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USAGE OF MECHANISM CALIBRABILITY IN DESIGN OF PARALLEL KINEMATICAL STRUCTURES

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Summary: *The paper deals with the research of the influence of design parameters on calibration of the parallel kinematic machines. The calibrability is introduced as a mechanism property describing the sensitivity of dimension accuracy and end effector positioning accuracy with respect to accuracy of sensor measurements. The optimization of calibrability leads to minimization of the error transfer from sensors to end-effector. The calibrability can be used as a design criterion within the design process of a new redundant or nonredundant parallel kinematical structures as well as for modification of the existing structures. The calibrability has been applied to two practical examples. Firstly the positioning accuracy of the spindle of machine tool Trijoint 900H is improved by modification of selected parameters. Secondly the calibrability is used as a main design criterion of a redundant parallel calibrating and measuring machine RedCaM.*

1. Introduction

Even despite very accurate manufacture of machine tool it is not possible in case of parallel kinematics to use the design dimensions for the nonlinear kinematical transformation in control system. It is necessary to determine the really manufactured dimensions as accurate as possible. In case of parallel kinematics (non-cartesian one) it is not possible to determine the real dimensions by direct measurement therefore these dimensions must be computed from some indirect measurements (Stengele, 2002). This process can be generally extended also to stiffness parameters of the system (Ecorchard, Neugebauer, Maurine, 2006).

The accuracy of the resulting positioning is influenced by the machine structure, its geometrical parameters, the positioning of sensors, the calibration pose set and the measuring accuracy of the sensors. Therefore several observability indices have been introduced (Wampler, Hollerbach, Arai, 1995), (Hollerbach, Wampler, 1996). These indices are usually used for the optimization of measurement pose sets in order to improve the accuracy of the calibrated parameters (Zhuang, Yan, Masory, 1998).

This paper deals with the concept of the calibrability that is an observability index. This index is however used for the optimization of the machine tool design (the machine structure, its geometrical parameters, the positioning of sensors) in order to improve the ultimate positioning accuracy of machine end-effector. Therefore the calibrability is becoming an important design criterion.

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2. Calibration of machines

The basic algorithm of the calibration problem formulation will be ilustrated by calibration of the machine tool Trijoint 900H (Fig. 1). Horizontal machine centre TRIJOINT 900 H is a machine tool of hybrid concept (Valášek, Čížek, Petrů, 2002) developed in cooperation of KOVOSVIT MAS Inc. Sezimovo Ustí and Department of Mechanics, Faculty of Mechanical Engineering CTU in Prague.

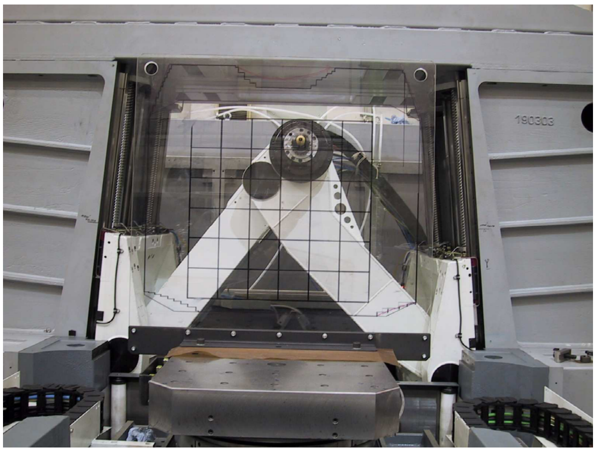
2.1. Basic algorithm of machine calibration

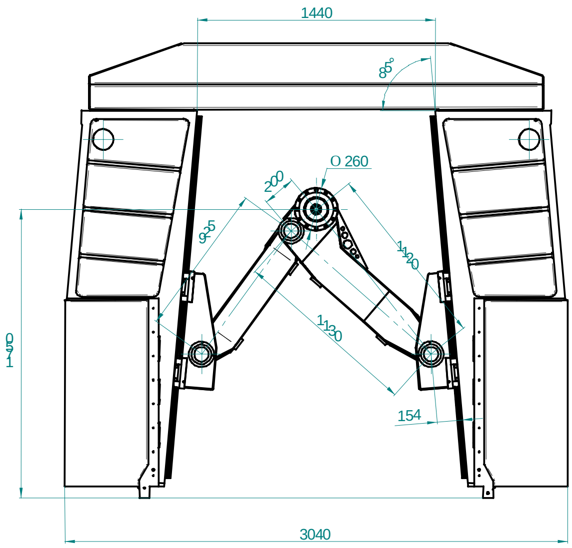
The kinematic transformation between the coordinates of drive (the positions of carriages $s_{12} = s_{12}$ (t), $s_{15} = s_{15}$ (t)), the dimensions of the mechanism d=[x_{1P2}, y_{1P2}, x_{1P5}, y_{1P5}, β_2, β_5, l_3, l_4, x_{4V}, y_{4V}] and the positions of the cutting tool on the machine platform ($x_V = x_V$ (t), $y_V = y_V$ (t)) (Fig. 2).

$$[x_{1V}, y_{1V}] = \mathbf{f}_{KT}(s_{12}, s_{15}, \mathbf{d}) \tag{1}$$

It is the direct kinematical solution of the mechanism. In the case of TRIJOINT it is solvable in closed analytical form. The algorithm of non-redundant calibration is based on the solution of equation (1) for the unknown dimensions d on the basis of measurements of positions of cutting tool V by an external device and simultaneous measurement of drive coordinates s_{12}, s_{15}.

$$\mathbf{d} = \mathbf{f}_{KT}^{-1}(s_{12}, s_{15}, x_{1V}, y_{1V}) \tag{2}$$





a) mechanism of cutting tool part b) design scheme

Fig. 1 Machine tool TRIJOINT 900H

The proper calibration algorithm (Stengele, 2002) uses Newton's or modified Newton's procedure modified for overconstrained system of nonlinear algebraic equations (more equations than unknowns) describing the relationship between the unknown dimensions **d** and the measurements **v** of the cutting tool V positions

$$\mathbf{f}(\mathbf{d}) = \mathbf{v}$$
$$\mathbf{J}\delta\,\mathbf{d} = \mathbf{v} - \mathbf{f}(\mathbf{d}) = \delta\,\mathbf{v} \tag{3}$$

In the ***i-th*** iteration step of Newton's method there are computed the following dimension corrections

$$\delta\mathbf{d}_i = (\mathbf{J}_i^{\mathbf{T}}\mathbf{J}_i)^{-1}\mathbf{J}_i^{\mathbf{T}}\delta\mathbf{v}_i\,, \tag{4}$$

where $\mathbf{J}_i$ is the Jacobi matrix of partial derivatives of kinematical transformation (1),(2),(3) with respect to the calibrated dimensions **d** and $\delta\mathbf{v}_i$ is the vector of deviations of positions of cutting tool V computed and measured. The convergence has been improved by the usage of symbolically computed partial derivatives. The new values of dimensions are then computed as

$$\mathbf{d}_{i+1} = \mathbf{d}_i + \delta\mathbf{d}_i. \tag{5}$$

Based on them there are computed new values $\delta\mathbf{d}_{i+1}$ and $\mathbf{J}_{i+1}$ and iterations continues until the deviations are being decreased.

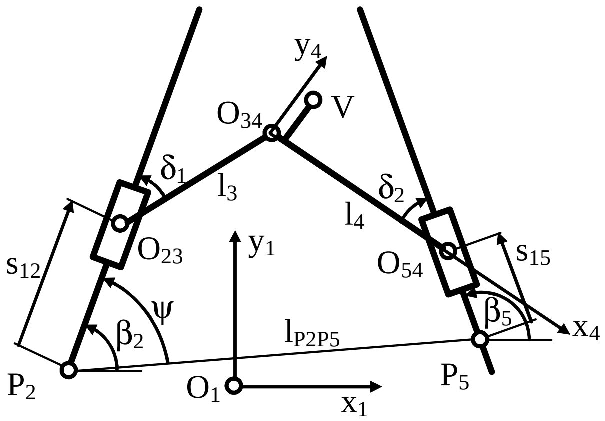


Fig. 2 Kinematical description of the machine

2.2. Mapping of non-sensitivity region within parameter space

The basic calibration procedure described within section 2.1. provides us with unique solution for given data. This solution is unique for very broad region of initial guess of parameters of iterative solution by Newton method. Nevertheless during the calibration experiments we found out, that parameters (dimensions of mechanism) determined from different realisations of calibration measurements show large variances. The fundamental reason of this phenomenon is an interaction of inferior conditionality of linear systems solved during iterations of Newton method, measurement errors and errors of model simplifications regarding real machine. Consequently it is very useful to acquire deeper insight into relations between parameter space and space of calibration results. The crucial step towards efficient mapping of the parameter space is singular value decomposition (SVD) of system matrices of iterations (4) of Newton method

$$\mathbf{J}_i^{\mathrm{T}}\mathbf{J}_i = \mathbf{U}_i\mathbf{S}_i\mathbf{V}_i^{\mathrm{T}}. \tag{6}$$

The matrices $\mathbf{U}_i$ and $\mathbf{V}_i$ are orthonormal ($\mathbf{U}_i^{-1} = \mathbf{U}_i^{\mathrm{T}}, \mathbf{V}_i^{-1} = \mathbf{V}_i^{\mathrm{T}}$) and $\mathbf{S}_i$ is diagonal matrix of singular values sequenced in descending order. For example the singular values of TRIJOINT calibration are typically in range from $2*10^2$ to $2*10^{-4}$. Considering SVD, equation (4) can be rewritten into form

$$\mathbf{U}_i\mathbf{S}_i\mathbf{V}_i^{\mathrm{T}}\delta\mathbf{d}_i = \mathbf{J}_i^{\mathrm{T}}\delta\mathbf{v}_i. \tag{7}$$

The singular value decomposition introduces vector of auxiliary variables $\mathbf{y}_i = \mathbf{V}_i^{\mathrm{T}}\delta\mathbf{d}_i$, which are generally evaluated from equation

$$\mathbf{S}_i\mathbf{y}_i = \mathbf{U}_i^{\mathrm{T}}\mathbf{J}_i^{\mathrm{T}}\delta\mathbf{v}_i \ . \tag{8}$$

If the rank of system matrix is reduced by *r* (matrix is singular, last *r* singular values are zeros), the last *r*-tuple of elements of auxiliary vector $\mathbf{y}_i$ serves as free parameters of solution. Unique solution is replaced by *r*-parametric solution. This is not the case of TRIJOINT calibration, however the lowest singular values identify the subspace of parameters with the weakest influence on computational calibration errors. The region with the weakest influence on computational calibration errors will be denominated as non-sensitivity region. The mapping of the region has been performed as follows.

1. We consider a few of iterations of Newton method. Experience indicate that two iterations are enough for reaching solution from reasonable starting point within parameter space (deviated in mm from solution point).
2. The last (corresponding to lowest singular values) elements of auxiliary vectors $\mathbf{y}_i$ (i=1,2) are considered as free optimisation parameters, whereas the rest of elements is computed standardly from system (8).
3. The appropriate objective function is put together, for example the mean value of sum of absolute values of computational errors in x and y direction for calibration pins $F = \sum_{j=1}^{n}(|dx_j| + |dy_j|)/n$. Where n is number of pins and dx_j, dy_j are final computational errors for j-th pin.
4. The genetic optimisation method is used for minimisation of F because of its natural mapping of solution space within favourable region.

As has already been noted, during the calibration experiments we found out, that parameters (dimensions of mechanism) determined from different realizations of calibration measurements show large variances. These findings initiate development of mapping procedure described in previous subparagraph. The figure 3 shows correlation of minimized objective function with other reasonable criterions of region of the best results. Symbol j means again index of calibration pin/point.

The example of visualisation of non-sensitivity region is on Fig. 4. The displayed points within parameter space fullfield conditions for criterions from Fig.3 ($F = \sum_{j=1}^{n}(|dx_j| + |dy_j|)/n < 4e-6$, $\max_j(|dx_j|) < 10e-6$ and $\max_j(|dy_j|) < 10e-6$). The objective function expressing computational calibration errors for TRIJOINT is very flat (Fig. 4), especially with respect to some parameters (l_3, y_{4V}).

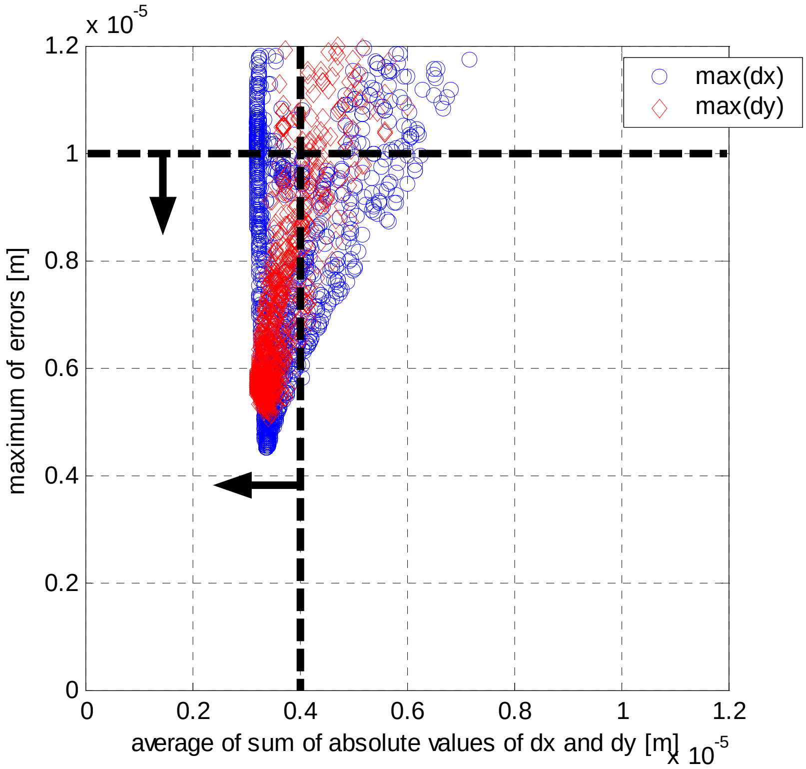


Fig. 3 Results of non-sensitivity region mapping (limiting of subregion of best results)

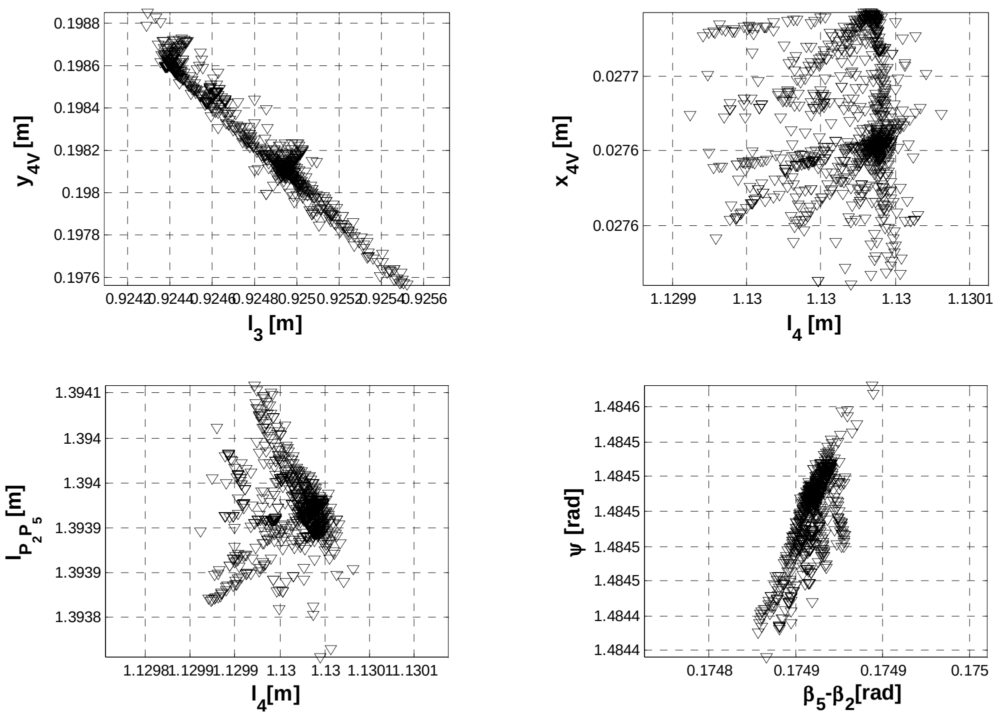


Fig. 4 The non-sensitivity region for values of objective functions delimited in Fig. 3

3. Calibrability

The basic calibration procedure provides us with the unique solution for the given data. This solution is unique for very broad region of initial guesses of parameters of iterative solution by Newton’s method. Nevertheless during the practical calibration of different machine tools it has been found out, that the parameters (dimensions of the mechanism) determined from different realizations of calibration measurements vary considerably. The fundamental reason of this phenomenon is an interaction of the inferior conditionality of linear systems solved during the iterations of Newton’s method, measurement errors, and errors of model simplifications regarding real machine. Consequently it is very useful to acquire a deeper insight into the relations between the parameter space and the space of the calibration results. Based on that the concept of the calibrability is introduced and the measure of calibrability is defined as

$$C = cond(\mathbf{J}_i^{\mathbf{T}}\mathbf{J}_i) \tag{9}$$

The smaller value of the calibrability C the more accurate determination of unknown real values of the manufactured dimensions $\mathbf{d}$ and the more accurate determination of the output coordinates $\mathbf{v}$ from the input coordinates $\mathbf{s}$, i.e. smaller resulting measurement errors (Fig. 5). The influence of the machine parameters is ilustrated by influence of relative spindle position x_{4V}, y_{4V} of Trijoint 900H (see Fig. 2).

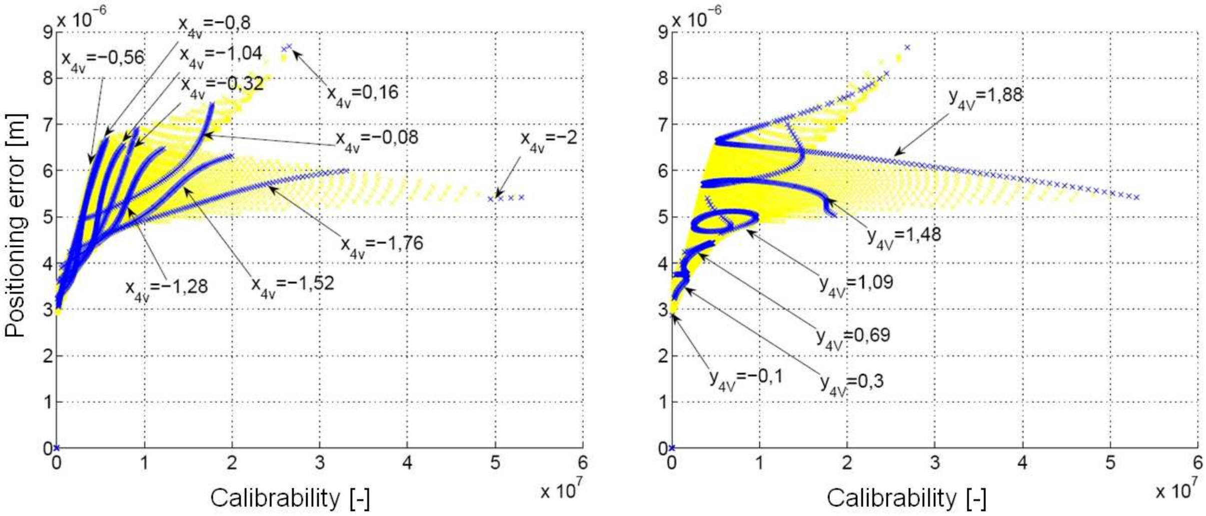
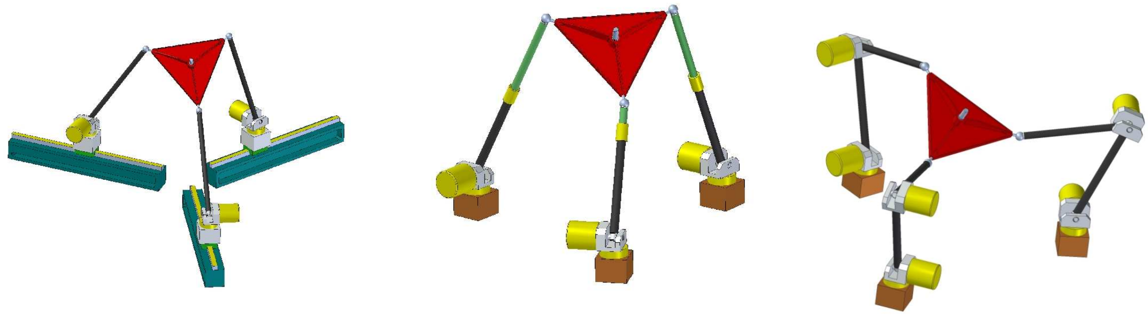


Fig. 5 Dependence of output positioning error and calibrability on spindle position x_{4V}, y_{4V}

4. Usage of calibrability – synthesis of calibration mechanism RedCaM

The appropriate application of the calibrability for the machine design has been the synthesis of calibration and measuring machine RedCaM (Redundant Calibration Machine) for 6 degrees of freedom. The good calibrability is the basis for accurate measurement of the platform position and angular displacement as well as for the correct calibration of the RedCaM.

The RedCaM is based on the redundant parallel structure without drives (Valášek, Šika, Hamrle, 2007), (Šika et al., 2006). The drives are replaced by sensors for measurement of relative motions in separate kinematical couples. The detailed analysis has been realized for three structural variants (Fig. 6). All of them use the triangular platform whose position in space is determined indirectly from sensors. The platform is connected to frame by triple of different type legs (slider, telescopic, robotic arm). The motion of 6 DOF mechanisms is measured redundantly by 9 sensors (rotational or linear).



(a) slider variant (b) telescopic variant (c) robotic arm variant

Fig. 6 Final structural variants of RedCaM

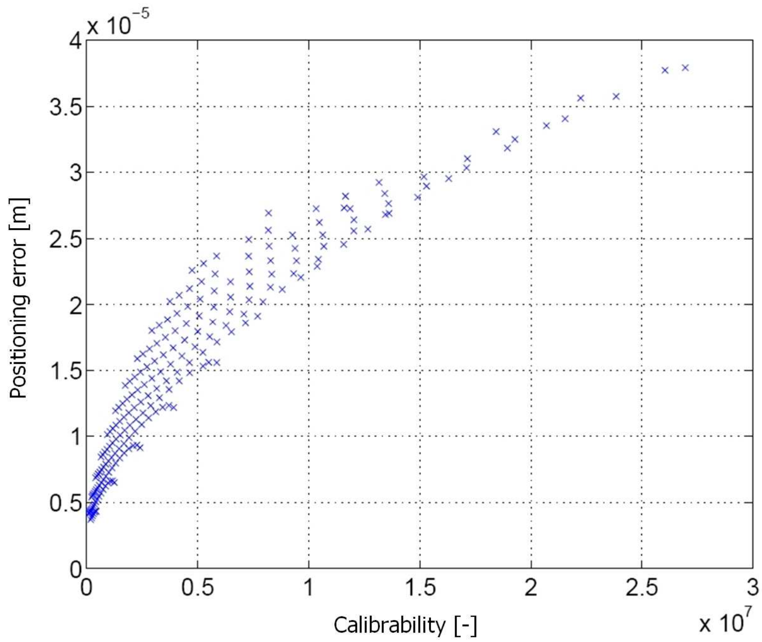


Fig. 7 Relation between calibrability and platform positioning error for final variant

The parameters of mechanisms have been optimized using the calibrability criterion. Dependence of the total positioning error on calibrability for final machine variant is shown in Fig. 7. The results with low positioning error are conditional on low values of calibrability. Each point corresponds to particular set of parameters. The error transfer between particular sensor errors and total positioning error of the end effector is approximately 1:1. This is very good result for the spatial mechanism.

The important condition of the accurate calibration is also the appropriate choice of the calibration points. The number of points as well as their position within the workspace has an important influence on the calibrability (Hollerbach, Wampler, 1996), (Wampler, Hollerbach, Arai, 1995). The relation between calibrability and positioning error of randomly distributed sets of calibration points is in Fig. 8. The number of calibration points in sets varies from 50 to 2000. Each point in figure represents one random set of calibration points.

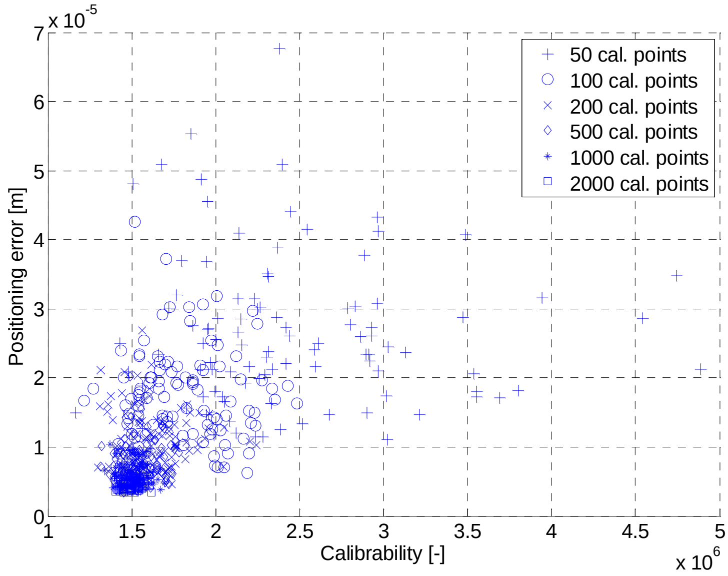


Fig. 8 Calibrability versus platform positioning error for random calibration points set

The correctness of the simulations has been tested using functional model (Fig. 9) of the RedCaM mechanism. The error transfer sensors-platform was again in ratio 1:1 as for simulations. Therefore the experiments proved the favorable error transfer of the designed structure as well as the previously investigated (Valášek, Šika, Štembera, 2004), (Valášek et al., 2005) importance of the redundant sensing of motion.

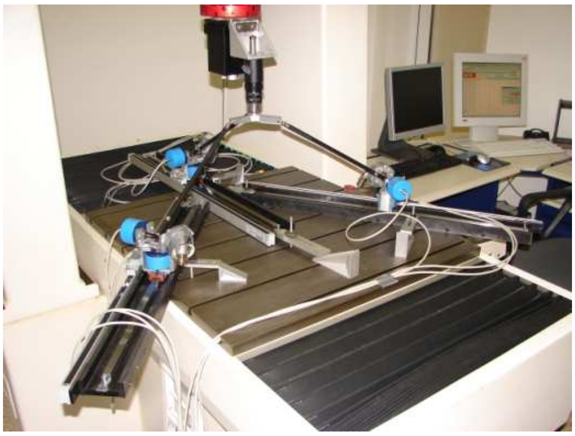


Fig. 9 Testing of functional model of slider variant of RedCaM

5. Conclusion

The influence of geometric parameters to calibration of parallel robots has been studied. The calibration of the machine tool Trijoint shows, that the accuracy of determination of particular dimensions mutually markedly differ. The accuracy is globally influenced by the conditionality of Jacobi matrix $\mathbf{J}_i$ of partial differentials of constraints with respect to parameters or alternatively $C = cond(\mathbf{J}_i^{\mathbf{T}}\mathbf{J}_i)$. This quantity has been introduced as calibrability. The calibrability describes the sensitivity of the kinematical structure to errors. It has been used as a design criterion. The important usage of the calibrability has been the synthesis of redundant parallel calibration and measuring machine RedCaM. The calibrability has been its main design criterion. The RedCaM is passive device attached to calibrated machine. The research of RedCaM has been finished by building of functional model and experiments. The optimization of its structure brings the ratio between sensor errors and end effector position errors near to 1:1 proved also experimentally. The calibrability has been proven as a new additional design criterion alongside other well known criterions like dexterity, modal properties and stiffness.

6. Acknowledgement

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