ENGINEERING MECHANICS 2009
National Conference with International Participation
Svratka, Czech Republic, May 11 – 14, 2009
pp. 1159–1183
Paper #287

# PARAMETRIC SENSITIVITY ANALYSIS OF PNEUMATIC FILTERS

**F. Siemieniako*, K. Peszyński****

**Summary:** *The course of amplitude characteristics of physical systems filtration capacity is its representative measure. Therefore, this paper focuses on an analysis on parametric sensitivity of exactly these characteristics. Research of cascade systems used for pneumatic filtration of determined periodical signals reveal lack of systematization as well as not identicalness in meaning of obtained research results. Analysis of parametric sensitivity, however is carried out on the basis of linear mathematical models, also makes it possible to evaluate the model sensitivity to nonlinear components.*

## 1. Introduction

The scope of problems which are commonly referred to in literature as 'sensitivity analysis' is extremely wide. Sensitivity theory application range covers problems from theoretical research methodology through practical industrial issues. Linear model concept is inseparably connected with the problem of dynamic properties parametric sensitivity. A mathematical linear model of cascade pneumatic systems is a system of ordinary differential equations for system with lumped-parameters. Analysis of dynamic properties of this systems can be made on the bases eigenvalues, transient or frequency characteristics. Research of cascade systems used for pneumatic filtration of determined periodical signals reveal lack of systematization as well as not identicalness in meaning of obtained research results. Among periodical signals the most frequently used in pneumatic devices [3, 7] are periodical rectangular signals.

$$p(t)=\begin{cases} p_0 & \text{for } nT<t<nT+\tau \\ 0 & \text{for } nT+\tau<t<T(n+1) \end{cases} \tag{1}$$

where: $p_0$ is amplitude, $T$ – period, $\tau$ – duration time of pulse in period.

Information transfer through rectangular pulse sequence can be performed by modulation of amplitude $p_0\,(p_0>0)$, fulfillment coefficient $\gamma=\tau/T$ $(\gamma=0\div1)$ or frequency $f=1/T$ $(f>0)$. Next the modulated periodical rectangular signal undergoes demodulation which involves reconstruction of the constant component of the filter output signal.

* Prof. Ing. Franciszek Siemieniako, DrSc, Department of Automation Technology, Bialystok Technical University, ul. Wiejska 45A, 15-351 Białystok, Poland, phone +48 85 746 92 00, fax + 48 85 746 92 10, e-mail: frank@pb.bialystok.pl
**Asistant prof. Ing. Kazimierz Peszyński, CSc, Department of Control and Machinery Design, University of Technology and Life Sciences, ul. Prof. S. Kaliskiego 7, 85-796 Bydgoszcz, phone +48 52 340 8248, fax + 48 52 340 82 45, e-mail: kazimierz.peszynski@mail.utp.edu.pl

The filter output $p_w$ signal properties (Siemieniako F. 1995) are represented by:

$p_{out}/p_0 = f(\gamma)$ – the filter static characteristic

$T_u$ – output signal setting time with accuracy $\pm d$ $\left(\text{for}\, t \geq T_{set}\ \left|p_0\gamma - p_{out}(t)\right| \leq d\right)$

Transmission bend. The course of ideal filters amplitude characteristics is shown in Fig. 1. In the pass bend $|G| = f(\omega)$ is equal to 1, but in stop band it is equal 0. Obviously in real filters existence of $|G|$ determination error is allowed.

The course of amplitude characteristics of physical systems filtration capacity is its representative measure. Therefore, this paper focuses on an analysis on parametric sensitivity of exactly these characteristics.

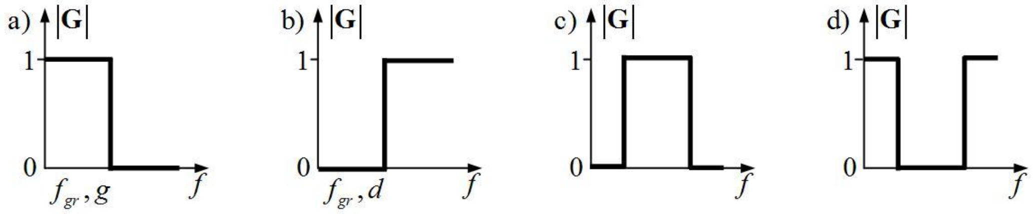



Fig. 1 Filters ideal amplitude characteristics a) low-pass filter, b) high-pass filter, c) band-pass filter, d) band-stop filter

## 2. Mathematical model

### 2. 1. Assumptions

Cascade pneumatic system operation is accompanied by many physical phenomena which condition acceptance of proper assumptions. The most important are:

- Air is treated as ideal gas
- Pressure distribution in particular chambers is homogenous, according to the condition

$$\frac{\mathrm{d}p_k}{p_k \mathrm{d}t} << \frac{a_{air}}{\varepsilon_k} \tag{2}$$

where: $a_{air} = \sqrt{\kappa R \Theta_k}$ is sonic speed in the air (disturbance propagation), $\varepsilon_k$ – maximal linear dimension of $k^{th}$ chamber

- In systems with changeable volume bellows undergo only strain and their effective surface is constant, hence if mobile mass connected with bellows surface is respected, equation balance forces acting on bellows $k^{th}$ chamber has a form

$$M_k \frac{d^2x}{dt^2} + C_k \cdot x = A_k \left(p_k - p_z\right) \tag{3}$$

- Flow continuity equation in $k^{th}$ chamber determine mass changes as difference inflows and outflows, and it has a form:

$$\frac{\mathrm{d}m_k}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{m}_{ki} - \sum_{j=1}^{m} \dot{m}_{kj} \tag{4}$$

- Equation of energy balance transported by air and exchanged with environment can be expressed as

$$E_{ki} - E_{kj} - E_k = \frac{\mathrm{d}U_k}{\mathrm{d}t} + L \tag{5}$$

- Mass changes $k^{th}$ chamber can be determined from the state equation in the following way

$$\frac{\mathrm{d}m_k}{\mathrm{d}t} = \frac{1}{R\Theta}\left( V_k \frac{\mathrm{d}p_k}{\mathrm{d}t} + p_k \frac{\mathrm{d}V_k}{\mathrm{d}t} \right) \tag{6}$$

  For initial conditions $p_{k0}$, $\Theta_{k0}$, $V_{k0}$ after having respected polytrophic equation $p_k V_k^{\kappa} = const$. There was received:

$$m_k = \frac{1}{R\Theta_{k0}} p_{k0}^{(\kappa-1)/\kappa} V_k p_k^{l/\kappa} \tag{7}$$

  The concept of the medium constant mass is justified in many cases significantly simplifies energy balance and flow continuity equations.
- Pneumatic resistance flow characteristics resulting from the description of throttling flowing air phenomenon are treated as linear or nonlinear. The condition of obtaining linear characteristics is forming laminar flow in the resistor channel.

In engineering calculations a simplification is frequently used involving introducing pneumatic conductivity which numerically corresponds to air flow intensity through the resistor, with the resistor pressure drop

$$\dot{m}_{ij} = U_{ij}\left( p_i - p_j \right) \tag{8}$$

2.2 Model of Cascade stiff systems

Generally cascade stiff system (Fig. 2) can consists of chambers of $n$ volumes $V_1, \ldots, V_n$ connected by resistors with pneumatic conductivity $U_{12}, U_{13}, \ldots, U_{1n}, U_{23}, U_{24}, \ldots, U_{2n}, \ldots, U_{(n-1)n}$. Moreover each chamber can be connected with the initial input signal source respectively, by resistors with pneumatic conductivity $U_{01}, U_{02}, \ldots, U_{0n}$. For each cascade chamber a first order differential ordinary equation with constant coefficients were obtained from mass balance [INFLOW]-[OUTFLOW]=[ACCUMULATION] after necessary transformations, with assumptions (2), (4), (6), (7). Hence, for $n$ chambers a system of $n$ equations can be obtained:

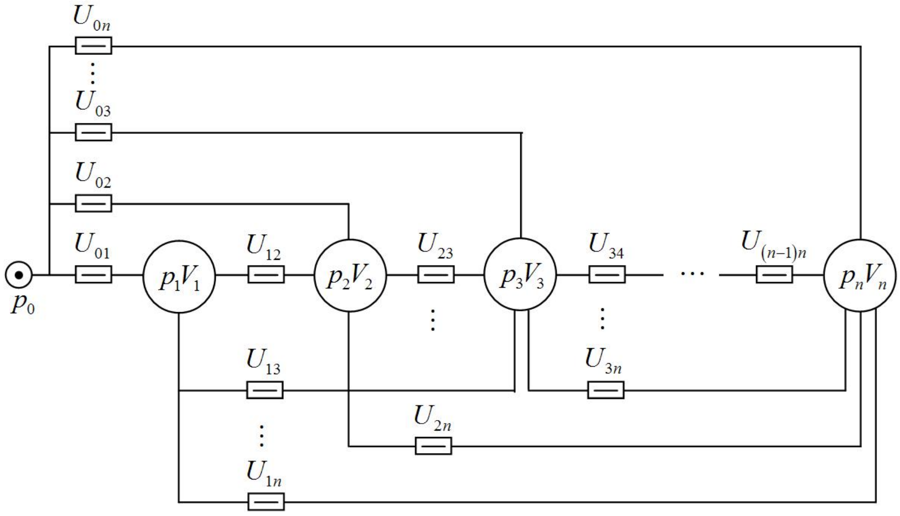



Fig. 2 Physical model of cascade stiff system

$$\begin{aligned}
\frac{V_1}{\kappa R\Theta}\frac{\mathrm{d}p_1}{\mathrm{d}t} &= \sum_{i=2}^{n} U_{i1}\left(p_i - p_1\right) + U_{01}\left(p_0 - p_1\right) \\
\frac{V_2}{\kappa R\Theta}\frac{\mathrm{d}p_2}{\mathrm{d}t} &= \sum_{i=2}^{n} U_{i2}\left(p_i - p_2\right) + U_{02}\left(p_0 - p_2\right) \\
&\vdots \\
\frac{V_n}{\kappa R\Theta}\frac{\mathrm{d}p_n}{\mathrm{d}t} &= \sum_{i=2}^{n} U_{in}\left(p_i - p_n\right) + U_{0n}\left(p_0 - p_n\right)
\end{aligned} \tag{9}$$

If $k_{ik} = U_{ik}\left(\kappa R\Theta / V_k\right)$ denotes, mathematical model of the considered system is a state equation

$$\dot{\mathbf{U}} = \mathbf{AU} + \mathbf{Bp_0} \tag{10}$$

where: $\dot{\mathbf{U}} = \begin{bmatrix} \frac{\mathrm{d}p_1}{\mathrm{d}t} & \frac{\mathrm{d}p_2}{\mathrm{d}t} & \dots & \frac{\mathrm{d}p_n}{\mathrm{d}t} \end{bmatrix}^T$, $\mathbf{U} = \begin{bmatrix} p_1 & p_2 & \dots & p_n \end{bmatrix}^T$, $\mathbf{B} = \begin{bmatrix} k_{01} & k_{02} & \dots & k_{0n} \end{bmatrix}^R$

$$\mathbf{A} = \begin{bmatrix} -\sum_{i=2}^{n} k_{i1} & k_{21} & \dots & k_{n1} \\ k_{12} & -\sum_{i=1,3}^{n} k_{i2} & \dots & k_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ k_{1n} & k_{2n} & \dots & -\sum_{i=1}^{n-1} k_{in} \end{bmatrix}$$

2.3. Model of flexible cascade

In flexible cascade systems, in many applications, movable elements masses $\left(M_n\right)$ are significant and cannot be neglected in their mathematical model. In Fig. 3 one chamber of flexible cascade is presented as a fragment of a cascade system

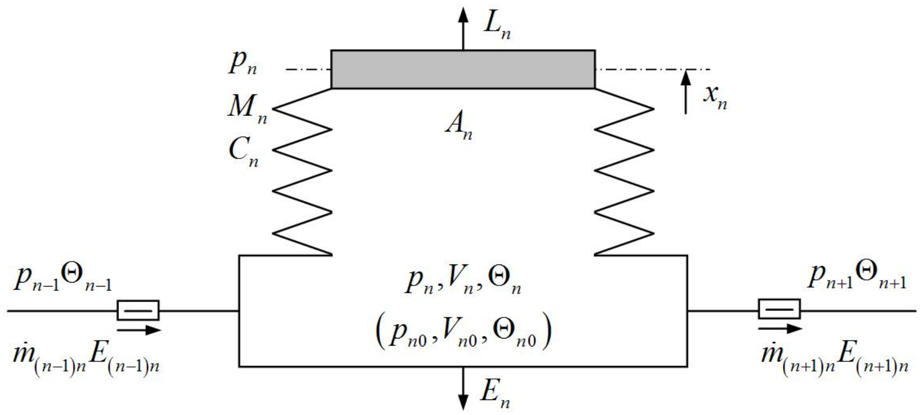



Fig. 3 Physical model of cascade flexible system

On the bases of equations (3), (4), (5), and (6) having respected the well known dependencies $E = iQ = c_p\Theta$, $U = c_v m\Theta$, $L = p\left(\mathrm{d}V/\mathrm{d}t\right)$, the following equation system describing the cascade dynamics has been received

$$\begin{cases} \kappa R\left(\Theta_{n-1}\dot{m}_{(n-1)n} - \Theta_n\dot{m}_{n(n+1)}\right) - (\kappa-1)E_z = \kappa p_n\dfrac{\mathrm{d}V_n}{\mathrm{d}t} + V_n\dfrac{\mathrm{d}p_n}{\mathrm{d}t} \\ R\Theta_n\dot{m}_{(n-1)n} - R\Theta_n\dot{m}_{n(n+1)} + \dfrac{p_nV_n}{R\Theta_n}\dfrac{\mathrm{d}\Theta_n}{\mathrm{d}t} = \kappa p_n\dfrac{\mathrm{d}V_n}{\mathrm{d}t} + V_n\dfrac{\mathrm{d}p_n}{\mathrm{d}t} \\ V_n = V_{n0} + A_nx_n \\ M_n\dfrac{\mathrm{d}^2x_n}{\mathrm{d}t^2} + C_nx_n = A_n\left(p_n - p_z\right) \end{cases} \tag{11}$$

If $E_z = \alpha\left(\Theta_n - \Theta_z\right)$, where $\alpha = f\left(\text{flow rate, density, air temperature}\right)$ and medium constant quantity (7 are accepted, the equations system describing the cascade will be as follows:

$$\begin{cases} \dfrac{p_{n0}^{(\kappa-1)/\kappa}}{R\Theta_{n0}}\left[p_n^{1/\kappa}\dfrac{\mathrm{d}V_n}{\mathrm{d}t} + \dfrac{1}{\kappa}V_np_n^{(\kappa-1)/\kappa}\dfrac{\mathrm{d}p_n}{\mathrm{d}t}\right] = \dot{m}_{(n-1)n} - \dot{m}_{n(n+1)} \\ V_n = V_{n0} + A_nx_n \\ M_n\dfrac{\mathrm{d}^2x_n}{\mathrm{d}t^2} + C_nx_n = A_n\left(p_n - p_z\right) \end{cases} \tag{12}$$

Respecting equation (3) of the system (2.11) in equation (2) and assuming that the flows $\dot{m}_{(n-1)n}$, $\dot{m}_{n(n+1)}$ are non linear pressure function linearization should be performed. After expanding to Taylor series and after neglecting high-order terms we receive

$$\frac{1}{\kappa R\Theta_{n,0}}\left[\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_0+\frac{1-\kappa}{\kappa}\frac{V_{n,0}+A_n x_{n,0}}{p_{n,0}}\left(\frac{\mathrm{d}p_n}{\mathrm{d}t}\right)_0\right]p_n+\frac{V_{n,0}+A_n x_{n,0}}{\kappa R\Theta_{n,0}}\frac{\mathrm{d}p_n}{\mathrm{d}t}+\frac{A_n}{\kappa R\Theta_{n,0}}\left(\frac{\mathrm{d}p_n}{\mathrm{d}t}\right)_0+$$
$$+\frac{p_{n,0}A_n}{R\Theta_{n,0}}\frac{\mathrm{d}x_n}{\mathrm{d}t}=\left[\left(\frac{\partial\dot m_{(n-1)n}}{\partial p_{n-1}}\right)_0-\left(\frac{\partial\dot m_{n(n+1)}}{\partial p_n}\right)_0\right]p_n-\left(\frac{\partial\dot m_{n(n+1)}}{\partial p_{n+1}}\right)_0 p_{n+1},$$
$$M_n\frac{\mathrm{d}^2x_n}{\mathrm{d}t^2}+C_n x_n=A_n\left(p_n-p_z\right)$$

where variables $x_n$, $p_{n-1}$, $p_{n+1}$, and $p_z$ are defined with respect to coordinate system with a centre $x_{n0}$, $p_{(n-1)0}$, $p_{n0}$, $p_{(n+1)0}$, and $p_{z0}$. Accepting

$$\left(\frac{\mathrm{d}x_n}{\mathrm{d}t}\right)_0=0,\ \left(\frac{\mathrm{d}p_n}{\mathrm{d}t}\right)_0=0,\ x_{n,0}=0$$

assuming that

$$\left(\frac{\partial\dot m_{(n-1)n}}{\partial p_{n-1}}\right)_0=-\left(\frac{\partial\dot m_{(n-1)n}}{\partial p_n}\right)_0=U_{n-1,n},$$
$$\left(\frac{\partial\dot m_{(n-1)n}}{\partial p_{n-1}}\right)_0-\left(\frac{\partial\dot m_{n(n+1)}}{\partial p_n}\right)_0$$

and introducing denotations $\mathrm{d}x_n/\mathrm{d}t=v_{n2}$ after transformation mathematical model of the cascade (Fig. 3) was obtained in the form of linear state equation

$$\begin{cases}\dfrac{\mathrm{d}p_n}{\mathrm{d}t}=-\dfrac{\kappa R\Theta_{n,0}}{V_{n,0}}\left(U_{(n-1)n}+U_{n(n+1)}\right)p_n+\dfrac{p_{n,0}A_n}{R\Theta_{n,0}}v_n+\dfrac{\kappa R\Theta_{n,0}}{V_{n,0}}U_{(n-1)n}p_{n-1}+\dfrac{\kappa R\Theta_{n,0}}{V_{n,0}}U_{(n-1)n}p_{n+1}\\ \dfrac{\mathrm{dx}_n}{\mathrm{d}t}=v_n\\ \dfrac{\mathrm{d}v_n}{\mathrm{d}t}=\dfrac{A_n}{M_n}p_n-\dfrac{C_n}{M_n}x_n-\dfrac{A_n}{M_n}p_z\end{cases}\tag{13}$$

The above formula can be easily adjusted for cases when a cascade contains more than two resistors (is connected by resistors with more chambers) or when it contains more than one flexible bellows.

For each chamber we can write an equation in the form (13) and accept the notations

$$\begin{aligned}&\overline{\mathbf{P}}=\left[p_1,\ \ldots,p_n\right] &&\overline{\mathbf{U}}=\left[\overline{\mathbf{P}}\quad\overline{\mathbf{X}}\quad\overline{\mathbf{V}}\right]^T\\&\overline{\mathbf{X}}=\left[x_1,\ \ldots,x_n\right] &&\overline{\mathbf{W}}=\left[w_1,\ldots,w_n\right],\overline{\mathbf{W}}\in\overline{\mathbf{U}}\\&\overline{\mathbf{V}}=\left[v_1,\ \ldots,v_n\right]\end{aligned}$$

the dynamics of the considered multi chamber system can be expressed by linear state equation (14)

$$\begin{aligned}&\frac{\mathrm{d}\overline{\mathbf{U}}}{\mathrm{d}t}=\mathbf{A}\overline{\mathbf{U}}+\mathbf{B}\overline{p}_0\\&\overline{\mathbf{W}}=\mathbf{C}\overline{\mathbf{U}}\end{aligned}\tag{14}$$

where

$$\mathbf{C} = \operatorname{diag}\left[\{0,1\}\right]$$

$$\text{diam} \quad \mathbf{C} = 3n \times 3n$$

$$\text{diam} \quad \mathbf{A} = 3n \times 3n$$

$$\text{diam} \quad \mathbf{B} = 3n$$

Entries of matrixes $\mathbf{A}$ and $\mathbf{B}$ are functions of the system constructional parameters.

## 3. System parametric sensitivity

3.1. Sensitivity concept

Dynamic parameters of physical systems depend on theirs structures and constructional parameters. Linear mathematical models of these systems can be presented in a form of state space equation (14). If changes of the system constructional parameters vector are written in the form

$$\overline{p}(t) = \overline{p}_0(t) + ev(t) \tag{15}$$

where:

$\overline{p}(t)$ – nominal vector of the system constructional parameters,

$v(t)$ – the system parameters vector deviation function,

$e$ – small value

the sensitivity function can be described by the dependence

$$s(\overline{p}, \overline{v}, t) = \lim_{e \to 0} \frac{\overline{F}(\overline{p} + e\overline{v}, t) - \overline{F}(\overline{p}, t)}{e} \tag{16}$$

If the vector deviation kind and initial conditions are constant the system dynamic properties will be functions of its constructional parameters $F(\overline{p}_i) = F(p_1, p_2, \ldots, p_n)$. This function analytic with respect to $\overline{p}_i$ vector parameters in the surroundings of a point defined by nominal vector of parameters $\overline{p}_0$ can be expanded into Taylors series. First order term of this expansion

$$\delta_i^l = \left.\frac{\partial F}{\partial p_i}\right|_{\overline{p}_i = \overline{p}_0} = \delta_i^l(F, p_i) \tag{17}$$

is called first order "absolute parametric sensitivity". Absolute sensitivity of the function $F(\overline{p}_i)$ is defined as a function of parameters $\overline{p}_i$. Thus the function $F(\overline{p}_i)$ variable value is equal to

$$\Delta F = \sum_{i=1}^{n} \delta_i^1 \cdot \Delta p_i \tag{18}$$

Comparison of sensitivity of a given function $F(\overline{p}_i)$ to the system parameters changes is possible due to introduction of first order semi-absolute sensitivity.

$$\xi_i^l * = \left.\frac{\partial F}{\partial \ln p_i}\right|_{\overline{p}_i=\overline{p}_0} = p_i \delta_i^l = \xi_i^l * (F, p_i)$$

or (19)

$$\xi_i^l = \left.\frac{\partial F}{\partial \ln p_i}\right|_{\overline{p}_i=\overline{p}_0} = \frac{1}{F}\delta_i^l = \xi_i^l (F, p_i)$$

Comparison of sensitivity of the system different functions representing the system properties (e.g.: chamber pressure, acceleration of the bellows bottom in pneumatic systems) to its different parameters is possible through analyzing relative sensitivities

$$v_i^l = \left.\frac{\partial \ln F}{\partial p_i}\right|_{\overline{p}_i=\overline{p}_0} = \frac{p_i}{F}\sigma_i^l = v_i^l (F, p_i) \tag{20}$$

hence, relative deviation with respect to function $F$ can be expressed:

$$\frac{\Delta F}{F} = \sum_{i=1}^{n} v_i^1 \frac{\Delta p_i}{p_i} \tag{21}$$

The function relative sensitivities to the system different parameters changes depend on each other. Relevant dependences can be found by summing them.

The above equations prove that there are univocal relations between the mentioned kinds of sensitivities

$$\begin{aligned} \xi_i^1 &= p_i \delta_i^l \\ v_i^1 &= \frac{p_i}{F}\delta_i^l = \frac{1}{F}\xi_i^1 \end{aligned} \tag{22}$$

Hence, the choice of sensitivity kinds in examining physical systems depends on the needs.

### 3.2. Engine values and their sensitivity

The study on eigenvalues allows not only for indirect definition of parametric sensitivity as they are also helpful in examinations of time or frequency sensitivity characteristics. Matrix of eigenvalues $\mathbf{\Lambda} = [\lambda_1, \ldots, \lambda_n]^T$ of real matrix $\mathbf{A}_{n\times n}$ (for a free system $\dot{\mathbf{U}} = \mathbf{AU}$) is derived from solutions of the equation (Kaczorek T., 1976):

$$\det[\mathbf{A} - \lambda \mathbf{I}] = 0 \tag{23}$$

For each eigenvalue there exists at least one solution of linear equations:

$$\begin{aligned} \mathbf{A}\overline{k}_i &= \lambda_i \overline{k}_i \\ \overline{w}_i \mathbf{A} &= \overline{w}_i \lambda_i \end{aligned} \tag{24}$$

Solution $\overline{\mathbf{k}}_i = \mathrm{col}\left[\overline{k}_1^{(1)} \ldots \overline{k}_n^{(1)}\right] = \mathbf{Y}$ is called 'eigenvalue matrix' (eigenvector of matrix $\mathbf{A}$), $\overline{\mathbf{W}}_i = \left[\overline{W}_1^{(1)} \ldots \overline{W}_n^{(1)}\right] = \mathbf{Y}^{-1}$ is called eigenvalues row of $\mathbf{A}$. Matrix of eigenvectors $\mathbf{Y}_{n\times n}$ is similarity matrix diagonalizing matrix $\mathbf{A}$:

$$\mathbf{\Lambda} = \mathbf{Y}^{-1}\mathbf{AY} \tag{25}$$

Parametric sensitivity of eigenvalues, with the assumption that all the values of the examined system are isolated, can be determined from the dependence:

$$\frac{\partial \mathbf{\Lambda}}{\partial p_i} = \mathbf{Y}^{-1} \frac{\partial \mathbf{A}}{\partial p_i} \mathbf{Y} \tag{26}$$

Derivatives of matrix $\mathbf{A}$ in relation to particular parameters can be determined from central subtraction:

$$\frac{\partial \mathbf{A}}{\partial p_i} = \frac{\mathbf{A}\left(\left[p_1 \dots p_i + \delta p_i \dots p_n\right]\right) - \mathbf{A}\left(\left[p_1 \dots p_i + \delta p_i \dots p_n\right]\right)}{2\delta p_i} \tag{27}$$

3.3 Parametric sensitivity of frequency characteristics

For shaping characteristics of complex amplitude systems-with numerous parameters, exerting a similar influence on properties, it is advantageous to use sensitivity analysis, firs of all, due to standardization of examinations, transparency of results, and a possibility of defining permissible changes of constructional parameters of the examined systems.

Determination of spectral transfer functions matrixes $G(j\omega)$ of a system described by equation of state (14) is possible after having performed Fourier transformation of this equation with zero initial conditions.

$$j\omega \cdot \bar{U}(j\omega) = A\bar{U}(j\omega) + B\bar{p}_0(j\omega)$$
$$\bar{W}(j\omega) = C\bar{U}(j\omega)$$

hence,

$$\bar{U}(j\omega) = \left[j\omega I - A\right]^{-1} B\bar{p}_0(j\omega)$$

and then

$$\bar{W}(j\omega) = C\left[j\omega I - A\right]^{-1} B\bar{p}_0(j\omega)$$

According to transmittance definition, complex matrix of frequency characteristics has the following form:

$$G(j\omega) = \frac{\bar{W}(j\omega)}{\bar{p}_0(j\omega)} = C\left[j\omega I - A\right]^{-1} B \tag{28}$$

If $\mathbf{Y}^{-1}\mathbf{A}\mathbf{Y} = \mathbf{\Lambda}$ is to be respected

$$\left(j\omega\mathbf{I} - \mathbf{A}\right)^{-1} = \mathbf{Y}\mathbf{Y}^{-1}\left(j\omega\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{Y}\mathbf{Y}^{-1} = \mathbf{Y}\mathrm{diag}\left[\frac{1}{j\omega - \lambda_1} \cdots \frac{1}{j\omega - \lambda_n}\right] =$$
$$= \left[\bar{k}_1 \dots \bar{k}_n\right]\mathrm{diag}\left[\frac{1}{j\omega - \lambda_1} \cdots \frac{1}{j\omega - \lambda_n}\right]\left[\bar{w}_1 \dots \bar{w}_n\right] = \bar{\mathbf{k}}\mathrm{diag}\left[\frac{1}{j\omega - \lambda_1} \cdots \frac{1}{j\omega - \lambda_n}\right]\bar{\mathbf{w}}$$

Thus, determination of compound frequency characteristics is reduced to determining values of eigenvectors and eigenrows of real matrix $\mathbf{A}$. Dependence of elements of this matrix $\left[g_{ij}(j\omega)\right]$ on frequency are frequency characteristics:

$$A_{ij}(\omega) = \left| g_{ij}(j\omega) \right|$$
$$L_{ij}(\omega) = 20 \log A_{ij}(\omega) \tag{29}$$
$$\varphi_{ij}(\omega) = \arg g_{ij}(j\omega)$$

Sensitivity of frequency characteristics in relation to parameter $p_i$ can be expressed:

$$\frac{\partial G(j\omega)}{\partial p_i} = \left[ \frac{\partial}{\partial p_i} \left( \left[ j\omega \mathbf{I} - \mathbf{A} \right]^{-1} \right) \mathbf{B} + \left[ j\omega \mathbf{I} - \mathbf{A} \right]^{-1} \frac{\partial \mathbf{B}}{\partial p_i} \right] \tag{30}$$

Significant simplification of calculation can be obtained after earlier determination of parametric eigenvalues and after including $\frac{\partial \mathbf{A}}{\partial p_i} = \mathbf{Y}^{-1} \frac{\partial \mathbf{A}}{\partial p_i} \mathbf{Y}$ and expression $\left[ j\omega \mathbf{I} - \mathbf{A} \right]^{-1}$ into transformations, by means of eigenvectors and eigenrows.

Denoting $\frac{\partial G(j\omega)}{\partial p_i} = \left[ \frac{\partial g_{ij}(j\omega)}{\partial p_i} \right] = \left[ a_{ji} + jb_{ji} \right]$, parametric sensitivity of real frequency characteristics can be expressed as follows:

$$\frac{\partial A_{ij}(\omega)}{\partial p_i} = \frac{1}{\left| g_{ij} \right|} \left( a_{ij} \operatorname{Re} g_{ij} + b_{ij} \operatorname{Im} g_{ij} \right)$$
$$\frac{\partial L_{ij}(\omega)}{\partial p_i} = \frac{20}{\ln 10 \left| g_{ij} \right|^2} \left( a_{ij} \operatorname{Re} g_{ij} + b_{ij} \operatorname{Im} g_{ij} \right) \tag{31}$$
$$\frac{\partial \varphi_{ij}(\omega)}{\partial p_i} = \frac{\operatorname{Im} g_{ij} \cdot \operatorname{Re} g_{ij}}{\left| g_{ij} \right|^2} \left( \frac{b_{ij}}{\operatorname{Im} g_{ij}} - \frac{a_{ij}}{\operatorname{Re} g_{ij}} \right)$$

## 4. Scope and method of calculations

Spectral transmittances of the examined systems were determined by using a method based on solving eigenvalues of mathematical models. During determining values of eigenvectors and eigenrows there was used an algorithm involving using a series of transformations of matrix $\mathbf{A}$ similarity, reducing it to the upper part of Hessenberg shape, and then transforming $\mathbf{QR}$ into the quasi-triangular shape. Calculation of eigenvalues and eigenrows of matrixes $\mathbf{A}$ in this way, is based on similarity transformation properties. If $\mathbf{Y}_A$ is matrix of eigenvalues of square matrix $\mathbf{A}$, and $\mathbf{Y}_B$ is a matrix of eigenvectors of square matrix $\mathbf{B}$, then there exists dependence $\mathbf{Y}_A = \mathbf{P}\mathbf{Y}_B$. Derivatives of matrix $\mathbf{A}$ towards particular parameters were determined from central subtraction (27).

The scope of the cascade constructional parameters change accepted for the research, corresponding to values occurring in pneumatic devices, allowed for evaluation of the influence of these changes on the course of amplitude logarithmic characteristics, which in turn, enabled assessment of parametric sensitivity of the examined cascades in their application for periodical signals filtration. Values of particular parameters changed within the following range: $V = 5\cdot10^{-6} \div 1\cdot10^{-4}\,\mathrm{m}^3$, $U = 0.1\cdot10^{-8} \div 0.6\cdot10^{-8}\,\mathrm{kg\cdot s^{-1}\cdot Pa^{-1}}$, $S = 3\cdot10^{-4} \div 9\cdot10^{-4}\,\mathrm{m}^2$, $M = 0.7 \div 0.5\,\mathrm{kg}$, $C = 250 \div 1000\ \mathrm{N\cdot m^{-1}}$.

For simulation tests and analysis of the systems parametric sensitivity Marix VI.10[9] program was used.

The program makes it possible to:

- solve the eigenvalue of matrix of state
- determine trajectory of the model solution with assigned initial conditions
- (graphic presentation) in the period of time establised by the user,
- determine spectral trnsmittance and drafting frequency characteristics (phase, amplitude, and amplitude-logarythmic ) within the frequency range established by the user,
- determine parametric sensitivity of the above mentioned dynamic properties in a graphic form ( for time courses and frequency characeristics ),
- record and configure the examined system on the disc and its repeated reading (own format of data),

## 5. Study results of selected structures

### 5.1. General information

During design of pneumatic filters of priodical sygnals, the choice of a proper flow circuit containing active resistsances , interances and capacities is of big importance. The choice of the system appropriate structure and its parameters should provide desired characteristics of filter systems frequencies. The desired characteristics are characteristics of such a system that damps the amplitude of particular harmonics of the considered course, with desired accuracy, at the same time, damping the harmonics involves smoothing(cutting out) courses with higher frequencies: within the band of defined frequencies with lower or higher frequencies from certain boundary frequency.

The purpose of initial examinations of pneumatic filter systems was to establish structures and the nominal vector of parameters of these systems accomplishing the task of:

- low-pass filters,
- high-pass filters,
- band-pass filters.

The results of parametric sensitivity analysis of selected pneumatic systems are presented below.

### 5.2. Pneumatic low pass filter

The task of pneumatic low pass filter can be performed by a system built from constant resistance and a chamber with constant volume linked in a series manner.

If filtration quality requirements are defined (Siemieniako F.,1995) then the low pass filter structures must be sought among multi-chamber structures and appropriate link of chambers through pneumatic resistance of different values.

In this paper there have been presented results of analysis of parametric sensitivity of a two-chamber cascade amplitude characteristics presented in figure 4.

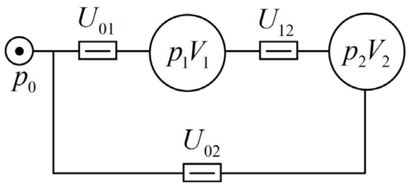



Fig 4 Pneumatic two-chamber cascade

For the system as in figure 5.1, it can be written:

$$\begin{cases} \dfrac{V_1}{\kappa R\Theta_0}\dfrac{\mathrm{d}p_1}{\mathrm{d}t} = U_{01}\left(p_0 - p_1\right) + U_{12}\left(p_2 - p_1\right) \\ \dfrac{V_2}{\kappa R\Theta_0}\dfrac{\mathrm{d}p_2}{\mathrm{d}t} = U_{02}\left(p_0 - p_2\right) + U_{12}\left(p_1 - p_2\right) \end{cases} \tag{32}$$

Hence, the matrix equation of state has the form:

$$\dot{\mathbf{U}} = \mathbf{AU} + \mathbf{B}p$$

where:

$$\dot{\mathbf{U}} = \begin{bmatrix} \dfrac{\mathrm{d}p_1}{\mathrm{d}t} & \dfrac{\mathrm{d}p_2}{\mathrm{d}t} \end{bmatrix}^T, \ \mathbf{U} = \begin{bmatrix} p_1 & p_2 \end{bmatrix}, \ p = \begin{bmatrix} p_0 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \dfrac{\kappa R\Theta_0}{V_1}U_{01} & \dfrac{\kappa R\Theta_0}{V_1}U_{02} \end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} -\dfrac{\kappa R\Theta_0}{V_1}\left(U_{01} + U_{12}\right) & \dfrac{\kappa R\Theta_0}{V_1}U_{12} \\ \dfrac{\kappa R\Theta_0}{V_2}U_{12} & -\dfrac{\kappa R\Theta_0}{V_2}\left(U_{02} + U_{12}\right) \end{bmatrix}^T$$

For parameters $V_1 = V_2 = 1.0 \cdot 10^{-4}\,\mathrm{m}^3$, $U_{01} = U_{02} = U_{12} = 0.4 \cdot 10^{-8}\,\mathrm{kg \cdot s^{-1} \cdot Pa^{-1}}$, $\kappa R\Theta_0 = 117724.4\ \mathrm{N \cdot m \cdot kg^{-1}}$ results of calculations are presented in figures 5,6,7.

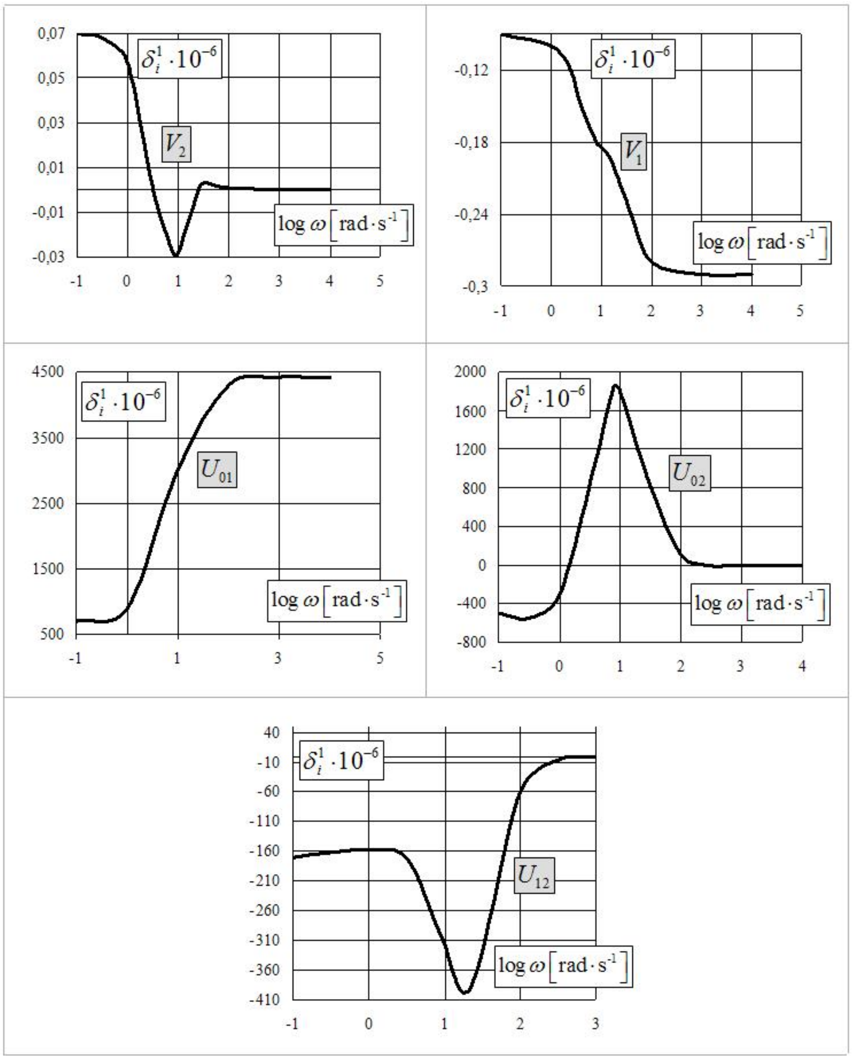



Fig. 5 Absolute sensitivity of logarithmic characteristics of amplitude cascade from fig.4, input signal is $p_1$

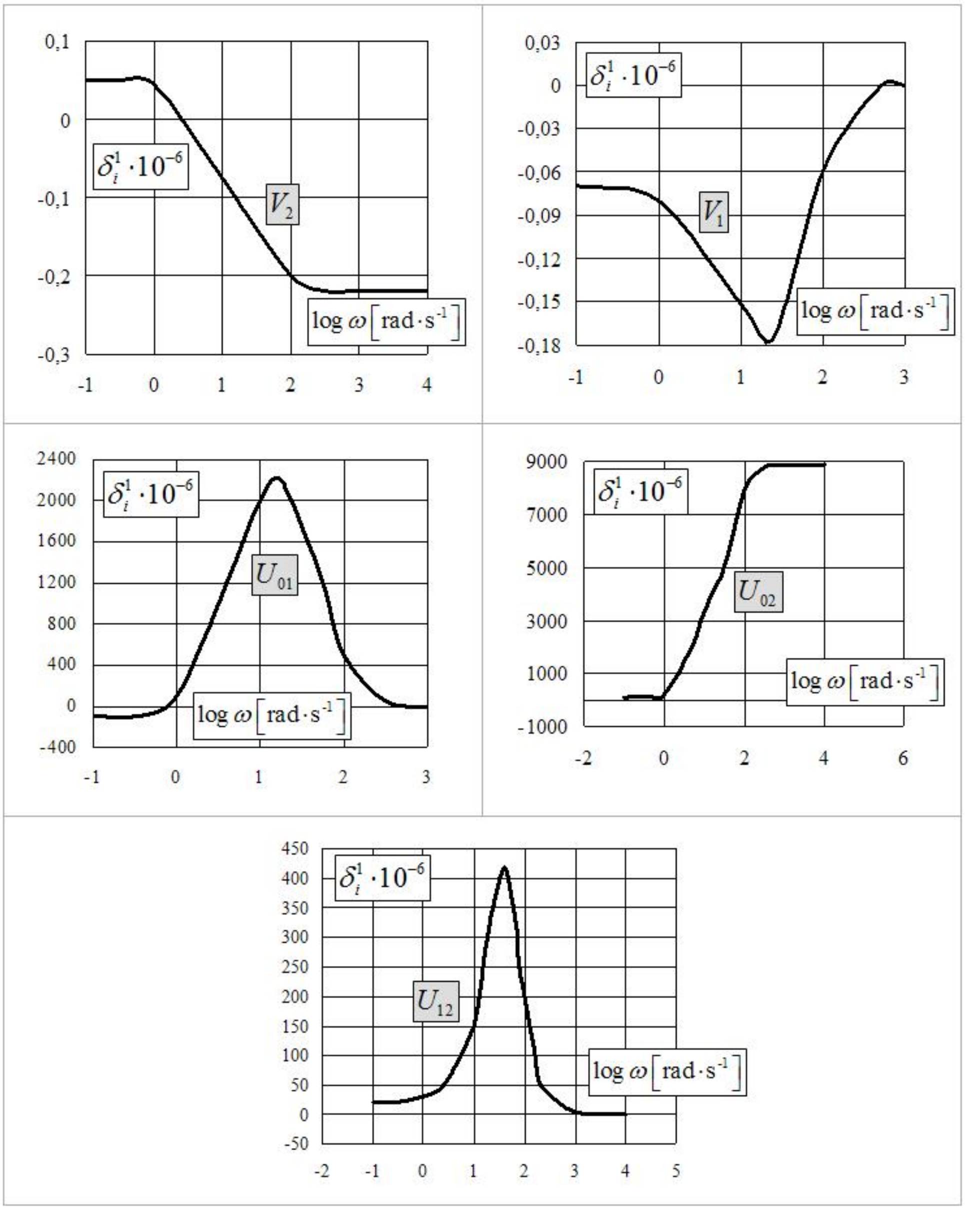



Fig. 6 Absolute sensitivity of the cascade logarithmic characteristics from figure 4, output signal is $p_2$

According to figure 6 (input signal is $p_2$),eg. for $\omega = 10\,\mathrm{rad\cdot s^{-1}}$ absolute sensitivity of logarithmic amplitude characteristics for particular parameters will be as follows: $\Delta L = -0.075\cdot 10^{-6}\Delta V_2$, $\Delta L = -0.17\cdot 10^{-6}\Delta V_1$, $\Delta L = 2000\cdot 10^{-6}\Delta U_{01}$, $\Delta L = 3500\cdot 10^{-6}\Delta U_{02}$, $\Delta L = 150\cdot 10^{-6}\Delta U_{12}$.

Comparison of the influence of particular parameters on the course of amplitude characteristics of the system from figure 4 can be made on the basis of semi-absolute sensitivity (fig. 7). Eg. for $\omega > 100\,\mathrm{rad\cdot s^{-1}}$ the influence of $U_{01}$, $U_{12}$, $V_1$ as compared to the influence of $U_{02}$ and $V_2$, on the course of amplitude characteristics of this system, is to be neglected.

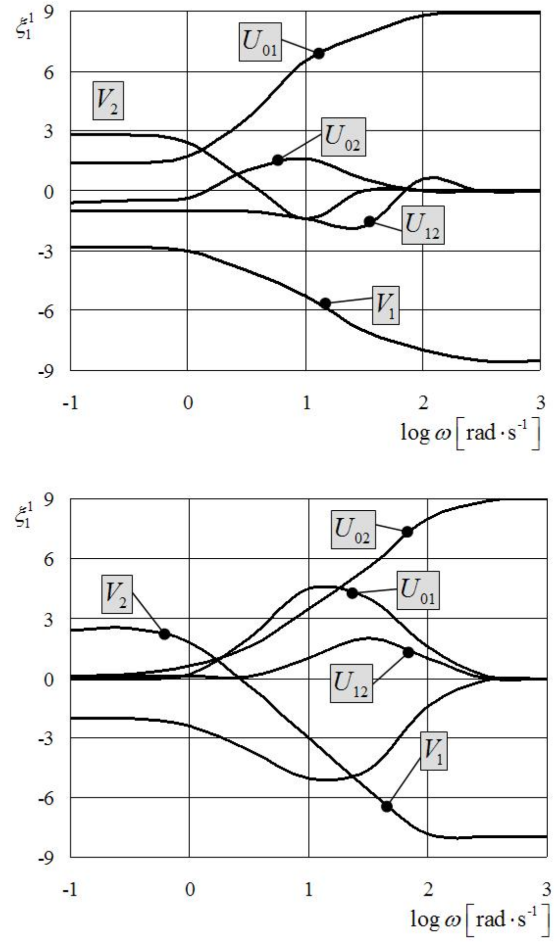



Fig.7 Semi-absolute sensitivity of logarithmic amplitude characteristics:
a- input signal is $p_1$, output signal is $p_2$

5.3. Pneumatic high pass filter

Search for high pass filter was carried out among pneumatic two and three-chamber systems. Among two-chamber cascades no structure performing the task of high pass with specified filtration quality has been found. However, three-chamber systems make it possible to build many structures with considerabely varying parameters ($U$, $V$, $C$, $M$, $S$), which can perform the tasks of high pass filter. In the present stage of research it is not possible to make a classification of structures or present a table of parameters of a given structure in the function of quantities representing the quality of filtration (e.g. lower limit of not damped periodicities). Therefore, in this paper there have been presented examination results of analysis of sensitivity for a selected pneumatic three-chamber cascade (fig. 8).

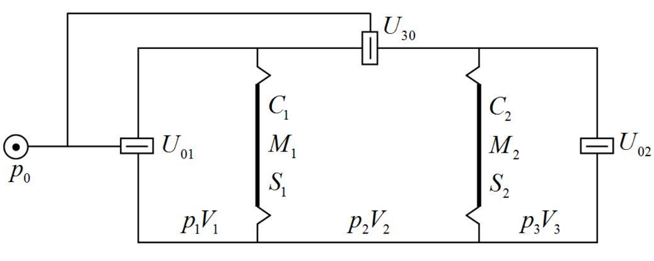



Fig. 8 Scheme of a three- chamber cascade pneumatic structure

The above pneumatic cascade can be described by the follwing equation:

$$\begin{cases} \dfrac{\mathrm{d}p_1}{\mathrm{d}t} = -\dfrac{\kappa R\Theta_0}{V_1}U_{01}p_1 - \dfrac{p_{10}\kappa S}{V_1}\vartheta_1 + \dfrac{\kappa R\Theta_0}{V_1}U_{01}p_0 \\ \dfrac{\mathrm{d}p_2}{\mathrm{d}t} = -\dfrac{\kappa R\Theta_0}{V_2}U_{02}p_2 - \dfrac{p_{20}\kappa S}{V_2}\vartheta_1 - \dfrac{p_{20}\kappa S}{V_2}\vartheta_2 + \dfrac{\kappa R\Theta_0}{V_2}U_{02}p_0 \\ \dfrac{\mathrm{d}p_3}{\mathrm{d}t} = -\dfrac{\kappa R\Theta_0}{V_3}U_{30}p_3 - \dfrac{p_{30}\kappa S}{V_3}\vartheta_2 \\ \dfrac{\mathrm{d}x_1}{\mathrm{d}t} = \vartheta_1 \\ \dfrac{\mathrm{d}x_2}{\mathrm{d}t} = \vartheta_2 \\ \dfrac{\mathrm{d}\vartheta_1}{\mathrm{d}t} = \dfrac{S}{M_1}p_1 - \dfrac{S}{M_1}p_2 - \dfrac{C_1}{M_1}x_1 \\ \dfrac{\mathrm{d}\vartheta_2}{\mathrm{d}t} = \dfrac{S}{M_2}p_2 - \dfrac{S}{M_2}p_3 - \dfrac{C_2}{M_2}x_2 \end{cases}$$

Hence, matrix equation of state has the form:

$$\dot{\mathbf{U}} = \mathbf{AU} + \mathbf{B}p$$

where:

$$\dot{\mathbf{U}} = \begin{bmatrix} \dfrac{\mathrm{d}p_1}{\mathrm{d}t} & \dfrac{\mathrm{d}p_2}{\mathrm{d}t} & \dfrac{\mathrm{d}p_3}{\mathrm{d}t} & \dfrac{\mathrm{d}x_1}{\mathrm{d}t} & \dfrac{\mathrm{d}x_2}{\mathrm{d}t} & \dfrac{\mathrm{d}\vartheta_1}{\mathrm{d}t} & \dfrac{\mathrm{d}\vartheta_2}{\mathrm{d}t} \end{bmatrix}^T,$$

$$\mathbf{U} = \begin{bmatrix} p_1 & p_2 & p_3 & x_1 & x_2 & \vartheta_1 & \vartheta_2 \end{bmatrix},\ p = \begin{bmatrix} p_0 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \dfrac{\kappa R\Theta_0}{V_1}U_{01} & \dfrac{\kappa R\Theta_0}{V_1}U_{02} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} -\dfrac{\kappa R\Theta_0}{V_1}U_{01} & 0 & 0 & 0 & 0 & -\dfrac{p_{10}\kappa S}{V_1} & 0 \\ 0 & -\dfrac{\kappa R\Theta_0}{V_2}U_{02} & 0 & 0 & 0 & \dfrac{p_{20}\kappa S}{V_2} & \dfrac{p_{20}\kappa S}{V_2} \\ 0 & 0 & -\dfrac{\kappa R\Theta_0}{V_3}U_{30} & 0 & 0 & 0 & \dfrac{p_{30}\kappa S}{V_3} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \dfrac{S}{M_1} & -\dfrac{S}{M_1} & 0 & -\dfrac{C_1}{M_1} & 0 & 0 & 0 \\ 0 & \dfrac{S}{M_2} & -\dfrac{S}{M_2} & 0 & -\dfrac{C_2}{M_1} & 0 & 0 \end{bmatrix}$$

The course of logarithmic amplitude characteristics has been defined for different values of particular parameters. Presented in fig. 9. courses were obtained for parameters presented in table 1 and for $M_1 = M_2 = 0.2\,\text{kg}$, $V_1 = V_2 = V_3 = 1\cdot 10^{-5}\,\text{m}^3$, $U_{01} = 2\cdot 10^{-10}\,\text{kg}\cdot s^{-1}\cdot \text{Pa}^{-1}$, $C_1 = 500\ \text{N}\cdot\text{m}^{-1}$

Table 1

| Sample number | $S_1 = S_2$ | $C_2$ | $U_{02}$ | $U_{03}$ |
|---|---|---|---|---|
| | $\text{m}^2$ | $\text{N}\cdot\text{m}^{-1}$ | $\text{kg}\cdot s^{-1}\cdot\text{Pa}^{-1}$ | $\text{kg}\cdot s^{-1}\cdot\text{Pa}^{-1}$ |
| 1 | $1\cdot 10^{-1}$ | $1\cdot 10^{1}$ | $5\cdot 10^{-8}$ | $1\cdot 10^{-9}$ |
| 2 | $1\cdot 10^{-2}$ | $1\cdot 10^{-1}$ | $1\cdot 10^{-7}$ | $3\cdot 10^{-9}$ |
| 3 | $1\cdot 10^{-3}$ | $1\cdot 10^{-4}$ | $2\cdot 10^{-7}$ | $5\cdot 10^{-9}$ |

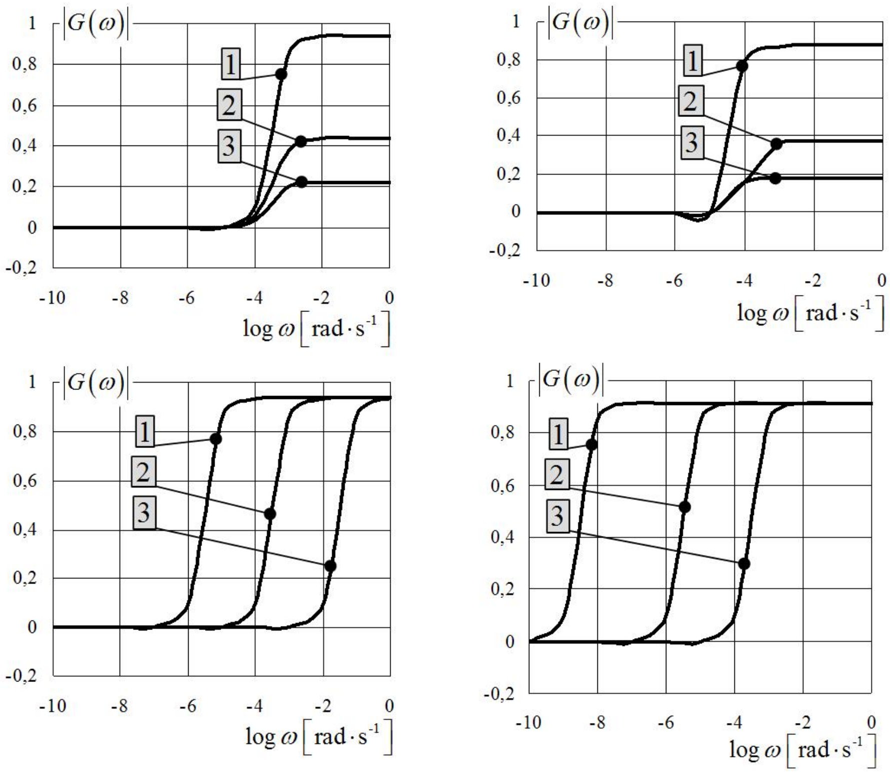



Fig. 9 Courses of logarithmic amplitude characteristics of the system from fig. 8 (output signal is $p_2$): a) variable $U_{02}$, b) variable $U_{30}$, c) variable $S$, d) variable $C_2$ (with change of a given parameter, the other ones were treated as consant, marked in table 1)

Examination of sensitivity of semi-relative parametric logarithmic characteristics of the system from figure 8 was carried out for nominal vectors of parameters

$\bar{\mathbf{W}}[M_1 \quad M_2 \quad S_1 \quad S_2 \quad C_1 \quad C_2 \quad V_1 \quad V_2 \quad V_3 \quad U_{01} \quad U_{02} \quad U_{30}]$

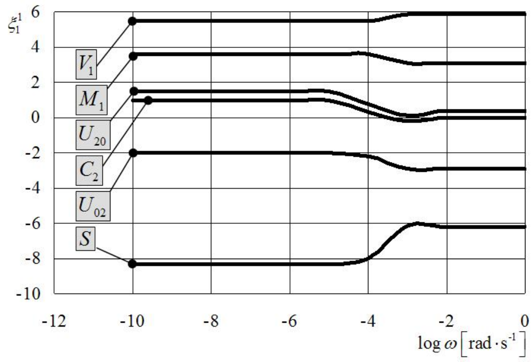
– for vector marked $\bar{\mathbf{W}}_1$

$M_1 = M_2 = 0.2\,\text{kg}$, $S_1 = S_2 = 1\cdot 10^{-2}\,\text{m}^2$, $C_1 = 500\,\text{N}\cdot\text{m}^{-1}$, $C_2 = 10\,\text{N}\cdot\text{m}^{-1}$, $V_1 = V_2 = 1\cdot 10^{-5}\,\text{m}^3$, $V_3 = 1\cdot 10^{-6}\,\text{m}^3$, $U_{01}$=$2\cdot 10^{-10}\text{kg}\cdot s^{-1}\cdot\text{Pa}^{-1}$, $U_{02}$=$5\cdot 10^{-8}\text{kg}\cdot s^{-1}\cdot\text{Pa}^{-1}$, $U_{30}$=$5\cdot 10^{-9}\text{kg}\cdot s^{-1}\cdot\text{Pa}^{-1}$

– for vector marked $\bar{\mathbf{W}}_2$

$M_1 = 0.1\,\text{kg}$, $M_2 = 0.2\,\text{kg}$, $S_1 = S_2 = 1\cdot10^{-1}\,\text{m}^2$, $C_1 = 500\,\text{N}\cdot\text{m}^{-1}$, $C_2 = 10\,\text{N}\cdot\text{m}^{-1}$, $V_1 = 1\cdot10^{-5}\,\text{m}^3$, $V_2 = 2\cdot10^{-5}\,\text{m}^3$, $V_3 = 1\cdot10^{-6}\,\text{m}^3$, $U_{01}$=$2\cdot10^{-10}\text{kg}\cdot s^{-1}\cdot\text{Pa}^{-1}$, $U_{02}$=$2\cdot10^{-7}\text{kg}\cdot s^{-1}\cdot\text{Pa}^{-1}$, $U_{30}$=$5\cdot10^{-8}\text{kg}\cdot s^{-1}\cdot\text{Pa}^{-1}$

Courses of sensitivity of semi-relative logarithmic amplitude characteristics $\xi = f(\omega)$ for particular parameters of the system (output signal is $p_2$) have been shown in fig. 10



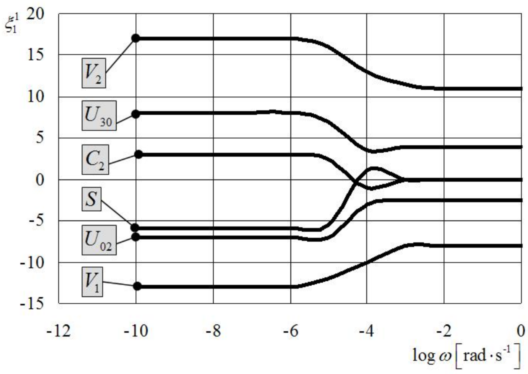




Fig. 10 Sensitivity of semi-relative logarithmic amplitude characteristics:
a) vector of parameters $\bar{\mathbf{W}}_1$, b) vector of parameters $\bar{\mathbf{W}}_2$

5.4. Pneumatic band-pass filter filter

Search for pneumatic band-pass filter was carried out in the scope of two and three-chamber systems. Values of particular parameters were changed in the following ranges:

$M = 0.1 \div 0.9\,\text{kg}$, $S = 2\cdot10^{-2} \div 2\cdot10^{-5}\,\text{m}^2$, $C = 100 \div 1000\,\text{N}\cdot\text{m}^{-1}$, $V = 1\cdot10^{-1} \div 1\cdot10^{-5}\,\text{m}^3$, $U$=$5\cdot10^{-6} \div 1\cdot10^{-11}\text{kg}\cdot s^{-1}\cdot\text{Pa}^{-1}$

The paper deals with results of examinations of two- chamber cascade presented in figure 11 in a schematic way.

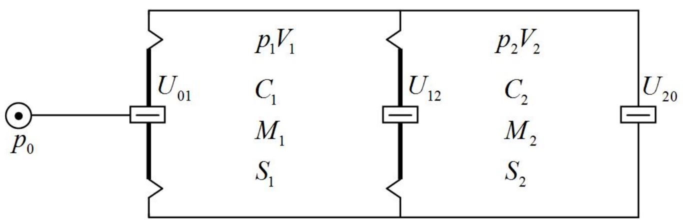



Fig. 11 Scheme of pneumatic two-chamber cascade

For the presented system it can be written

$$\begin{cases} \dfrac{\mathrm{d}p_1}{\mathrm{d}t} = -\dfrac{\kappa R\Theta_0}{V_1}\left(U_{01}+U_{12}\right)p_1 - \dfrac{p_{10}\kappa S_1}{V_1}\vartheta_1 + \dfrac{\kappa R\Theta_0}{V_1}U_{12}p_2 - \dfrac{p_{10}\kappa S}{V_1}\vartheta_2 + \dfrac{\kappa R\Theta_0}{V_1}U_{01}p_0 \\ \dfrac{\mathrm{d}p_2}{\mathrm{d}t} = -\dfrac{\kappa R\Theta_0}{V_2}U_{12}p_1 - \dfrac{\kappa R\Theta_0}{V_2}\left(U_{12}+U_{20}\right)p_2 - \dfrac{p_{20}\kappa S_2}{V_2}\vartheta_2 \\ \dfrac{\mathrm{d}x_1}{\mathrm{d}t} = \vartheta_1 \\ \dfrac{\mathrm{dx}_2}{\mathrm{d}t} = \vartheta_2 \\ \dfrac{\mathrm{d}\vartheta_1}{\mathrm{d}t} = \dfrac{S_1}{M_1}p_1 - \dfrac{C_1}{M_1}x_1 \\ \dfrac{\mathrm{d}\vartheta_2}{\mathrm{d}t} = \dfrac{S_2}{M_2}p_1 - \dfrac{C_2}{M_2}x_2 - \dfrac{S_2}{M_2}p_2 \end{cases}$$

Hence, the matrix equation of state has the form:

$$\dot{\mathbf{U}} = \mathbf{AU} + \mathbf{B}p$$

where:

$$\dot{\mathbf{U}} = \begin{bmatrix} \dfrac{\mathrm{d}p_1}{\mathrm{d}t} & \dfrac{\mathrm{d}p_2}{\mathrm{d}t} & \dfrac{\mathrm{d}x_1}{\mathrm{d}t} & \dfrac{\mathrm{d}x_2}{\mathrm{d}t} & \dfrac{\mathrm{d}\vartheta_1}{\mathrm{d}t} & \dfrac{\mathrm{d}\vartheta_2}{\mathrm{d}t} \end{bmatrix}^T,$$

$$\mathbf{U} = \begin{bmatrix} p_1 & p_2 & x_1 & x_2 & \vartheta_1 & \vartheta_2 \end{bmatrix},\ p = \begin{bmatrix} p_0 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \dfrac{\kappa R\Theta_0}{V_1}U_{01} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} -\dfrac{\kappa R\Theta_0}{V_1}\left(U_{01}+U_{12}\right) & 0 & -\dfrac{p_{10}\kappa S_1}{V_1} & -\dfrac{\kappa R\Theta_0}{V_1}U_{12} & 0 & -\dfrac{p_{10}\kappa S_2}{V_1} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \dfrac{S_1}{M_1} & -\dfrac{C_1}{M_1} & 1 & 0 & 0 & 0 \\ \dfrac{\kappa R\Theta_0}{V_2}U_{12} & 0 & 0 & \dfrac{\kappa R\Theta_0}{V_2}\left(U_{12}+U_{20}\right) & 0 & -\dfrac{p_{20}\kappa S_2}{V_2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \dfrac{S_2}{M_2} & 0 & 0 & -\dfrac{S_2}{M_2} & -\dfrac{C_2}{M_1} & 0 \end{bmatrix}$$

Logarithmic amplitude characteristics (fig. 12) have been determined for the following parameters: $M_1 = M_2 = 0.2\,\mathrm{kg}$, $S_1 = S_2 = 2\cdot10^{-4}\,\mathrm{m}^2$, $C_1 = C_2 = 500\,\mathrm{N\cdot m^{-1}}$, $V_1 = V_2 = 1\cdot10^{-5}\,\mathrm{m}^3$, $U_{01}$=$2\cdot10^{-6}\mathrm{kg}\cdot s^{-1}\cdot\mathrm{Pa}^{-1}$, $U_{20}$=$1\cdot10^{-7}\mathrm{kg}\cdot s^{-1}\cdot\mathrm{Pa}^{-1}$, $U_{12}$=$1\cdot10^{-10}\mathrm{kg}\cdot s^{-1}\cdot\mathrm{Pa}^{-1}$

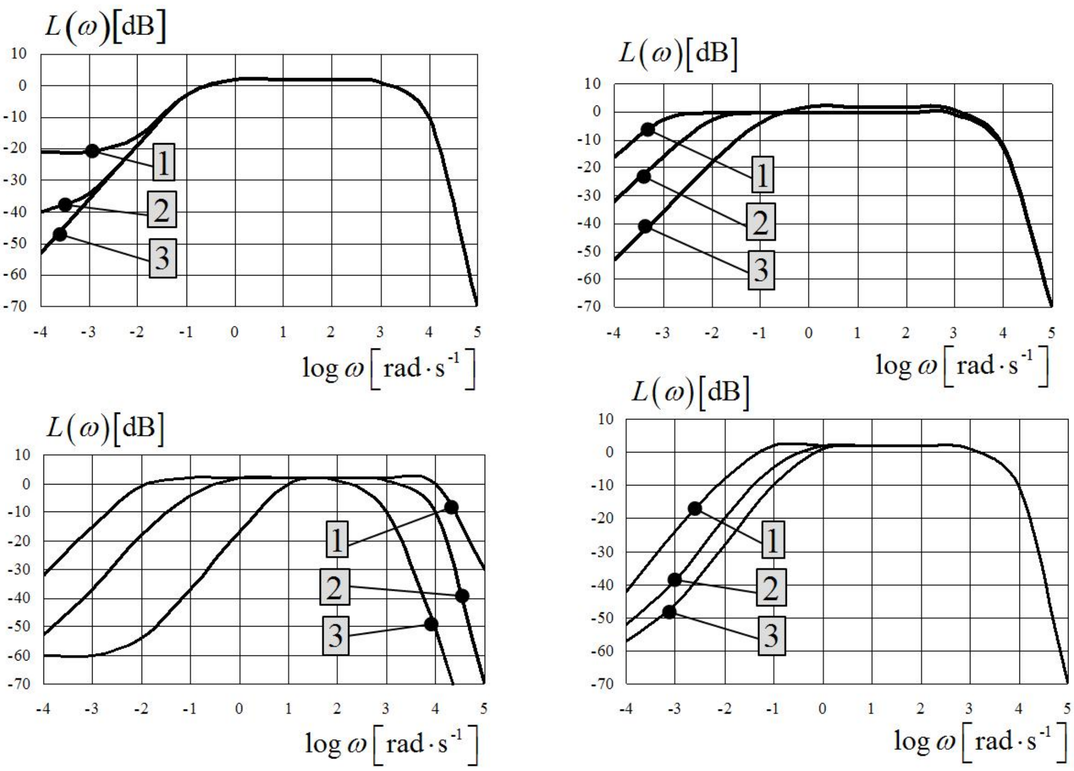



Fig.12. Logarithmic amplitude characteristics of the cascade from figure 12 (input signal is $p_1$): a) variable $U_{12}$, b) variable $U_{20}$, c) variable $S_2$, d) variable $C_2$

Parametric, semi-relative sensitivity of logarithmic characteristics to particular parameters for vector of nominal parameters:
$M_1 = M_2 = 0.2\,\text{kg}$, $S_1 = S_2 = 2\cdot 10^{-2}\,\text{m}^2$, $C_1 = C_2 = 500\,\text{N}\cdot\text{m}^{-1}$, $V_1 = V_2 = 1\cdot 10^{-5}\,\text{m}^3$,
$U_{01}$=$1\cdot 10^{-6}\text{kg}\cdot s^{-1}\cdot\text{Pa}^{-1}$, $U_{20}$=$1\cdot 10^{-7}\text{kg}\cdot s^{-1}\cdot\text{Pa}^{-1}$, $U_{12}$=$1\cdot 10^{-10}\text{kg}\cdot s^{-1}\cdot\text{Pa}^{-1}$.

Results have been presented in figure 13.

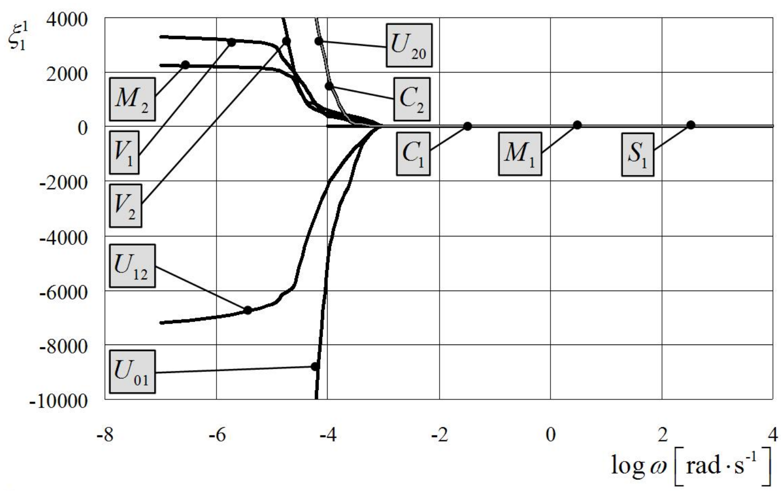



Fig. 13 Semi-relative sensitivity of logarithmic amplitude characteristics (output signal is $p_2$)

## 6. Summary

6.1. Conclusion

- Analysis of parametric sensitivity of pneumatic filters was carried out with the assumption of linear approximation of pneumatic resistance flow characteristics. The obtained results are of importance in terms of quality. It is also important to say that the effects of such an assumption are evaluated through assessment of the divergence between the real and linear courses of resistance flow characteristics or through comparing the system certain properties obtained on the basis of its linear mathematical model and properties determined for the nonlinear model (or experimentally).
- Presented results reveal that the system filtration capability (course of amplitude characteristics) can be easily shaped through a change of some parameters, whereas by means of others it is hardly possible or not possible, at all.
- Sensitivity of the function to changes of filtration parameter $p_1$ is variable, and depends on the value of remaining parameters. This means that for one vector of nominal parameters the function can be sensitive to changes of parameter $p_1$, and for another its sensitivity can be equal to zero.
- Analysis of parametric sensitivity, however is carried out on the basis of linear mathematical models, also makes it possible to evaluate the model sensitivity to nonlinear components.
- The choice of structure for a given filter type was made using the method of successive trials, and the parameters values were accepted on the basis of analysis of the elaborated results. This problem can be solved by the method of structural analysis. Briefly speaking the purpose of structural sensitivity analysis is to facilitate easy choice of a fairly accurate model of a given phenomenon. If models of the same phenomena have different structures and simultaneously one of their properties within the examined area is the same, then, it can be said that the structures are equiponderant (structural stability in relation to this property).
- Determination of parameters for a system with a defined structure, having defined properties, is the next important issue which can be solved be the method of sensitivity analysis. This task is referred to in literature as modification.

6.2. Shaping the filter static characteristics

In impulse devices (with periodical signal), working in the range of medium pressures, pneumatic resistances realize turbulent flow (having nonlinear flow characteristics). Hence, the static characteristics of the filter, both in the function of filling coefficient $\lambda_i$ and the function of amplitude p $p_0$ of the input system, is nonlinear. There arises a question whether there is a possibility of obtaining nonlinear flow characteristics?

The so far existing analysis as well as results of other researchers allow to state that the influence of particular pneumatic parameters of periodical signals ( $p_0$ , $f$ , $\gamma$ ) on the value of constant component of input signal is different. Generally, it can be written:

$$p_{n,0} = p_0\gamma + p_0k_1 \pm f \cdot k_2 \tag{33}$$

Although coefficients $k_1$ and k $k_2$ are not constant in reality, they will be neglected in further considerations due to their low value (to 0,02) and, thus, small changes (-0,01). In figure 15 there is presented a course of characteristics $p_{1,0} = f(\gamma)$ for $p_0 = 20, 60, 100\,\text{kPa}$ with frequency of input signal $f = 4\,\text{Hz}$.

Periodical signal with established parameters ($p_0$, $f$, $\gamma$) is supplied through the main resistance (eg. $d_{01}$ or $d_{01}$ and $d_{12}$). The second supplying signal with parameters ($p_0{*}, \gamma{*} = (1-\gamma), f$) is supplied to the chamber through correction resistance (e.g. $d_{01k}$ or $d_{02k}$). The range of parameters changes of the examined one-chamber cascade, as well as parameters of input signals are presented in table 2, and the course of static characteristics is presented in figure 15.

Similarly, the range of changes of the examined two-chamber cascade was presented in table 3, and the course of static characteristics is shown in figure 16.

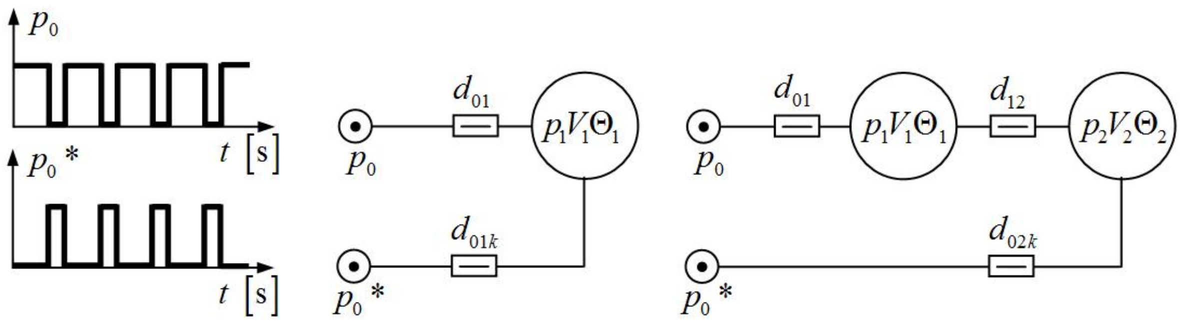



Fig. 14 The examined cascade structures with correction resistances
a) one-chamber cascade, b) two-chamber cascade

Table 2.

| On curve | 1 | 2 | 3 | 4' | 5' | 6’ | 4" | 5" | 6" |
|---|---|---|---|---|---|---|---|---|---|
| $d_{01}$ [mm] | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 |
| $d_{01k}$ [mm] | 0 | 0 | 0 | 0.38 | 0.43 | 0.51 | 0.38 | 0.43 | 0.51 |
| $p_0$ [kPa] | 100 | 60 | 20 | 100 | 100 | 100 | 100 | 100 | 100 |
| $p_0$ * [kPa] | 0 | 0 | 0 | 100 | 100 | 100 | 91 | 91 | 91 |

Table 3.

| On curve | 1' | 2' | 3' | 4' | 1" | 2" | 3" | 4" | 5 | 6 |
|---|---|---|---|---|---|---|---|---|---|---|
| $d_{01}$ [mm] | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 |
| $d_{01k}$ [mm] | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 |
| $d_{02k}$ [mm] | 0.38 | 0.43 | 0.51 | 0.60 | 0.38 | 0.43 | 0.51 | 0.60 | 0 | 0 |
| $p_0$ [kPa] | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| $p_0$ * [kPa] | 100 | 100 | 100 | 100 | 91 | 91 | 91 | 91 | 0 | 0 |

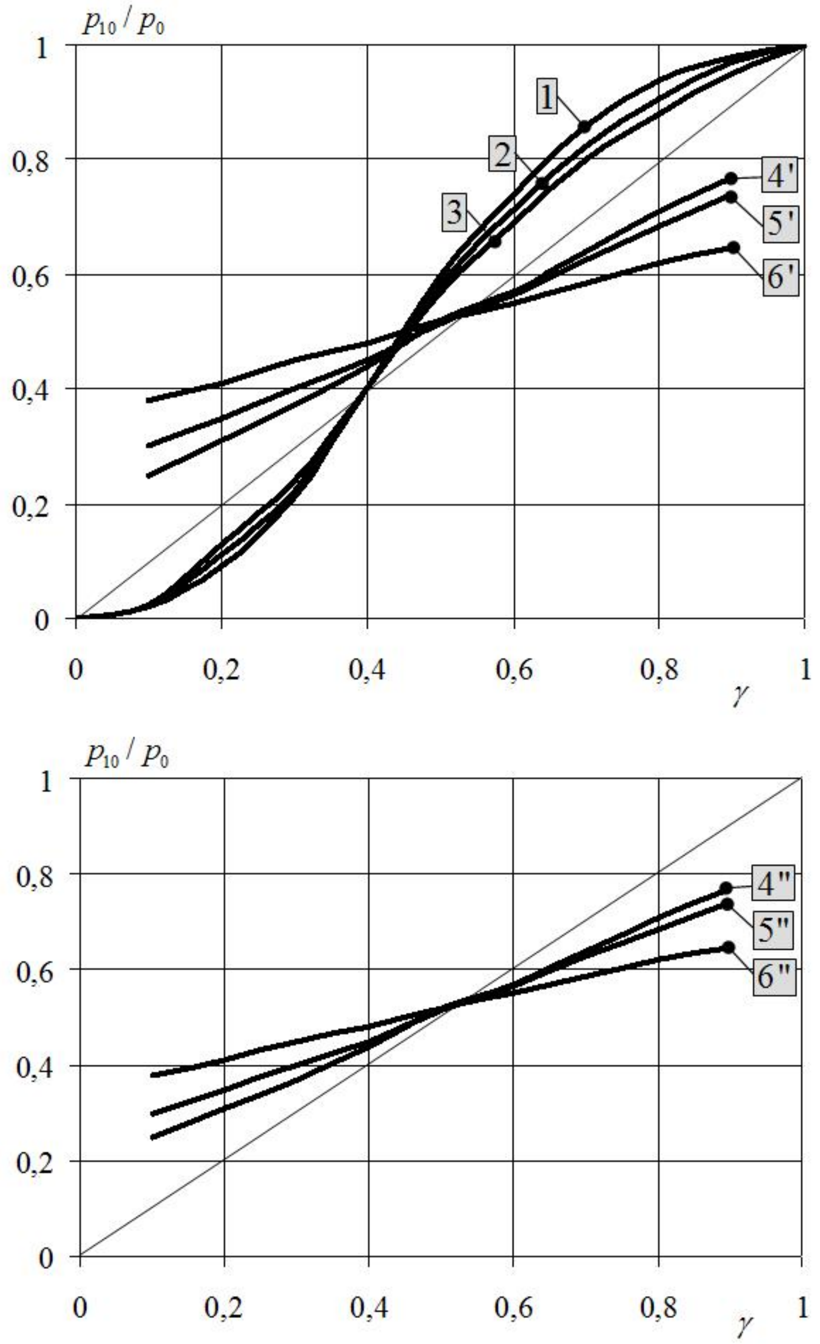
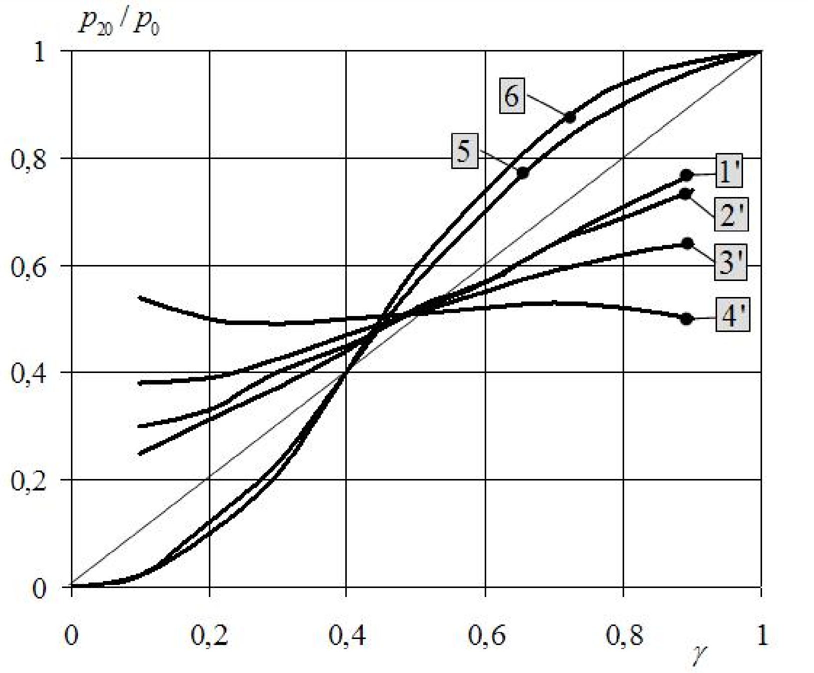



Fig. 15 Course of static characteristics of one-chamber filter



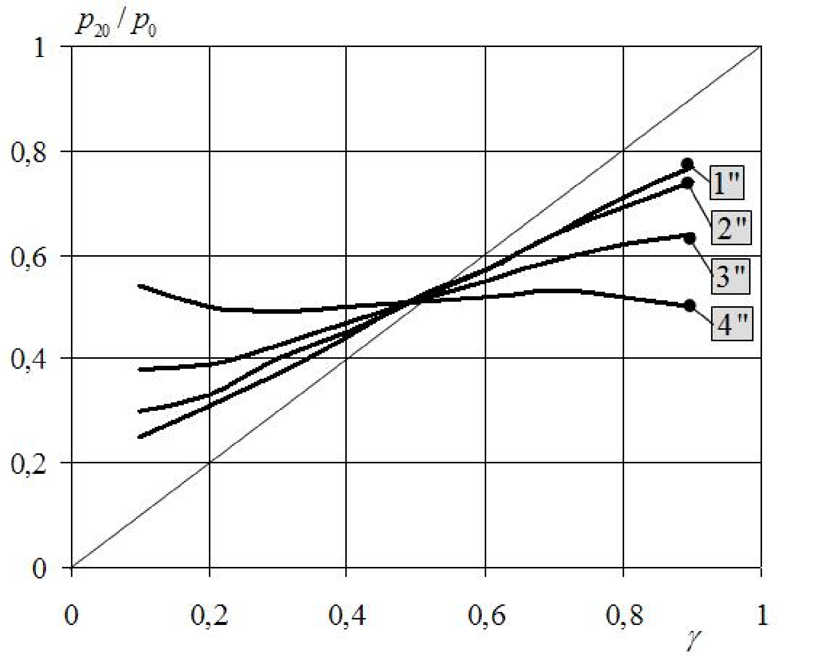




Fig. 16 Course of static characteristics of two-chamber filter

## 7. References


Geher K. (1976) Teoria tolerancji i wrażliwości układów elektronicznych (In English: Electronic Systems Sensitivity and Tolerance Theory). WNT, Warszawa.

Kaczorek T. (1976) Teoria układów regulacji automatycznej (In English: Control Systems Theory). WNT, Warszawa.

Kamiński L. M. (1980) Pneumatyczny cyfrowy modulator szerokości impulsów (In English: Pneumatic Digital Modulator of Pulse Width). Patent P-170341.

Kisilowski I. (1983) Analiza wrażliwości parametrycznej wartości własnych układu mechanicznego (In English: Analysis of Parametric Sensitivity of Mechanical System Eigenvalues). Archiwum Budowy Maszyn.

Lamel L., Osiadacz A. (1976) Elementy pneumatyczne w automatyce (In English: Pneumatic Elements in Automation). WNT, Warszawa.

Siemieniako F. (1995) Pneumatyczne filtry dolnoprzepustowe, analiza i synteza (In English: Pneumatic Low-Pass Filters, Analysis and Synthesis). Rozprawy naukowe nr 30. Wydawnictwa PB, Białystok.

Werszko M., Wroński J., Ferber J. (1982) Płynowy przetwornik ciśnienia wewnątrzczaszkowego (In English: Fluidic Sensor of Inner Braincase Pressure). Patent nr 131236.

Winogrodzki W. (1987) Wykorzystanie metod analizy wrażliwościowej do badania i syntezy pneumatycznych układów kaskadowych (In English: Using of Sensitivity Analysis Methods for Cascade Systems Testing and Synthesis). Rozprawa doktorska, Politechnika Warszawska.

Winogrodzki W. (1990) Program MATRIX V.1.10, T-matic (In English: MATRIX V.1.10 Software, T-matic). Białystok.

Żuchowski A. (1980) O problemie optymalizacji filtrów stosowanych w układach rozwijających (In English: About Problem of Filters Optimization Used in Unrolling Systems). PAK nr 6.