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KINEMATICAL CALIBRATION OF SLIDING STAR

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Summary: *The paper deals with the different calibration procedures of parallel kinematical structure Sliding Star. The different calibration procedures differ by the number of sensors and considered model of kinematical model of joints. They result into different resulting accuracy of TCP positioning.*

1. Introduction

The mechanisms based on the parallel kinematical structures have many advantages (Valasek 2004). However, one of their problems is the necessity to calibrate them. These machines are designed with certain (nominal) dimensions and are manufactured with certain real dimensions that differ from the nominal dimensions by the always present manufacturing deviations. It is necessary to identify these deviations after the manufacture and enter their values (or directly the values of real dimensions) into computer control system that enables to compensate these deviations during the machine motion and so on to reach its positioning with substantially increased accuracy compared to the usage of only nominal dimensions. The accuracy improvement is routinely by 1-2 orders.

The calibration of serial kinematical structure is relatively simple because particular parts (dimensions) of kinematic chain can be calibrated independently and in sequence subsequently sequentially. Nevertheless with the transition to the 5 axes machining the sequential decomposition of calibration into particular dimensions is very difficult. For parallel kinematical structures the decomposition of calibration into particular dimensions is not possible and it is necessary to calibrate all parts (dimensions) simultaneously. Moreover the dependence of calibration accuracy on the knowledge of real machine dimensions is for parallel kinematic structures much higher than for serial kinematic structures, because for their motion control the computer control system must in real time realize nonlinear kinematic transformation between the platform position and the drive positions.

The previous research has proven that the results of calibration and subsequent accuracy of TCP (Tool Center Position) positioning can be significantly influenced by the structure, dimensions and the placement of the sensors of the kinematical structure of the calibrated mechanism. This property has been called calibrability (Valasek et al. 2007). This paper investigates the influence of the number and placement of sensors for the calibration of parallel kinematical structure Sliding Star.

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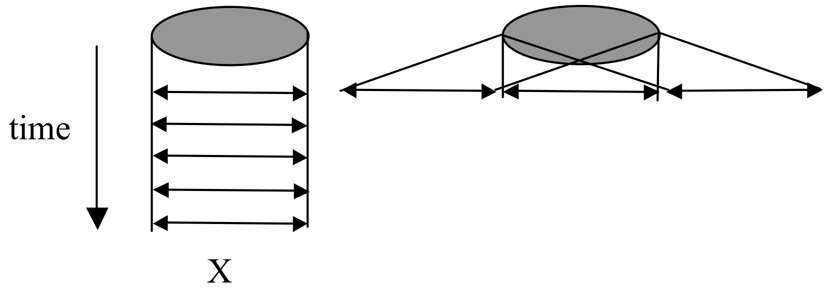
2. Principle of Redundant Measurement

The basic idea of the principle of redundant measurement (Valasek 2004) is simple. If it is necessary to increase the accuracy of a physical measurement then the number of measurements is increased in time (Fig. 1a) and the results are statistically processed. The methods of mechatronics with current cheap electronics enable to realize the same effect by carrying out simultaneously many measurements in one instantaneous time instant (Fig. 1b) and again by subsequent statistical processing. But in the case of parallel kinematic structures it arises an additional effect of limiting the resulting error by intersection of uncertainty intervals. Another view on this calibration process is that all manufactured components and all carried out measurements are always affected by manufacturing imprecision, but if the measurements are redundant then the manufacturing inaccuracies can be determined from the mathematical processing of constraints of overdetermined measurements.



(a) (b)

Fig. 1 Principle of redundant measurement

3. Redundant calibration

The redundant measurement means that the position of the platform is measured by more sensors than it is necessary for its determination. The application of the principle of the redundant measurement for the calibration of parallel kinematical structures is the redundant calibration. The investigated kinematical structures include kinematical loops, at least virtual ones, otherwise they cannot include more measurements than DOFs. The kinematical loops are described by the kinematical constraints (Stejskal & Valasek 1996)

$$\mathbf{f}(\mathbf{d}, \mathbf{s}, \mathbf{v}) = \mathbf{0} \tag{1}$$

where $\mathbf{d}$ are the dimensions of the mechanism, $\mathbf{s}$ are the input (measured) coordinates in the joints and the guides and $\mathbf{v}$ are the output coordinates, i.e. the position of the end-effector. The basic calibration algorithm (Stengele 2002, Petru & Valasek 2004) uses Newton's method modified for overconstrained system of nonlinear algebraic equations (more equations than unknowns) that follow from the constraints (1) formulated for many instances of measurements. If j=1, …, n positions of the kinematical structure are considered (measured) then the constraint equations (1) are coupled into the constraint equations for the calibration

$$\mathbf{F}(\mathbf{d}, \mathbf{S}, \mathbf{V}) = \mathbf{0} \tag{2}$$

where for the position j the constraint $\mathbf{f}_j = \mathbf{f}(\mathbf{d}, \mathbf{s}_j, \mathbf{v}_j) = \mathbf{0}$ from the equation (1) holds and $\mathbf{F} = [\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_n]^T$, $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_n]^T$, $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n]^T$. In traditional (non-redundant) calibration approach the output coordinates $\mathbf{V}$ are measured by external devices. In redundant (self) calibration approach the used constraints (1) do not include the output coordinates $\mathbf{V}$. The equation (2) covers both approaches.

The calibration is based on the fact that the dimensions $\mathbf{d}$ are the same (constant) for all positions. Nevertheless the real values of the manufactured dimensions $\mathbf{d}$ differ from their design values $\bar{\mathbf{d}}$. Thus the only unknown variables in the equation (2) are the manufactured dimensions $\mathbf{d}$. The Newton method of the calibration is derived from the Taylor series of (2)

$$\mathbf{F}(\bar{\mathbf{d}}, \mathbf{S}, \mathbf{V}) + \mathbf{J}_{\mathbf{d}} \delta\, \mathbf{d} + \ldots = \mathbf{0} \tag{3}$$

with Jacobi matrix $\mathbf{J}_{\mathbf{d}}$ of partial derivatives of the kinematical constraints (2) with respect to the calibrated dimensions $\mathbf{d}$. Hence

$$\mathbf{J}_{\mathbf{d}} \delta\, \mathbf{d} = -\mathbf{F}(\bar{\mathbf{d}}, \mathbf{S}, \mathbf{V}) = \delta\, \mathbf{r} \tag{3}$$

and the i-th iteration step of Newton's method (Stengele 2002, Petru & Valasek 2004) is

$$\delta\mathbf{d}_i = (\mathbf{J}_{\mathbf{d}_i}{}^{\mathbf{T}} \mathbf{J}_{\mathbf{d}_i})^{-1} \mathbf{J}_{\mathbf{d}_i}{}^{\mathbf{T}} \delta\mathbf{r}_i \tag{4}$$

where $\mathbf{J}_{\mathbf{d}_i}$ is the Jacobi matrix and $\delta\mathbf{r}_i = -\mathbf{F}(\mathbf{d}_i, \mathbf{S}, \mathbf{V})$ is the vector of deviations computed from measured quantities and calibrated quantities $\mathbf{d}_i$ from the previous step. The new values of the dimensions are then computed

$$\mathbf{d}_{i+1} = \mathbf{d}_i + \delta\mathbf{d}_i \tag{5}$$

and the iterations continue until the deviations are decreasing.

The basic calibration procedure provides us with the unique solution for the given data. This solution is unique for very broad region of initial guesses of parameters of iterative solution by Newton's method. Nevertheless during the practical calibration of different machine tools (Valasek et al. 2005) it has been found out, that the parameters (dimensions of the mechanism) determined from different realizations of calibration measurements vary considerably. The fundamental reason of this phenomenon is an interaction of the inferior conditionality of linear systems solved during the iterations of Newton's method, measurement errors, and errors of model simplifications regarding real machine. Consequently it is very useful to acquire a deeper insight into the relations between the parameter space and the space of the calibration results. Based on that the concept of the calibrability is introduced and the measure of calibrability is defined (Valasek et al. 2007) as

$$C = cond(\mathbf{J}_{\mathbf{d}_i}{}^{\mathbf{T}} \mathbf{J}_{\mathbf{d}_i}) \tag{6}$$

The smaller value of the calibrability C the more accurate determination of unknown real values of the manufactured dimensions $\mathbf{d}$ and the more accurate determination of the output coordinates $\mathbf{v}$ from the input coordinates $\mathbf{s}$, i.e. smaller resulting measurement errors.

4. Sliding Star

Sliding Star is a functional model of hybrid machine tool, i.e. it includes both parallel and serial kinematical structures. The platform $B_1B_2B_3B_4$ with the spindle carries out the planar motion. It has 3 DOFs (x, y, φ_z in Fig. 2) but it is actuated by 4 drives (s_1, s_2, s_3, s_4 in Fig. 2). Therefore it is redundantly actuated parallel kinematical structure as it has more drives than DOFs. The kinematical parameters are in Fig. 2.

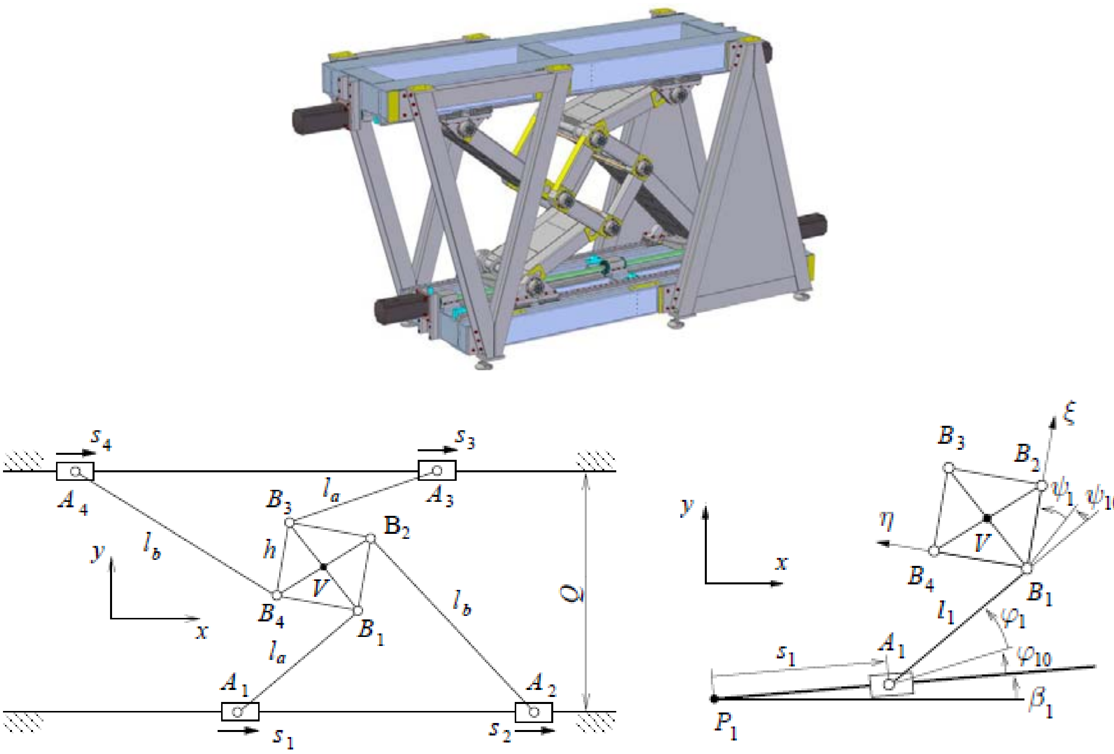


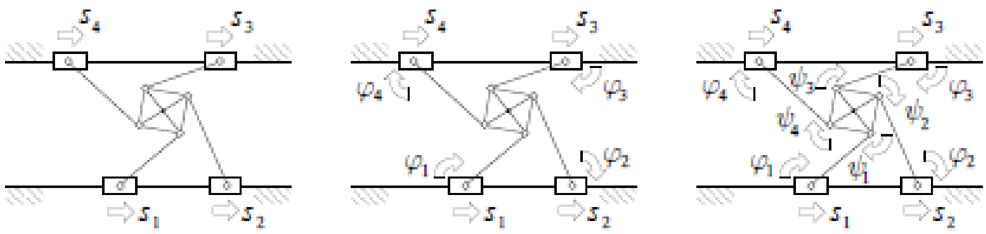
Fig.2 Sliding Star a) 3D model b) Kinematical structure c) Parameters of one leg

5. Calibration of Sliding Star

The basic calibration procedures of Sliding Star have been already investigated in Valasek et al. (2005). However, this paper investigates the influence of the chosen calibration procedure on the resulting accuracy of the positioning TCP.

Three different variants of redundant calibration have been investigated (Valasek et al. 2005). They differ by the number and placement of sensors (Fig. 3) and thus by the number of calibrated dimensions. The data from sensors are relative and therefore the zero positions of sensors belong to the calibrated parameters. The calibrated parameters on one leg are in Fig. 2c. They consist of the coordinates x_{Pi}, y_{Pi} of the initial point of linear measurement, of the angle β_i of the slider and the length l_i of the leg. The platform is determined by 5 calibrated parameters ξ_{B2}, ξ_{B3}, η_{B3}, ξ_{B4}, η_{B4}. The relative position of the spindle ξ_V, η_V with respect to the platform can be only calibrated by an external calibration device.

The first calibration variant (Fig. 3a) includes 4 sensors of the relative position of the carriages s_1, s_2, s_3, s_4 with respect to the frame. The values x_{P1}, y_{P1}, $\beta 1$ must be given. Altogether there are 18 unknown parameters: l_1, x_{P2}, y_{P2}, β_2 , l_2, ξ_{B2}, x_{P3}, y_{P3}, β_3 , l_3, ξ_{B3}, η_{B3}, x_{P4}, y_{P4}, β_4 , l_4, ξ_{B4}, η_{B4}.



(a) (b) (c)

Fig. 3 Schematic description of sensor placement for the variant with 4, 8 and 12 sensors

The second variant includes 8 sensors (Fig. 3b) for the relative position of the carriages s_1, s_2, s_3, s_4 with respect to the frame and for the relative angles of the legs φ_1, φ_2, φ_3, φ_4 with respect to the carriages. Again the values x_{P1}, y_{P1}, $\beta 1$ must be given. The new parameters are the initial values of the measurement of angles φ_{i0}. Altogether there are 22 unknown parameters: l_1, φ_{10}, x_{P2}, y_{P2}, β_2 , l_2, ξ_{B2}, φ_{20}, x_{P3}, y_{P3}, β_3 , l_3, ξ_{B3}, η_{B3}, φ_{30}, x_{P4}, y_{P4}, β_4 , l_4, ξ_{B4}, η_{B4}, φ_{40}.

The third variant includes 12 sensors (Fig. 3c) for the relative position of the carriages s_1, s_2, s_3, s_4 with respect to the frame and for the relative angles of the legs φ_1, φ_2, φ_3, φ_4 with respect to the carriages and for the relative angles of the legs ψ_1, ψ_2, ψ_3, ψ_4 with respect to the platform. Again the values x_{P1}, y_{P1}, $\beta 1$ must be given. The new parameters are the initial values of the measurement of angles ψ_{i0}. Altogether there are 26 unknown parameters: l_1, φ_{10}, ψ_{10}, x_{P2}, y_{P2}, β_2 , l_2, ξ_{B2}, φ_{20}, ψ_{20}, x_{P3}, y_{P3}, β_3 , l_3, ξ_{B3}, η_{B3}, φ_{30}, ψ_{30}, x_{P4}, y_{P4}, β_4 , l_4, ξ_{B4}, η_{B4}, φ_{40}, ψ_{40}.

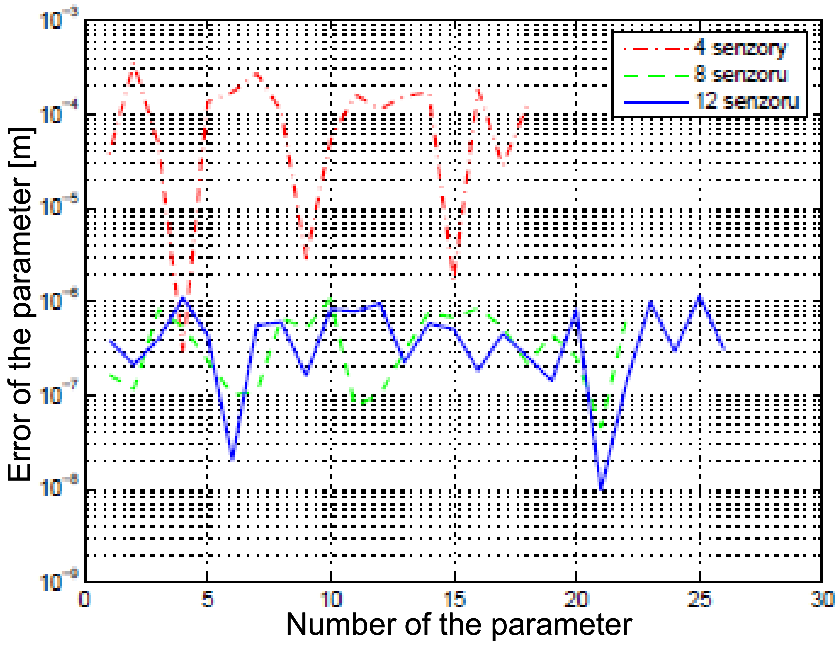


Fig. 4 The deviation of computed and real calibrated parameters

According to the procedure (1-5) the calibration for each variant has been carried out. One evaluation criterion was the deviation of the computed parameters from the real ones. It was also evaluated by the calibrability (6). The other evaluation criterion was the deviation of computed TCP positions V from the real ones.

The calibrability of the first variant was 3.10^8, of the second variant was 3.10^4, of the third variant was 4.10^4. This means that the calibrability is improved by 10^4 by the usage of 8 sensors instead of 4 and the error of parameters decreased by 10^2 (Fig. 4) and also the accuracy of TCP position is improved by 10^2 (Fig. 5-6). Then the influence of the increase of sensors from 8 to 12 was negligible (Fig. 4, Fig. 6-7).

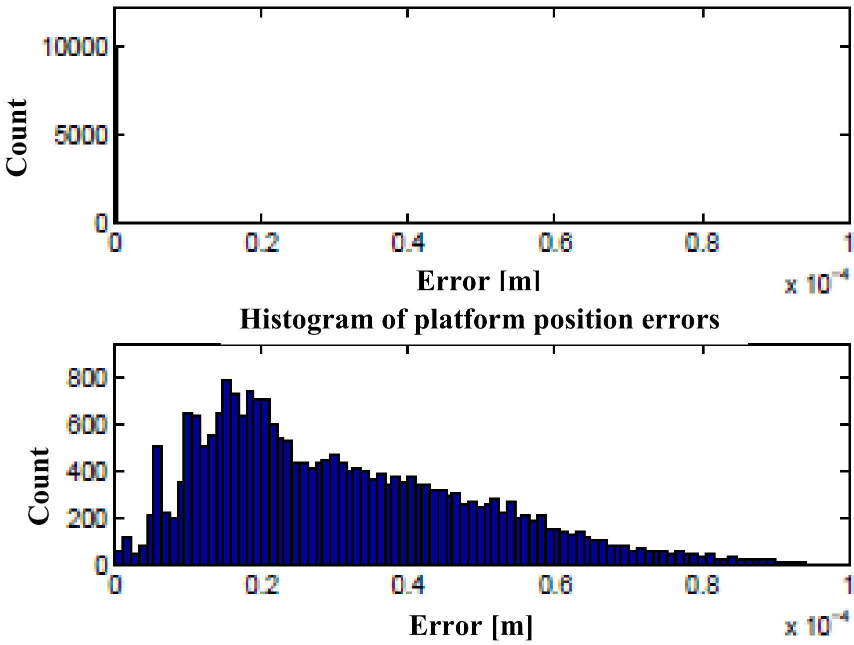


Fig. 5 The deviation of computed and real TCP positions for 4 sensors

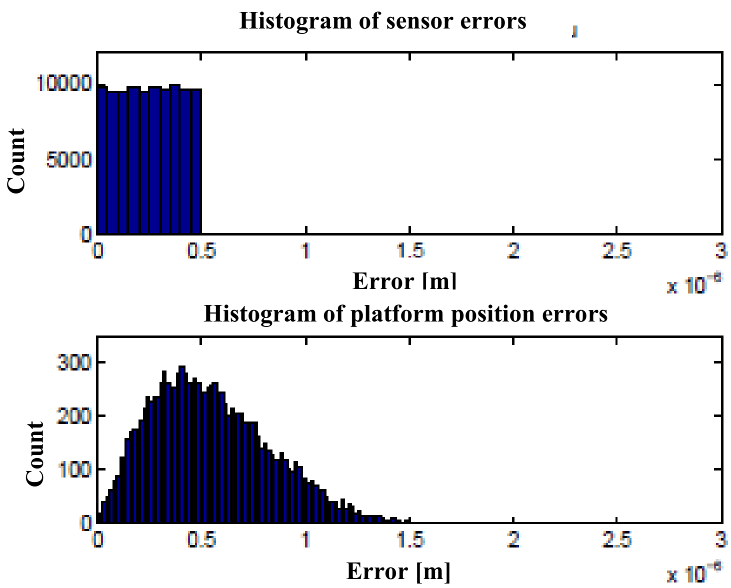


Fig. 6 The deviation of computed and real TCP positions for 8 sensors

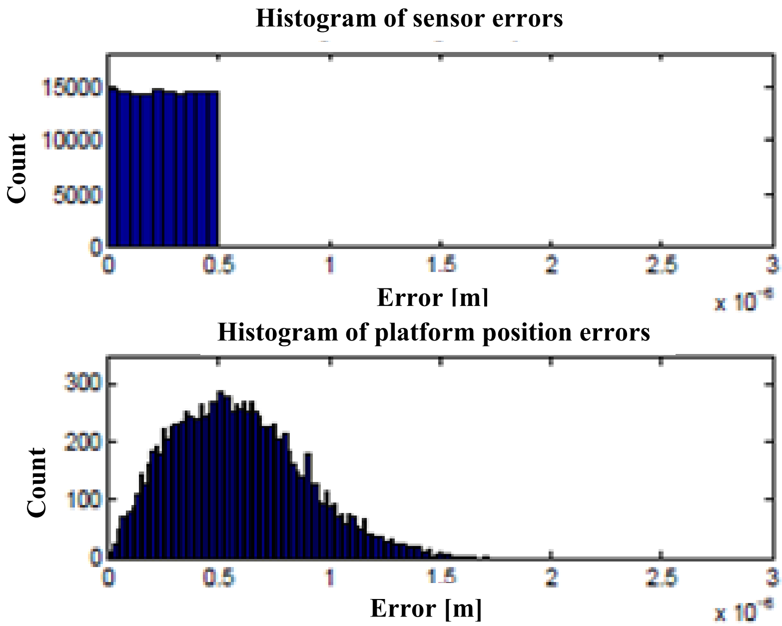


Fig. 7 The deviation of computed and real TCP positions for 12 sensors

6. Conclusion

The increase of redundancy of sensors has very favourable influence on the results of calibration process of parallel kinematical structures. The increase of accuracy of calibration can be easily 10^2 and more. The measure of calibrability can be used for the selection and optimization of calibration procedural variants. These approaches and results were demonstrated on the redundantly actuated parallel kinematical structure Sliding Star.

7. Acknowledgement

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8. References

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