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MODELLING OF THE TURBINE GENERATORS

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Summary: *The paper deals with the approach to modelling and optimization of turbine generators whose bodies have primarily non-symetrical geometry and stiffnesses in two perpendicular planes. To equalize these stiffnesses it is necessary to perform so called Lafoon's slits to the generator body. Using conclusions from fracture mechanics we can simulate rotor behaviour respecting slits and determine their optimal depth.*

1. Introduction

The different stiffnesses of the turbine generator body in two perpedicular planes paralell with generator axis can cause the parametric resonances and than the rise of unstability. The very frequently used way to avoid such unpleasant phenomena presents Lafoon's slits. Meaning of this slits can be understood as an instrument for proper stiffness decreasing in corresponding plane and equalizing both stiffnesses. Scheme of one turbine generator is depicted in fig. 1

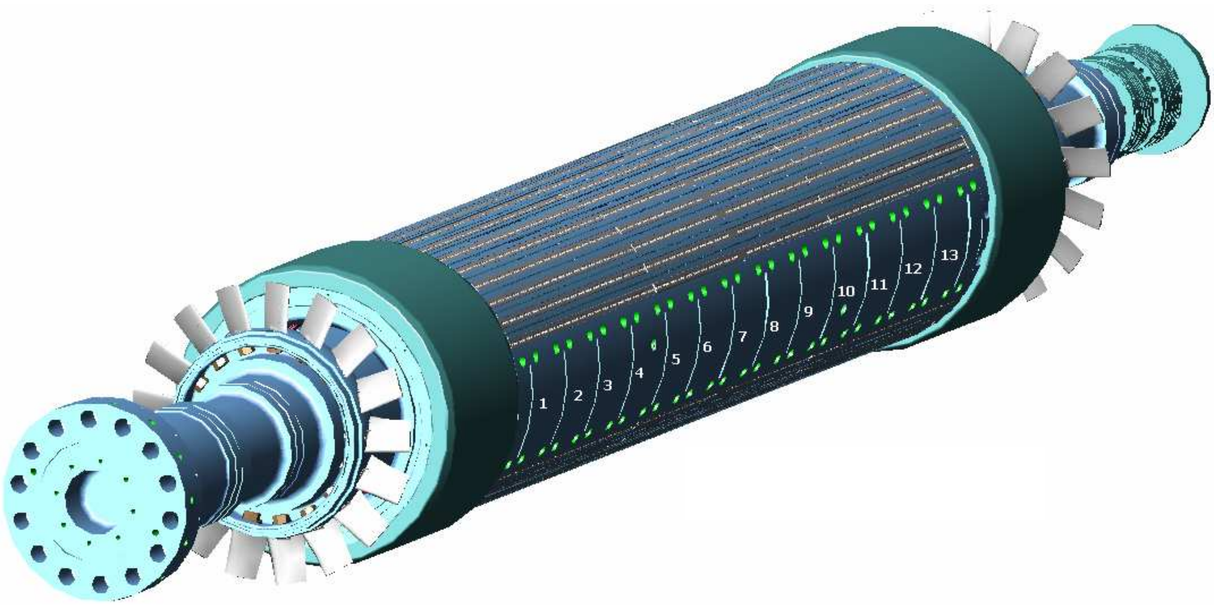

Figure 1. Scheme of the turbine generator

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2. Modelling of Lafoon's slits by means of crack finite element

The scheme of the cracked finite element is depicted in fig. 2

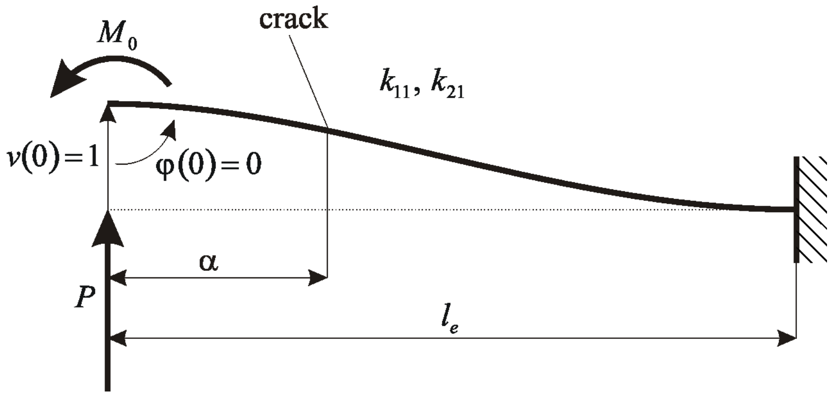

Figure 2. Scheme of cracked finite element

One of the possibilities to obtain the stiffness matrix of the element uses the stiffness influence coefficients. Deformation potential energy of the intact element can be expressed in form

$$U_0 = \frac{1}{2E}\int_0^l \frac{M^2(\xi)}{I_\zeta(\xi)}\,\mathrm{d}\xi, \qquad (1)$$

where $M(\xi)$ is bending moment and $I_\xi(\xi)$ means cross-sectional moment of inertia. The relation expressing energy opening crack can be obtained from the fracture mechanics experience in form (Kuruc 2008, Dupal 2001)

$$U_{prid} = \frac{4h}{E\pi}\frac{M^2(\alpha)}{\left(S_\zeta^2 - AI_\zeta\right)^2}\int_{\tilde{A}}\left(S_\zeta - \eta A\right)^2 d\tilde{A} = \frac{4h}{E\pi}\frac{M^2(\alpha)}{\left(S_\zeta^2 - AI_\zeta\right)^2}\left(S_\zeta^2\tilde{A} - 2S_\zeta\tilde{S}_\zeta A + \tilde{I}_\zeta A^2\right), \qquad (2)$$

where S_ζ, I_ζ and A means moment of area, cross-sectional moment of inertia and area, respectively. Analogous quantities corresponding to the crack area are marked by tilda.

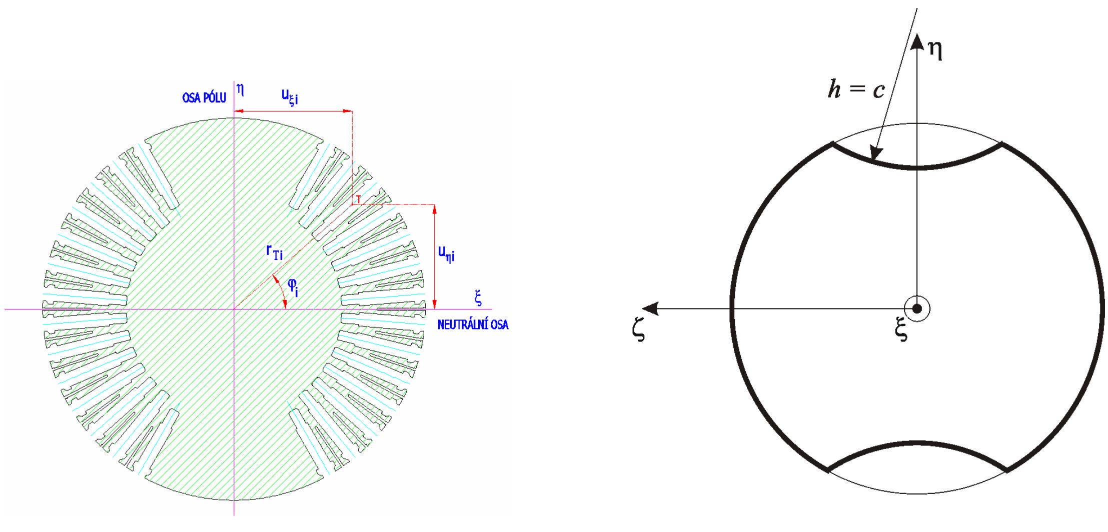

Figure 3. Cross-section of generator

Using Castigliano's rule e.g.

$$v_i = \frac{\partial U_0}{\partial P_i} + \frac{\partial U_{prid}}{\partial P_i} = \frac{1}{EI_\zeta}\int_0^l M(\xi)\frac{\partial M(\xi)}{\partial P_i}d\xi + 2kM(\alpha)\frac{\partial M(\alpha)}{\partial P_i}, \tag{3}$$

we can come to the time dependent periodical stiffness matrix of rotor finite element in form

$$\mathbf{K}_e^{CR}(t) = \mathbf{T}^T(t)\begin{bmatrix} \mathbf{K}_\zeta & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}\mathbf{K}_\eta\mathbf{P} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}\mathbf{T}(t) + \mathbf{K}_T \tag{4}$$

where $\mathbf{T}(t)$ is transformation matrix, $\mathbf{K}_\zeta$ and $\mathbf{K}_\eta$ are stiffness matrices expressed in rotating coordinate system $\xi\eta\zeta$ corresponding to the bending in plane $\xi\eta$ and $\xi\zeta$, respectively. The matrix $\mathbf{K}_T$ is coordinate invariant part corresponding to longitudinal and torsional deformations. The matrix $\mathbf{K}_e^{CR}(t)$ corresponds to the order of generalized coordinates as follows:

$$\mathbf{q}_e(t) = \begin{bmatrix} \mathbf{q}_1(t) \\ \mathbf{q}_2(t) \\ \mathbf{q}_3(t) \\ \mathbf{q}_4(t) \end{bmatrix}, \quad \mathbf{q}_1(t) = \begin{bmatrix} v_1(t) \\ \psi_1(t) \\ v_2(t) \\ \psi_2(t) \end{bmatrix}, \quad \mathbf{q}_2(t) = \begin{bmatrix} w_1(t) \\ \vartheta_1(t) \\ w_2(t) \\ \vartheta_2(t) \end{bmatrix}, \quad \mathbf{q}_3(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \quad \mathbf{q}_4(t) = \begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \end{bmatrix}. \tag{5}$$

The order of element displacements should be finally rearranged. Meaning of the individual symbols follows from the fig. 4

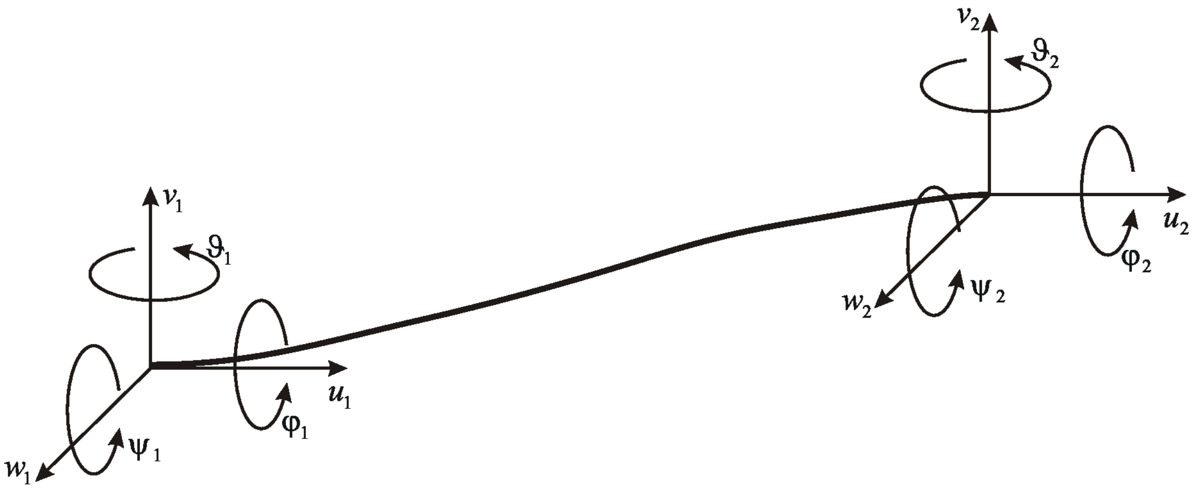

Figure 4. Element generalized displacements

3. Equation of motion

The special crack finite element matrices were developed for modelling of slits. Supposing the permanently open cracks we can come to the linear equation of motion with periodically time dependent matrices (1)

$$\begin{gathered}\mathbf{T}^T(t)\mathbf{M}\mathbf{T}(t)\ddot{\mathbf{q}}(t) + \left[\mathbf{G} + \mathbf{T}^T(t)\eta_V\mathbf{K}\,\mathbf{T}(t) + \mathbf{B}_S\right]\dot{\mathbf{q}}(t) + \\ + \left[\mathbf{K}_S + \mathbf{T}^T(t)(\eta_V\mathbf{K}\boldsymbol{\Omega} + \mathbf{K})\mathbf{T}(t)\right]\mathbf{q}(t) = \mathbf{f}(t).\end{gathered} \tag{1}$$

Individual matrices have meaning as follows: $\mathbf{T}(t) \in \mathbf{R}^{n,n}$ -transformation matrix, $\mathbf{T}^T(t)\mathbf{M}\mathbf{T}(t) \in \mathbf{R}^{n,n}$ -mass matrix, $\mathbf{G} \in \mathbf{R}^{n,n}$ -gyroscopic matrix, $\mathbf{T}^T(t)\eta_V \mathbf{K}\,\mathbf{T}(t) \in \mathbf{R}^{n,n}$ -matrix of proportinal radial damping, $\mathbf{T}^T(t)\eta_V \mathbf{K}\boldsymbol{\Omega}\,\mathbf{T}(t) \in \mathbf{R}^{n,n}$ -matrix of proportinal tangential damping, $\mathbf{K}_S$, $\mathbf{B}_S \in \mathbf{R}^{n,n}$ -stationary part of stiffness matrix and damping matrix, respectively, $\mathbf{T}^T(t)\mathbf{K}\,\mathbf{T}(t) \in \mathbf{R}^{n,n}$ -stiffness matrix.

The Floquet's theory can be used for stability assessment. We can firstly rewrite the eq. (1) without excitation into the brief form

$$\tilde{\mathbf{M}}(t)\ddot{\mathbf{q}}(t) + \tilde{\mathbf{B}}(t)\dot{\mathbf{q}}(t) + \tilde{\mathbf{K}}(t)\mathbf{q}(t) = \mathbf{0}, \tag{2}$$

where meaning of matrices in (2) follows from the comparison (1) and (2). Adding the identity

$$\tilde{\mathbf{M}}(t)\dot{\mathbf{q}}(t) - \tilde{\mathbf{M}}(t)\dot{\mathbf{q}}(t) = \mathbf{0} \tag{3}$$

to (2) we can transfer the original system of the ordinary differencial equations (ODE) of the second order to system of the first order in form

$$\mathbf{N}(t)\dot{\mathbf{x}}(t) - \mathbf{P}(t)\mathbf{x}(t) = \mathbf{0}. \tag{4}$$

The matrices in the last equation have form

$$\mathbf{N}(t) = \begin{bmatrix} \mathbf{B}(t) & \mathbf{M}(t) \\ \mathbf{M}(t) & \mathbf{0} \end{bmatrix}, \mathbf{P}(t) = \begin{bmatrix} -\mathbf{K}(t) & \mathbf{0} \\ \mathbf{0} & \mathbf{M}(t) \end{bmatrix}, \mathbf{x}(t) = \begin{bmatrix} \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) \end{bmatrix}. \tag{5}$$

Assuming regularity of $\mathbf{N}(t)$ we can come to

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t), \tag{6}$$

where

$$\mathbf{A}(t) = \mathbf{N}^{-1}(t)\mathbf{P}(t) \in \mathbf{R}^{2n,2n}. \tag{7}$$

The matrix $\mathbf{A}(t) = \mathbf{A}(t+T)$ is time periodical whose period is $T = 2\pi/\omega$ and ω is angular speed of generator. The measure of instability can be expressed by means of monodromy matrix eigenvalues. In case all the eigenvalues lie inside the unit circle in complex plane (included boundary) the system is stable and vice versa. In case of at least one of eigenvalues lying outside this circle, the system is unstable. Let introduce the fundamental matrix of solution starting from independent initial conditions (e.g. identity matrix)

$$\mathbf{X}(t) = [\mathbf{x}_1(t), \mathbf{x}_2(t), \ldots, \mathbf{x}_{2n}(t)]. \tag{8}$$

Monodromy matrix corresponds to the fundamental matrix expressed in time T. It means to solve the eq.

$$\dot{\mathbf{X}}(t) = \mathbf{A}(t)\mathbf{X}(t), \quad \mathbf{X}(0) = \mathbf{I}. \tag{9}$$

As a numerical example we can choose the generator of Riga turbine. The model of generator is depicted in fig. 5. Lafoon's slits are modelled by cracked finite elements. The dependence of maximal absolute value of the monodromy matrix eigenvalue on angular speed of revolution and depth of slits is depicted in fig. 6. The place corresponding to the unit absolute value of monodromy matrix eigenvalue for all angular speed values presents optimal depth of slits.

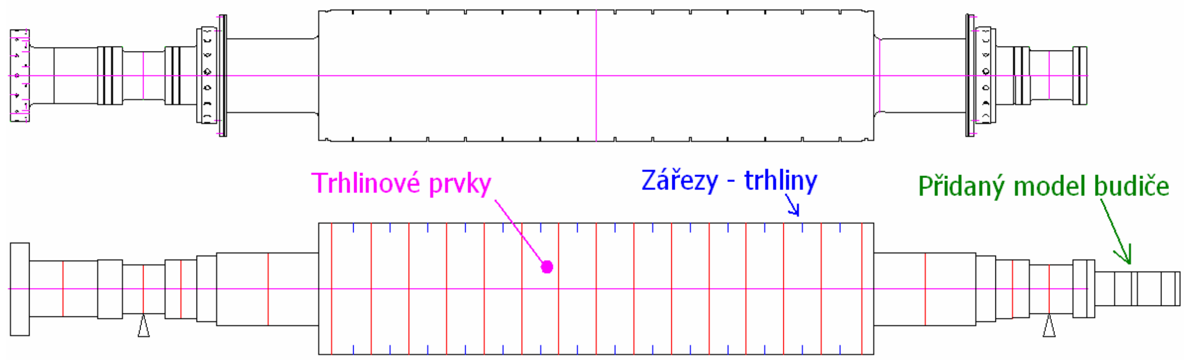

Figure 5. Model of generator Riga

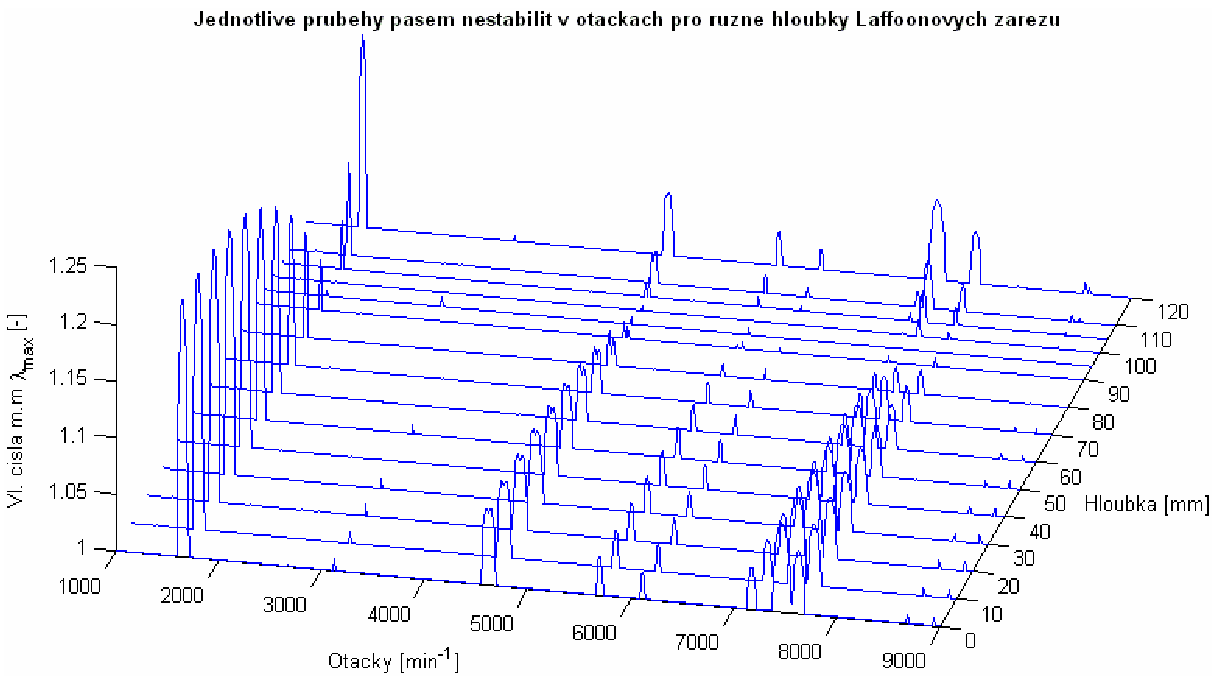

Figure 6. Dependence of the bands of instability of the Riga rotor on the depth of slits

4. Slit depth determination

One of the ways to the slit depth determination could be understood as an optimization process whose objective function will express measure of the difference between eigenfrequencies corresponding to the mode shapes of vibration in two perpendicular planes. The objective function can have form

$$\psi(s) = \boldsymbol{\Delta}^T \boldsymbol{\lambda}\,\mathbf{G}\,\boldsymbol{\Delta\lambda}, \qquad (10)$$

where the vector $\boldsymbol{\Delta\lambda}$ can be written down e.g. in these two ways

$$\mathbf{\Delta\lambda} = \begin{bmatrix} \Omega_{1\eta} - \Omega_{1\zeta} \\ \Omega_{2\eta} - \Omega_{2\zeta} \\ \Omega_{3\eta} - \Omega_{3\zeta} \end{bmatrix} \quad \text{or} \quad \mathbf{\Delta\lambda} = \begin{bmatrix} \mathrm{Im}\{\lambda_{1\eta}\} - \mathrm{Im}\{\lambda_{1\zeta}\} \\ \mathrm{Im}\{\lambda_{2\eta}\} - \mathrm{Im}\{\lambda_{2\zeta}\} \\ \mathrm{Im}\{\lambda_{3\eta}\} - \mathrm{Im}\{\lambda_{3\zeta}\} \end{bmatrix}, \tag{11}$$

and the weighting matrix is of diagonal form $\mathbf{G} = diag\{g_1,\ g_2,\ g_3\}$. To obtain the eigenvalues in (11) means to solve eigenproblem eq. (1) for $\mathbf{T}(0) = \mathbf{I}$. This optimization approach was applied retrospectivelly to the five already made up generators by company BRUSH SEM. Our results were written down to the last column of the tab. 1.

Table 1. Experimentally and computationally obtained depths of slits

Rotor	Slit depth [mm]				
	Experiment	M. Balda	C. Hoschl I.	C. Hoschl II.	New approach
RIGA	**104**	134,5	114,4	112,6	96,9
BDAX98	**95**	129,7	109,1	112,6	92,8
DAX7	**67,95**	99,2	86,6	98,7	63,0
GE9A5	**111,5**	114,2	86,1	87,8	87,6
GE7A6	**82,6**	109,9	85,1	97,8	76,7

5. Coclusion

As reader can see our results lie more closely to the experimentally determined depths then the results obtained by the application of three approaches used by two Czech notable experts. In addition these results lie always on the safe side of values. The method of stability assessment was very simplified and accelerated by the modal reduction. Despite of the fact that the system matrices are time dependent the reduction successfully came through by means of modal matrix calculated in the initial time $t = 0$.

6. Acknowledgement

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7. References

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