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APPROXIMATION OF VELOCITY DISTRIBUTION IN OPEN CHANNEL FLOW

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Summary: *The paper deals with approximation of spatial distribution of mean downstream velocity of sub-critical turbulent flow in an open rectangular channel with rough bed. New formulae of the local downstream velocity component* $V_x(y, z)$ *dependence on the vertical coordinate y and lateral coordinate z were introduced. The vertical velocity profile was approximated by log-law and the power law was used to describe the effect of channel walls on the horizontal velocity profile. The suggested formulae of spatial downstream velocity component distribution were calibrated and verified using the experimental data obtained by PIV method. The proposed approximation is in good agreement with experimental data, the difference between experimental and calculated data is in range of 5 - 10% except the area near the channel's corners and water level, where the maximal difference reaches 60% and 30% of experimental values, respectively.*

Introduction

A velocity distribution of one-dimensional flow in open channels is traditionally described by a log-law or by a power law. The log-law and the power law can be used not only for vertical velocity distribution, but also for a lateral distribution of local velocity.

In this paper we shall suppose that the spatial velocity distribution can be approximated by the log-law for the vertical velocity profile and by the power law for the horizontal velocity profile and for the description of the channel walls effect.

A logarithmic law and power-law (see, e.g. Schlichting, 1979):

$$u = \frac{u_*}{\kappa} \ln\left(\frac{y}{y_0}\right), \tag{1}$$

$$\frac{u}{u_{\max}} = \left(\frac{y}{\delta}\right)^b \tag{2}$$

are usually used for one-dimensional turbulent sub-critical flow in the wide channel, i.e. for the flow where the effect of channel walls can be neglected. In Eqs. (1) and (2) u is local horizontal velocity in downstream direction, u_* is shear velocity, $\kappa = 0.41$ is Kármán

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constant, y is vertical coordinate, $y_0 = 0.11\nu / u_* + 0.033 k_s$ is a distance from the wall, where u = 0 (sometimes called also hydraulic roughness length), v is kinematical viscosity, k_s is hydraulic roughness of the channel bed, δ is the distance, where velocity is maximum u_{max}, b is the exponent of power law. A value b changes from 1/6 to 1/10 depending on Reynolds number. For hydraulically smooth turbulent flow regime b = 1/7 is generally used. Based on these equations we suggest the below mentioned equations of spatial distribution of downstream velocity in the rectangular open channel.

The first suggested equation contains the log-law (Eq. (1)) and power law (Eq. (2)), the first for the description of vertical, the second for the horizontal velocity distribution

$$u = \frac{u_*}{\kappa} \ln\left(\frac{y}{y_0}\right)\left(1 - \left|\frac{z}{z_{\max}}\right|\right)^b , \quad z \in \left[-z_{\max}, z_{\max}\right], \tag{3}$$

where $z_{max} = B/2$ is half of channel width, z is lateral coordinate, and z = 0 is the centre of the channel, the parameter $b = b\ (Re)$ is a function of Reynolds number.

Cheng (2007) suggests the relationship $b = b\ (Re)$ for pipe flow

$$b = 1/(1.37 f^{-0.43}), \tag{4}$$

where $f = 0.316 Re^{-0.25}$ is Blasius friction factor, $Re = V_{av} D / v$ is Reynolds number, D is the pipe diameter, and V_{av} is average velocity.

Similarly Zagarola et al. (1997) determined the relationship $b = b\ (Re)$ for the pipe flow

$$b = \frac{1.085}{\ln Re} + \frac{6.535}{(\ln Re)^2}. \tag{5}$$

Both above mentioned dependences $b = b\ (Re)$ were developed for the pipe flow. We shall try to use them also for the open channel flow, however we shall define Reynolds number $Re = R_h V_{av} / v$, i.e. based on hydraulic radius R_h instead pipe diameter D.

The description of the velocity distribution in horizontal direction by Eq. (3) has one weakness - the Eq. (3) is not smooth in the centre of channel.

The second equation for the local downstream velocity distribution in open channel, which in accordance with the theory of turbulent flow is smooth in the centre of the channel, is

$$V_x(y,z) = \frac{u_*}{\kappa} \ln\left(\frac{y}{y_0}\right)\left(1 - \left|\frac{z}{z_{\max}}\right|^c\right), \quad z \in \left[-z_{\max}, z_{\max}\right]. \tag{6}$$

We assume that parameter c is also a function of Reynolds number, i.e. $c = c\ (Re)$ and its value can be evaluated from the evaluation of experimental data.

Unfortunately, Eq. (6) does not fit well the experimental points in the central part of the channel. It predicts significantly higher values than Eq. (3) and the experimentally determined velocity vertical and horizontal profiles, see Figures. (1) and (2). In order to decrease the difference between experimental and predicted values a quadratic term was subtracted from the equation Eq. (6)

$$V_x(y,z)=\frac{u_*}{\kappa}\ln\left(\frac{y}{y_0}\right)\left(1-\left|\frac{z}{z_{\max}}\right|^c\right)-a\left(\frac{z}{z_{\max}}\right)^2,\quad z\in\left[-z_{\max},z_{\max}\right] \tag{7}$$

The Eq. (7) is also smooth in the central part of the channel; the constant *a*, can be determined experimentally.

Velocity profile approximation

The measuring of the flow velocity in the rectangular channel with rough bed was conducted by the Particle Image Velocimetry (PIV) method. The local velocity values were measured only in the left half of the channel, since we supposed a symmetrical distribution of flow velocity in both channel's parts. A flow regime was a backwater.

The hydraulic parameters of the water flow in the channel were following: width of the channel was B = 0.25 m, depth of water in the channel was $h = 5.94\cdot 10^{-2}$ m, slope of the channel was S = 0.001, channel's ratio width/depth was B/h = 4.3, bed roughness was $k_s = 0.8\cdot 10^{-3}$ m, flow rate was Q = 4 l/s, water temperature was about 20°C, kinematical viscosity of water at 20°C was ν = $1.307\cdot 10^{-6}$ m²/s. Hydraulic radius of the channel was $R_h = \frac{Bh}{2h+B} = 4.03\cdot 10^{-2}$ m and average velocity V_{av} = $Q/(Bh)$ = 0.269 m/s, the Reynolds number was $Re = R_h V_{av}/\nu$ = 10800.

The vertical profiles of velocity are presented in Figure 1. By fitting log-law by least-squares method the values of shear velocity $u_* = (1.69 \pm 0.01)\cdot 10^{-2}$ m/s, and hydraulic roughness length $y_0 = 3.3\cdot 10^{-5}$ m were determined.

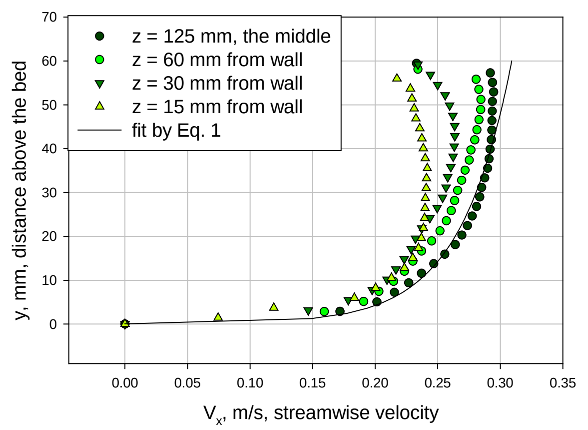

Figure 1. Vertical velocity profiles at different distances from the wall and fit by Eq. (1) with $u_* = 1.69\cdot 10^{-2}$ m/s, and $y_0 = 3.3\cdot 10^{-5}$ m.

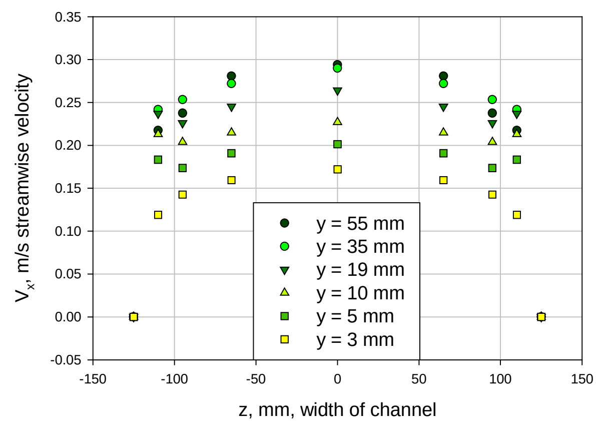

Figure 2. Horizontal velocity profiles at different horizontal planes.

Experimental horizontal velocity profiles at different levels are presented in Figure 2. Since we have at this time available only one complex measurement of the spatial velocity distribution in the channel under given conditions, we chose the measured points in the horizontal plane $y = 35$ mm (i.e. about 60% of the water depth) for calibration of the suggested Eqs. (3), (6), and (7). The remaining experimental data, i.e. measurement in horizontal planes at level $y = 55$ mm, 44 mm, and 26 mm, respectively, were used for the verifications. The least-squares method fits of experimental velocity values at the depth $y = 35$ mm by Eqs. (3), (6), and (7) are presented in Figure 3. We obtained values of the exponents $b = 0.082$, $c = 13.12$, and $a = 0.05$ mm.

The value of exponent b calculated according to Eqs. (4) and (5) is b = 0.164 and b = 0.193, respectively; the value of b obtained by the fit of experimental data is about two times less then those determined by calculation according to Eqs. (4) and (5).

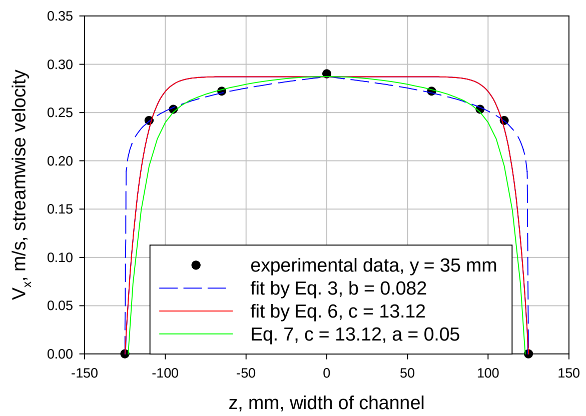

Figure 3. Experimental values of velocity and their fits by Eqs. (3), (6), and (7).

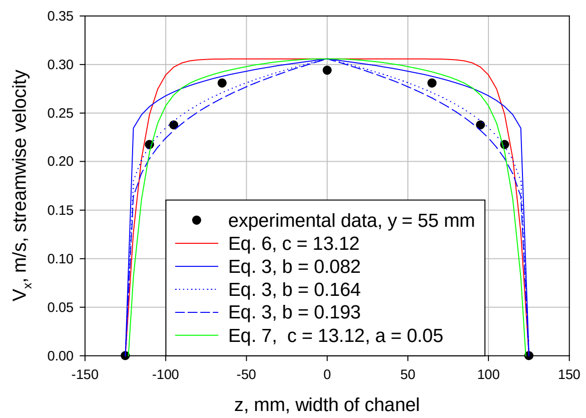

Figure 4. Velocity calculated according to Eqs. (3), (6), and (7).

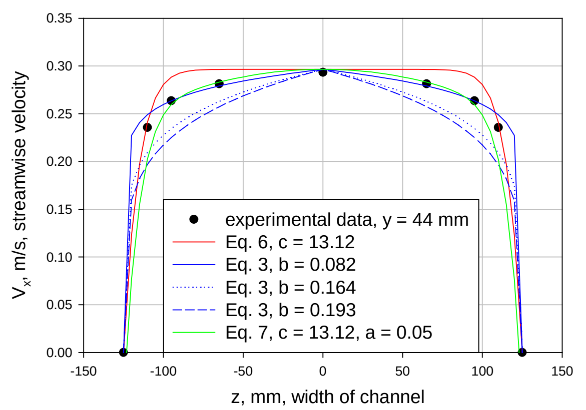

Figure 5. Velocity calculated according to Eqs. (3), (6), and (7).

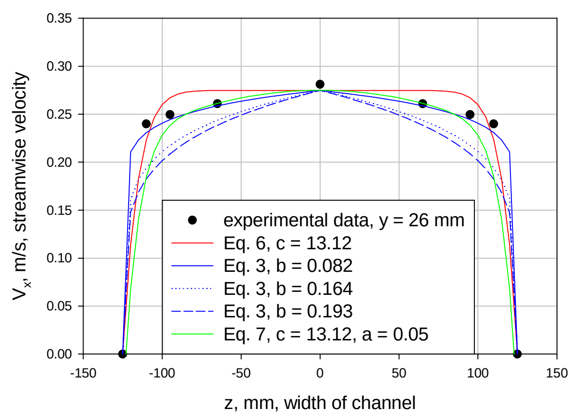

Figure 6. Velocity calculated according to Eqs. (3), (6), and (7).

Later the following values of exponents of Eqs. (3), (6), and (7), i.e.c = 13.12, b = 0.082, b = 0.164, and b = 0.193, and a = 0.05 will be used, and the horizontal velocity profile at various flow depth will be constructed. In Figures 4, 5, 6 the experimental values of the local velocity at $y = 55$ mm, $y = 44$ mm, and $y = 26$ mm and their approximation by Eqs. (3), (6), and (7) are illustrated.

From the Figures 4, 5, 6 it follows that the approximation of the velocity spatial distribution by Eq. (3) in horizontal plane at the level $y = 55$ mm (for calculated values of parameter b = 0.164, $b = 0.193$) agrees well with experimental data, the maximal difference is 4% and 6.6%, respectively. When the parameters of Eqs. (3), (6), and (7) are determined by fit of experimental data (b = 0.082, c = 13.12, and c = 13.12, $a = 0.05$), the maximal difference increases to 18%, 25%, and 13%, respectively. However, in the horizontal plane at the level y = 44 mm the approximation with parameter (b = 0.082) agrees well with experimental data, maximal difference is less than 6%, for the remaining values of the exponents ($b = 0.164$, $b = 0.193$, c = 13.12, and $c = 13.12$, $a = 0.05$) the maximal difference increases to 11%, 16%, 9.4%, and 14%, respectively. In the horizontal plane at the level y = 26 mm the approximation of the velocity distribution with parameters (b = 0.082, c = 13.12) agrees very well with experimental data, the maximal difference is 4% and 7%, respectively, for the approximation with parameters ($b = 0.164$, $b = 0.193$, and $c = 13.12$, $a = 0.05$) the maximal difference increases up to 19%, 24%, and 23%, respectively.

The following conclusion follows from the foregoing description: Eq. (3) with exponent $b = 0.082$, Eq. (6) with exponent c = 13.12, and Eq. (7) with exponent c = 13.12 and parameter $a = 0.05$ can be used for the description of spatial distribution of the local downstream velocity in open channel. Unfortunately, the curves with exponents calculated according to Eqs. (4) and (5), i.e. $b = 0.164$, $b = 0.193$ lie far from experimental points, and so they cannot be used for the description of spatial distribution of velocity.

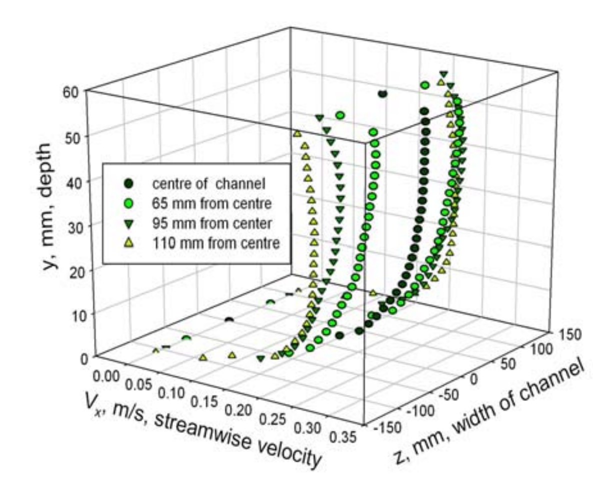

Figure 7. Experimental vertical profiles of the downstream velocity in open channel.

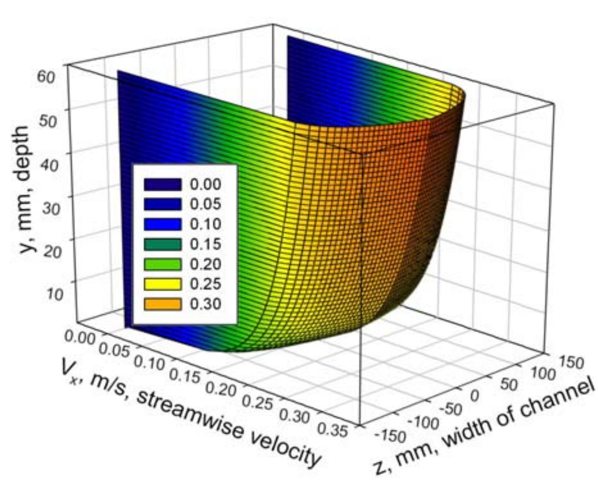

Figure 8. Approximation of the downstream velocity distribution by Eq. (3), $b = 0.082$.

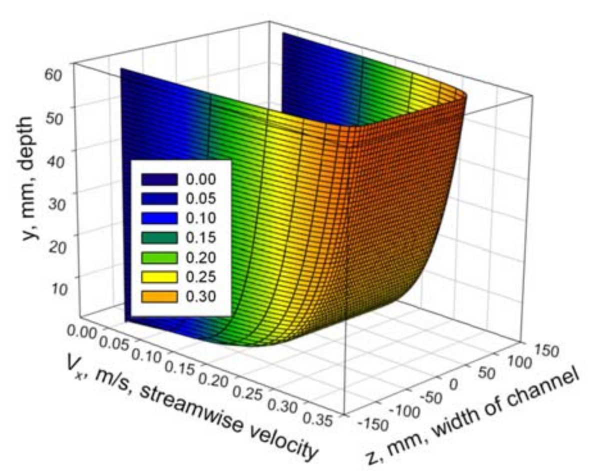

Figure 9. Approximation of the downstream velocity distribution by Eq. (6), $c = 13.12$.

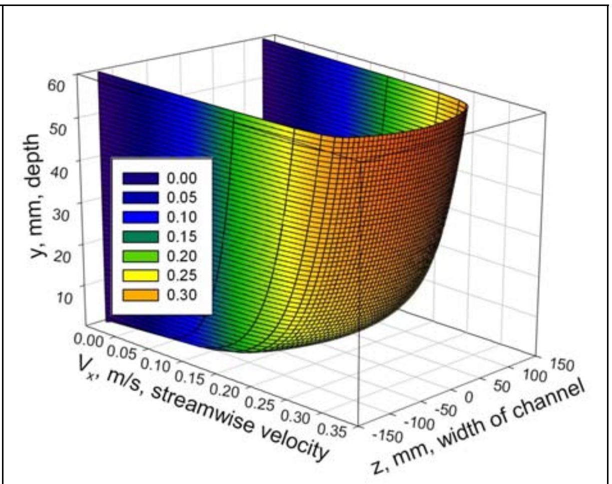

Figure 10. Approximation of the downstream velocity distribution by Eq. (7), $c = 13.12$, $a = 0.05$.

The spatial experimental profile and profile approximated by Eqs. (3), (6), and (7) are presented in the Figures 7, 8, 9, 10.

The isolines of differences between the experimental values V_{exp} and approximation of local downstream velocity V_{calc} (i.e. the values $100.(V_{calc} - V_{exp})/ V_{exp}$, [%]), are presented in Figure 11 for approximation according to Eqs. (3), (6), and (7).

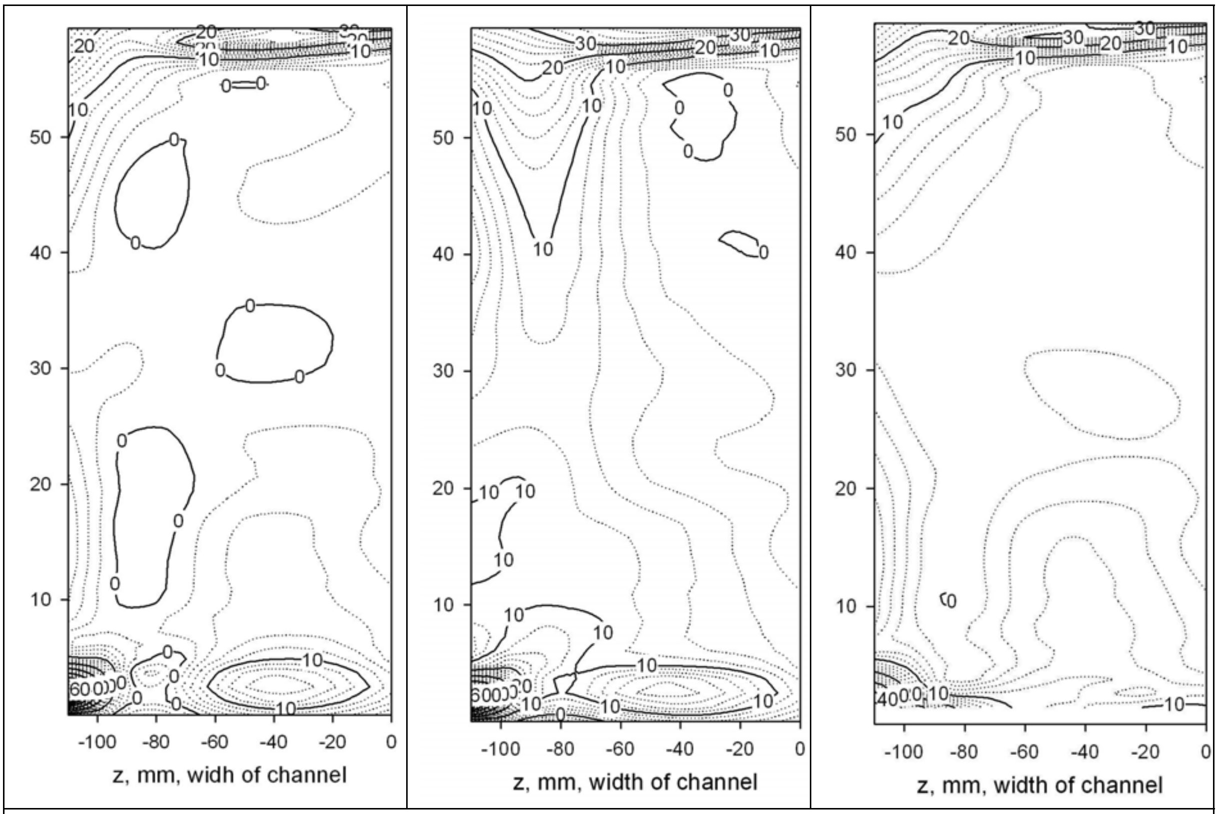

Figure 11. The difference of the experimental and approximated velocity values - left Eq. (3) $b = 0.082$; in the middle Eq. (6), $c = 13.12$; right - Eq. (7), $c = 13.12$, $a = 0.05$ (in %)

In the case of Eq. (6) the difference reaches 30% near the water-level and maximum 60% near the corners of channel; in the centre of channel (up to $z \approx 60$ mm) the difference is less than 5 %. In the case of Eq. (3) the difference is less than 5% even near the walls, except the region close to the channel bed and water level. In the case of equation (7) the difference in central part of the channel is less than 5%, near the water-level the differences reach about 20% (maximum 30%) similarly as for the other approximations. Near the corners of the channel the differences reach up to 40%, while for approximation by Eqs. (3) and (6) the difference reaches up to 60%. The considerable less values of the experimental local velocity in the corners is probably due to the retarding effect of the walls in the corners of channel.

It can be concluded that Eq. (7) described best the velocity profile in the open channel, if the parameters a and c were evaluated from the experimental data.

Conclusions

For the description of spatial downstream velocity distribution in open channel a function combined from a log-law (for the vertical velocity distribution) and a power law (for horizontal velocity distribution) was used.

Analysis of the local downstream velocity approximation by Eqs. (3), (6), and (7) shows that all three equations give a relatively good agreement with experiments if their parameters are determined experimentally. The difference between the experimental and calculated data is less than 5 % in the central part of the channel, in channel's corners and close to water level the differences increase up to 30 - 60%, respectively, probably due to the retarding effect of the walls.

Approximation of the downstream velocity profile by Eq. (7), which is the modified Eq. (6) with an added quadratic term, approximate very well the velocity distribution, and on the contrary to Eq. (3), it is smooth in centre of the channel.

The dependence of parameter c (in Eqs. (6) and (7)) on Reynolds number should be determined experimentally. The dependence of parameter b (Eqs. (4) and (5)) on Reynolds number was developed for the pipes flow only, they should be determined for the open channels to obtain more precise values. Also dependence of parameters c and a of Eqs. (6) and (7) on Reynolds number should be proved and determined experimentally. However, more experimental data will be necessary to determine the general relationship for the mentioned parameters.

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