable to determine the real elastic and viscose properties of such mixture. The results will be handed over to Technical University of Liberec, where at the end; they will be included into a mathematic model of a damper.

As first approach the state behaviour may be interpolated as a mixture of liquid substances (oils) at the equilibrium with properties resembling oily foams. Then to depict the meta-steady behaviour of air saturated oil is necessary to know its state behaviour at the equilibrium. To describe this interaction, chemical potential would be the primary dimension. Considering that a real liquid (oil) is taken in account, it will be necessary to know the activity, or the fugacity coefficients. These coefficients put real measured values and concentration in relation to values of pressures-fugacity as well as concentration-activity to define the chemical potential. The so defined chemical potential may be utilized for serious thermodynamic analysis. The fugacity and activity are mainly influenced by gases solubility, i.e. a precise value of Henry's constant and liquid surface tension. Utilizing the 1st transition theory of phase taking into account the high dependence of Henry's constant to temperature and the surface tension influenced by temperature and chemical composition (the so called Gibbs isotherm) it is possible to formulate the thermo-physical properties of two-phase mixtures in meta-stable state either.

The given project assumes that to measure gases solubility (precisely air) in hydraulic oils, in the range of at least 10 to 80°C and pressure between 0.1 – 5 MPa, equipment should be assembled. Such device is assumed to be more or less universal and with subtle modifications would allow measurements based on saturation or extraction principles, to determine the amount of dissolved gas at constant pressure as well as volume. It is also assumed that in the future this apparatus allows studying gases solubility in new materials such as ion liquids, which properties (thermal stability, non-flammability, and volatility) might be potentially suitable as hydraulic liquids. The results from measuring steady states, relevant Henry's constants, coefficients of activity and fugacity, elaborated in Institute of Chemical Technology Prague will be handed over to the Institute of Thermomechanics of the Academy of Sciences of the Czech Republic.

6. Conclusions

From the analyze of the executed models it follows that the prepared model with variable concentration of free air in the working liquid represents a qualitative improvement. The introduction of global and local temperatures will be necessary. This model will be multidisciplinary and its composition claims a cooperation of several research workplaces.

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