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SENSITIVITY ANALYSIS IN TROLLEYBUS VERTICAL DYNAMICS

P. Polach*, M. Hajžman*

Summary: *Vertical dynamic properties of the ŠKODA 21 Tr low-floor trolleybus were investigated on an artificial test track when driving with a real vehicle and when simulating driving with a multibody model on a virtual test track. Driving on the artificial test track was aimed at determining vertical dynamic properties of the real trolleybus and on the basis of them at verifying computer trolleybus models. Time histories and extreme values of the air springs relative deflections are the monitored quantities. Due to differences of the experiments and the computer simulations results the influences of the characteristics of the spring-damper structural elements of the axles suspension and the radial characteristics of the tires used in the trolleybus multibody model on the extreme values of the monitored quantities are evaluated.*

1. Introduction

Optimum dynamic properties of the vehicle intended for the public transport can usually be achieved in dependence on its structural design by the proper choice of suspension elements of axles (in some cases in combination with the proper choice of suspension elements of seats). The design must be the compromise of the requirements for the vehicle behaviour during driving manoeuvres, for the riding comfort and for the body and the chassis parts lifetime when driving on an uneven road surface, and for the passenger safety (e.g. Vlk, 2000).

Driving on the uneven road surface can provide valuable information about the vehicle vertical dynamic properties and about the suitability of the used suspension elements of axles. Especially time histories of relative deflections of springs, relative velocities in the shock absorbers, stress acting in the axles radius rods or radius arms and acceleration in various points in the vehicle interior are the monitored quantities (Gillespie & Karamihas, 2000). On the basis of those quantities it is possible to determine the forces acting in the suspension elements of axles, which can be utilized for the stress analysis of structures, for the prediction of the fatigue life of the body and of the chassis parts of the tested vehicle. The frequency domain responses of the acceleration in the vehicle interior can be used for the riding comfort evaluation. In order to evaluate the vertical dynamic properties of the vehicle when driving on the uneven road surface it is necessary to know the surface geometry or the surface characteristics, i.e. statistical properties of unevennesses of the surface (e.g. Vlk, 2000). The geometry of the uneven surface profile of the run through the section is known in test

* Dr. Ing. Pavel Polach, Ing. Michal Hajžman, Ph.D., Materials and Mechanical Engineering Research, ŠKODA VÝZKUM s.r.o.; Tylova 1/57; 316 00 Plzeň; tel.: +420.379 852 246, fax: +420.378 134 291; e-mail: {pavel.polach,michal.hajzman}@skodavyzkum.cz

polygons. Test tracks, which are created by distributing artificial vertical unevennesses (obstacles) on the smooth road surface, also are often used (e.g. Kepka et al., 1999).

Vertical dynamic properties of the empty ŠKODA 21 Tr low-floor trolleybus (see Fig. 1; its design concept is described in Polach & Hajžman, 2005b) were investigated on the artificially created test track when driving with the real vehicle and when simulating driving with the computer models on the virtual test track. Driving on the artificial test track was aimed at determining the vertical dynamic properties of the real trolleybus and on the basis of them at verifying computer models. The verified computer models will be further utilized for the simulations of driving on the virtual uneven road surfaces, which will be generated on the basis of the statistical evaluation of the measured quantities in the course of driving on the real city road with the real trolleybus (Hejman & Lukeš, 2005; Hejman & Lukeš, 2008).

Fig. 1 The ŠKODA 21 Tr low-floor trolleybus

Previous works (e.g. Polach & Hajžman, 2005b) showed that the results of the simulations and the experimental measurement are not completely identical (the extreme values of time histories of the air springs relative deflections are compared). Therefore the sensitivity analysis of the influence of various model parameters has to be performed.

Usually the sensitivity analysis is connected with the problems of the parameter selection for design optimization and with the problems of gradient calculations in gradient-based optimization procedures (Els et al., 2006). The sensitivity analysis is also a tool used in many applications in order to analyse qualitatively the behaviour of the chosen system. The general recursive approach to the calculation of sensitivities of multibody systems by means of direct differentiation is shown in Bae et al. (2001). The extension of this analytical approach for rigid-flexible systems is presented in Dias & Pereira (1997). However, in most practical cases of real multibody systems, the numerical approaches are the most suitable and efficient methods. Handling properties of road vehicles were investigated using the sensitivity analysis in Crolla et al. (1994). The sensitivity analysis for the tire wear evaluation was employed in Braghin et al. (2006).

The results of the sensitivity analyses of the multibody model of the ŠKODA 21 Tr low-floor trolleybus created in the **alaska 2.3** simulation tool (Maißer et al., 1998) at simulating driving on the virtual test track are given in Polach & Hajžman (2006), Polach & Hajžman (2008a) and Polach & Hajžman (2008b). The test track consisted of three standardized artificial obstacles (in compliance with the Czech Standard ČSN 30 0560 Obstacle II) – see Chapter 2. The parameters of the sensitivity analysis presented in Polach & Hajžman (2006) are the loading characteristics of the spring-damper structural elements of the axles suspension and the influence of changes of those characteristics on the extreme values of relative deflections of the air springs is monitored. The results of the sensitivity analysis of the influence of the different tire inflation are given in Polach & Hajžman (2008a). Sensitivity analyses in both cases were performed during the simulations of the trolleybus drive on the real test track at the trolleybus speed 44.13 km/h (the speed corresponds to the speed in the first documented test drive with the real trolleybus). The possibility of bounce of the tire from the road surface, which really occurs in the course of the vehicle relative speeding on the relatively demanding test track, is considered in the trolleybus multibody model. The influence of the change in the radial characteristics of the tires is not fully deterministic in the course of the simulation of drive on this test track and the results of the performed sensitivity analyses would be biased.

In order it may be possible to compare the influence of the loading characteristics of the spring-damper structural elements of the axles suspension and the radial characteristics of the tires on the results of the simulations, the speed 10 km/h, at which the tire bounce from the uneven road surface does not occur yet, was chosen at simulating the trolleybus drive on the test track (Polach & Hajžman, 2008b).

Another possibility how to reach keeping the tire-road surface contact during the run over the artificial obstacles is the topic of this paper. It consists in keeping the same trolleybus speed as at the experimental measurement (in this case 44.13 km/h) and virtual reducing the height of the obstacles. The height 16 mm instead of 60 mm is considered. It is the height of the artificial obstacles, at which the tire bounce from the uneven road surface does not occur yet at the multibody model speed 44.13 km/h.

2. Experimental measurements with the real trolleybus

The experimental measurements on the empty ŠKODA 21 Tr low-floor trolleybus were carried out in the depot of Hradec Králové Public City Transit Co. Inc. (Dopravní podnik města Hradce Králové, a.s.) in October 2004.

The test track consisted of three standardized artificial obstacles (in compliance with the Czech Standard ČSN 30 0560 Obstacle II: h = 60 mm, R = 551 mm, d = 500 mm – see Fig. 2) spaced out on the smooth road surface 20 meters one after another. The first obstacle was run over only with right wheels, the second one with both and the third one only with left wheels (see Fig. 3).

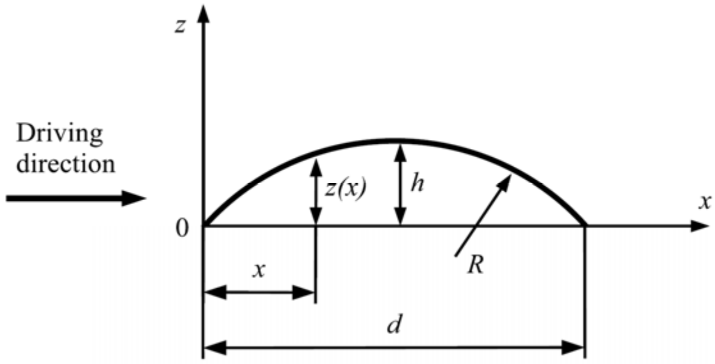

Fig. 2 The standardized artificial obstacle

In the course of the test drives the already mentioned time histories of the relative displacements between the axles and the chassis frame were recorded (altogether four displacement transducers, which were placed in the lateral direction approximately on the level of the air springs: on the left front half-axle, on the right front half-axle, on the rear axle to the left and on the rear axle to the right, were used). Further time histories of stress on twelve places of the trolleybus structure and time histories of the vertical acceleration on seven places of the trolleybus structure were recorded during the test drives. The records of the time histories of the measured quantities were made during three test drives. Trolleybus speed ranged within 43 km/h and 47 km/h at that drives.

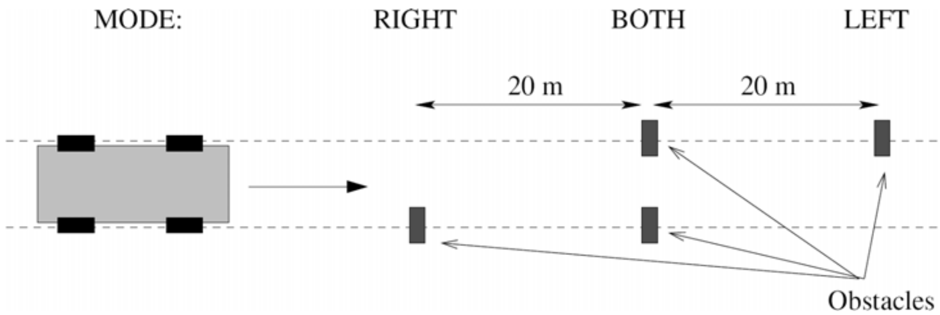

Fig. 3 A track scheme

3. Trolleybus multibody model

In order to simulate drives on the virtual test track, which correspond to the artificially created test track in the depot of Hradec Králové Public City Transit Co. Inc. with the modified obstacles height, the most complex multibody model (Polach, 2003) (see Fig. 4) created in the **alaska 2.3** simulation tool (Maißer et al., 1998) is used to investigate the influences of the loading characteristics of the spring-damper structural elements of the axles suspension and the radial characteristics of the tires.

3.1. Structure of multibody model

The multibody model of the ŠKODA 21 Tr low-floor trolleybus is formed by 35 rigid bodies and two superelements (2×4 bodies) mutually coupled by 52 kinematic joints. The rigid bodies correspond generally to the vehicles individual structural parts. The superelements correspond to the flexible parts of the chassis frame. The number of degrees of freedom in kinematic joints is 136. Rigid bodies are defined by inertia properties (mass, centre of mass co-ordinates and moments of inertia). Air springs and hydraulic shock absorbers in the axles suspension and bushings in the places of mounting some trolleybus structural parts are modelled by connecting the corresponding bodies by nonlinear spring-damper elements (Maißer et al., 1998). When simulating driving on the uneven road surface the contact point model of tires is used in the multibody model; radial stiffness and radial damping of tires are modelled by nonlinear spring-damper elements considering the possibility of bounce of the tire from the road surface (Kovanda et al., 1997).

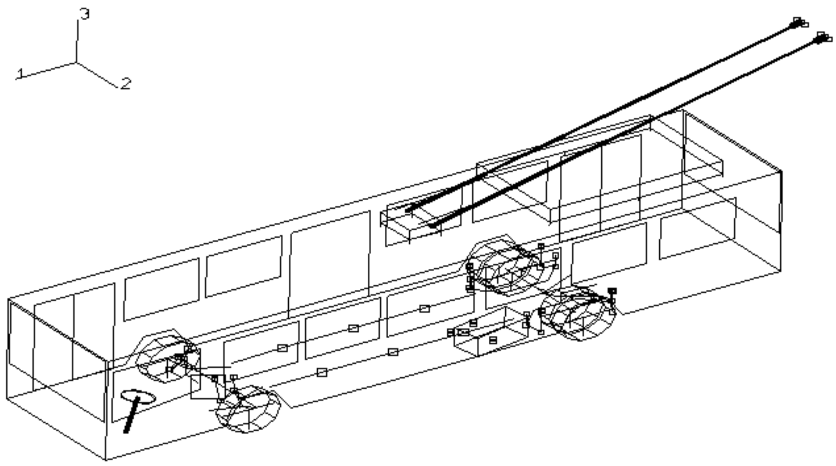

Fig. 4 Visualization of the multibody model of the ŠKODA 21 Tr low-floor trolleybus in the **alaska 2.3** simulation tool.

3.2. Characteristics of spring-damper structural elements

Dynamic properties of road vehicles are most influenced by the suspension springs, shock absorbers, bushings and tires (e.g. Blundell & Harty, 2004). In order that vehicle virtual computer model should reliably approximate kinematic and dynamic properties of the real vehicle, knowledge of the characteristics of those decisive spring-damper structural elements is the important presumption (besides the proper approach to the model creating and knowledge of all the relevant vehicle parameters).

The characteristics of the air springs (force in dependence on deflection) of the ŠKODA 21 Tr trolleybus were determined on the basis of the test reports of ŠKODA OSTROV s.r.o. and of the Hydrodynamic Laboratory of the Technical University of Liberec (Polach, 2003).

In the multibody model of the ŠKODA 21 Tr trolleybus the damping force in dependence on the relative velocity of compression and rebound of the shock absorbers is used as the shock absorbers characteristics. The characteristics were measured by BRANO a.s. (the shock absorbers producer) on the Schenck testing device (Polach & Hajžman, 2005a).

In the shock absorbers structure rubber bushings are used in the places of mounting to the chassis frame and to the axles of the trolleybus (Polach & Hajžman, 2005a). In the multibody model the bushings are modelled by means of spring elements, the nonlinear force-deformation characteristics of which were determined under the laboratory conditions (they are taken over from Kopenec, 2002) and which are coupled in series to the damping elements representing the hydraulic shock absorbers themselves.

The review of the tire models used in the field of vehicle multibody dynamics can be found in the monograph Pacejka (2002). The most important tire characteristics needed for solving the vehicle vertical dynamics tasks are their radial properties (Blundell & Harty, 2004). The used tire model for the vertical dynamics is the already mentioned contact point model based on the tire substitution by a single parallel spring and a damper. Radial stiffness and radial damping characteristics of the tires were experimentally measured in the Dynamic Testing Laboratory ŠKODA VÝZKUM s.r.o. The evaluation of the measured quantities for the purpose of generation of multibody models is given in Hajžman & Polach (2006).

4. Results of the simulations

As it has been already stated the results of the simulations at the same trolleybus speed as at the experimental measurements (in this case 44.13 km/h) and at the artificial obstacles height 16 mm during simulating the trolleybus drive on the test track are given in this paper. At this artificial obstacle height (at this trolleybus speed) the tire bounce from the uneven road surface does not occur yet.

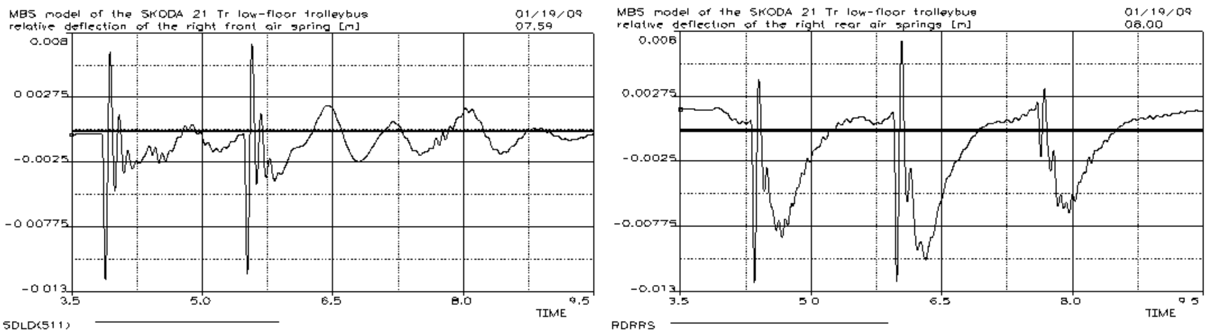

Fig. 5 Time histories of the right front and the right rear air springs relative deflection when simulating the test drive with the trolleybus multibody model at speed 44.13 km/h and at the artificial obstacles height 16 mm.

When simulating movement with the multibody models, nonlinear equations of motion, which are solved by means of numerical time integration, are generated. Results of the simulations were obtained using the Shampine-Gordon integration algorithm (Maißer et al., 1998). Fig. 5 shows the time histories of the air springs relative deflections at simulating the test drive with the trolleybus multibody model (with the consideration of the MICHELIN tire radial characteristics model at 100 % inflation – Polach & Hajžman, 2008a). The extreme values of the air springs relative deflections read from the time histories are in Tab. 1.

Tab. 1 Extreme values of the relative deflections of air springs

Obstacle	Value	Extreme values of relative deflection of air springs [mm]			
		Right front	Left front	Right rear	Left rear
1st	min.	-12	-2	-12	-6
	max.	6	1	4	3
2nd	min.	-12	-12	-12	-12
	max.	7	7	7	7
3rd	min.	-2	-11	-7	-12
	max.	2	7	3	4

5. Sensitivity analysis of the multibody model

The sensitivity analysis of the influence of the change of the spring-damper suspension elements characteristics and the tires radial characteristics in the trolleybus multibody model is performed

The sensitivity analysis of the influences of the selected parameters characterizing the system behaviour for the change in various system parameters is applied especially in the field of optimization, identification and correction of the mathematical models of the investigated systems. By means of that it is possible to determine which parameters influence the change of the chosen quantities most significantly and subsequently to select the parameters as the optimizing ones and to try to define them more precisely or to correct them.

As it was already mentioned the results of the computer simulations and the experimental measurements with the ŠKODA 21 Tr low-floor trolleybus, compared on the basis of the evaluation of the accordance of extreme values of the time histories of the air springs relative deflections with the measured extreme values of the relative displacements during the run on the test track, are not identical (especially in the course of the rebound stage of the rear axle suspension – e.g. Polach & Hajžman, 2006). It is obvious that this fact is influenced by the course of the characteristics of the spring-damper structural elements. That is why the sensitivity analysis of the influences of the force-velocity characteristics of the hydraulic shock absorbers, the force-deflection characteristics of the air springs and the force-deformation characteristics of the shock absorbers bushings on the extreme values of the time histories of the air springs relative deflections was performed in Polach & Hajžman (2006) and the results of the sensitivity analysis of the influence of the different tire inflation were given in Polach & Hajžman (2008a). In order it may be possible to compare the influence of the loading characteristics of the spring-damper structural elements of the axles suspension and the radial characteristics of the tires on the results of the simulations, the speed 10 km/h, at which the tire bounce from the uneven road surface does not occur yet, was chosen at simulating the trolleybus drive on the test track (Polach & Hajžman, 2008b).

As it has been already stated another possibility how to reach keeping the tire-road surface contact during the run over the artificial obstacles is to keep the same trolleybus speed as at the experimental measurement (in this case 44.13 km/h) and virtually reduce the height of the obstacles. The height 16 mm instead of 60 mm is considered. It is the height of the artificial obstacles, at which the tire bounce from the uneven road surface does not occur yet at the multibody model speed 44.13 km/h.

The influence of the changes in the parameters of the characteristics of the spring-damper suspension elements and the radial characteristics of the tires on the extreme values of the relative deflections of all the air springs at running over each obstacle of the artificial test track is monitored.

5.1. The sensitivity analysis of the dynamic response of the trolleybus multibody model

Like in most cases of the complicated multibody systems it is not possible to derive analytical relations to express the dynamic response of the given multibody model to the general excitation. Neither is it possible to derive analytical formulas to calculate the sensitivity of the dynamic response to the change in the system parameters. In order to express the partial derivative of the certain monitored quantity $y = y(\boldsymbol{p})$ regarding the vector of the S selected parameters of the system $\boldsymbol{p} = [p_1, p_2, \ldots, p_S]^T$ it is necessary to use relations for the numerical calculations of sensitivity, so called difference formulas (Gill et al., 1981).

Change Δy of the monitored quantity y can be expressed with a small change $\Delta\boldsymbol{p}$ of the initial parameters vector $\boldsymbol{p}_0$, when the specific conditions of the continuity of derivations of the monitored quantity y are fulfilled, using the Taylor formula (approx. by two terms)

$$\Delta y = y(\boldsymbol{p}_0 + \Delta\boldsymbol{p}) - y(\boldsymbol{p}_0) = \sum_{j=1}^{S} \frac{\partial y(\boldsymbol{p}_0)}{\partial p_j} \cdot \Delta p_j \quad . \tag{1}$$

After the modification of relation (1) it is obtained

$$\frac{\Delta y}{y(\boldsymbol{p}_0)} = \sum_{j=1}^{S} \frac{\partial y(\boldsymbol{p}_0)}{\partial p_j} \cdot \frac{p_{j0}}{y(\boldsymbol{p}_0)} \cdot \frac{\Delta p_j}{p_{j0}} \quad . \tag{2}$$

From relation (2) it is possible to get relative sensitivity $\Delta\bar{y}_\mathrm{j}$ of quantity y to the change in parameter p_j

$$\Delta\bar{y}_\mathrm{j} = \frac{\partial y(\boldsymbol{p}_0)}{\partial p_j} \cdot \frac{p_{j0}}{y(\boldsymbol{p}_0)} \quad . \tag{3}$$

Partial derivative in relation (3) is approximated using the finite difference

$$\frac{\partial y(\boldsymbol{p}_0)}{\partial p_j} = \frac{y(\boldsymbol{p}_0 + \Delta\boldsymbol{p}_j) - y(\boldsymbol{p}_0)}{\Delta p_j} \quad , \tag{4}$$

where vector $\Delta\boldsymbol{p}_j = [0, \ldots, 0, \Delta p_j, 0, \ldots, 0]^T$.

Then differential relation for the calculation of relative sensitivity $\Delta\bar{y}_j$ of quantity y to the change in parameter p_j, using relations (3) and (4), can be written in the final form

$$\Delta\bar{y}_j = \frac{y(\boldsymbol{p}_0 + \Delta\boldsymbol{p}_j) - y(\boldsymbol{p}_0)}{\Delta p_j} \cdot \frac{p_{j0}}{y(\boldsymbol{p}_0)} \quad . \tag{5}$$

Thus in case of the sensitivity analysis of the ŠKODA 21 Tr trolleybus multibody model when driving on the artificial test track the relative deflections of the air springs of axles are successively the monitored quantities y and the relative changes in the characteristics of the decisive spring-damper structural elements are the vectors of parameters $\boldsymbol{p}$.

5.2. Sensitivity analysis results

The loading characteristics of the spring-damper suspension elements and the tires radial characteristics were the parameters of the sensitivity analysis, during which the influence of parameter changes of those characteristics on the extreme values of relative deflections of the air springs was monitored.

The relative sensitivities of relative deflections of the air springs on the change in the individual parameters in the course of the simulations of driving on the virtual test track with the most complex multibody model (e.g. Polach, 2003) in the **alaska 2.3** simulation tool are given in Figs. 6a to 9a (Sha = influence of the force-velocity characteristics of the shock absorber; Spr = influence of the force-deflection characteristics of the air spring;

Bush = influence of the force-deformation characteristics of the shock absorber bushing; Tst = influence of the tire radial force-deformation characteristics; Tdm = influence of the tire radial damping coefficients; 1st to 3rd Obst = obstacle sequence; Comp = compression of air springs; Reb = rebound of air springs). For comparison in Figs. 6b to 9b there are given the relative sensitivities of relative deflections of the air springs at the trolleybus speed 10 km/h and at the obstacles height 60 mm, which are taken from Polach & Hajžman (2008b).

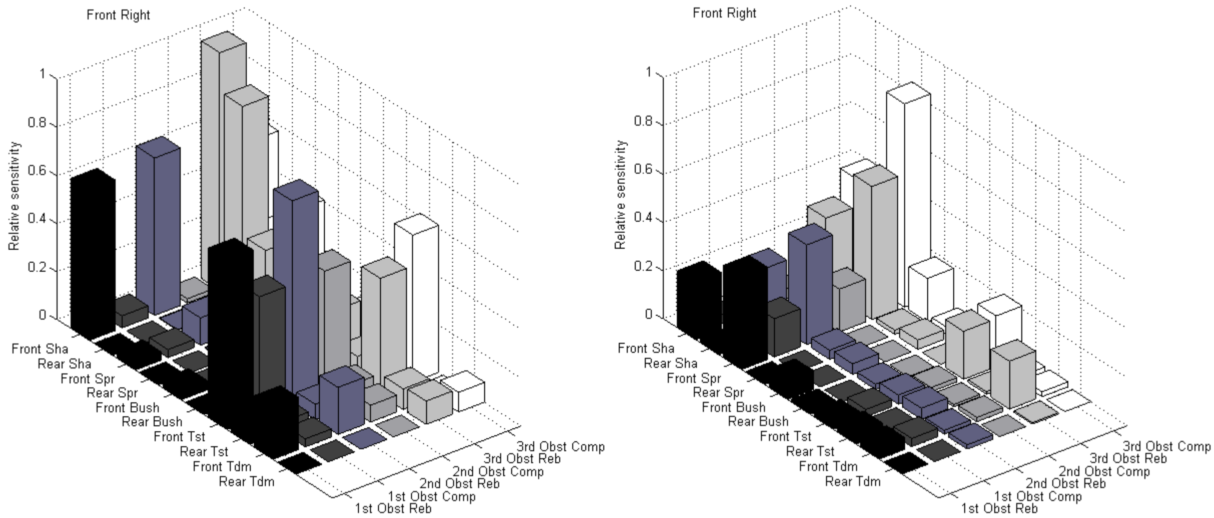

Fig. 6 Relative sensitivity of relative deflection of the right front air spring on the change in the individual parameters: a) at the trolleybus speed 44.13 km/h and at the obstacles height 16 mm; b) at the trolleybus speed 10 km/h and at the obstacles height 60 mm (taken from Polach & Hajžman, 2008b).

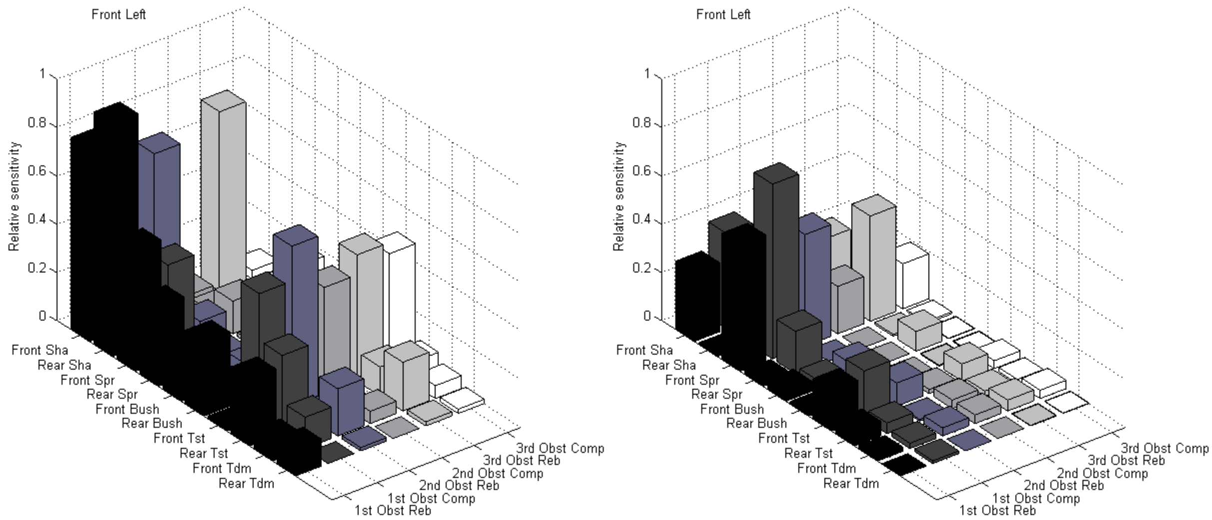

Fig. 7 Relative sensitivity of relative deflection of the left front air spring on the change in the individual parameters: a) at the trolleybus speed 44.13 km/h and at the obstacles height 16 mm; b) at the trolleybus speed 10 km/h and at the obstacles height 60 mm (taken from Polach & Hajžman, 2008b).

The results of the sensitivity analysis in the course of the simulations of driving on the virtual test track with the multibody model of the ŠKODA 21 Tr low-floor trolleybus (at the speed 44.13 km/h and at the obstacles height 16 mm) show that the influence of the force-velocity characteristics of the hydraulic shock absorbers and the force-deformation characteristics of the tires have greater influence on the results than the force-deflection characteristics of the air springs. Whereas in Polach & Hajžman (2008b) (i.e. at the trolleybus speed 10 km/h and at the obstacles height 60 mm) it was showed that the influence of the force-velocity characteristics of the hydraulic shock absorbers and the force-deflection characteristics of the air springs have a greater influence than the force-deformation characteristics of the tires.

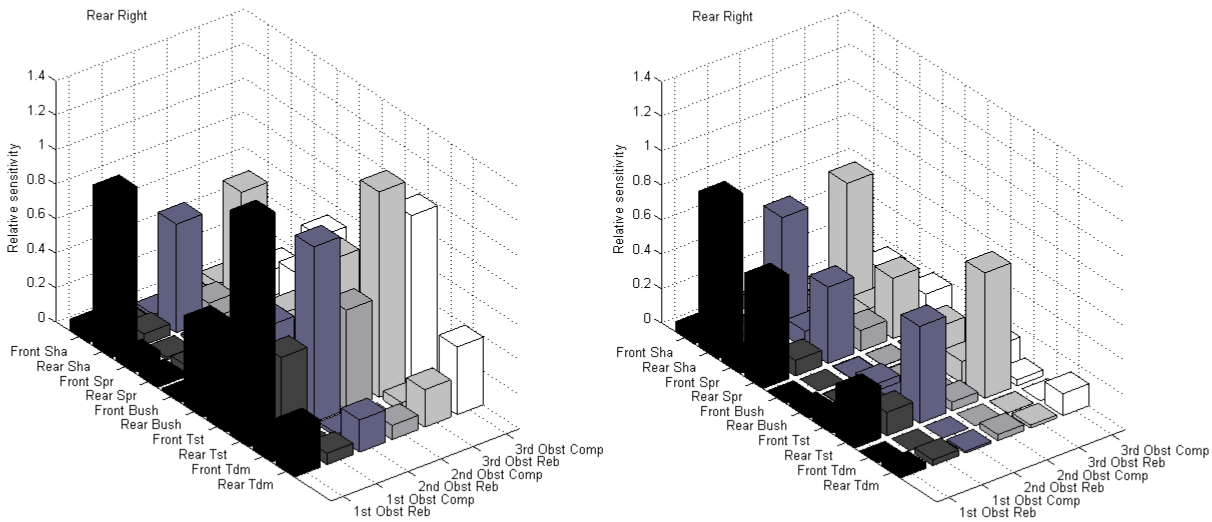

Fig. 8 Relative sensitivity of relative deflection of the right rear air springs on the change in the individual parameters: a) at the trolleybus speed 44.13 km/h and at the obstacles height 16 mm; b) at the trolleybus speed 10 km/h and at the obstacles height 60 mm (taken from Polach & Hajžman, 2008b).

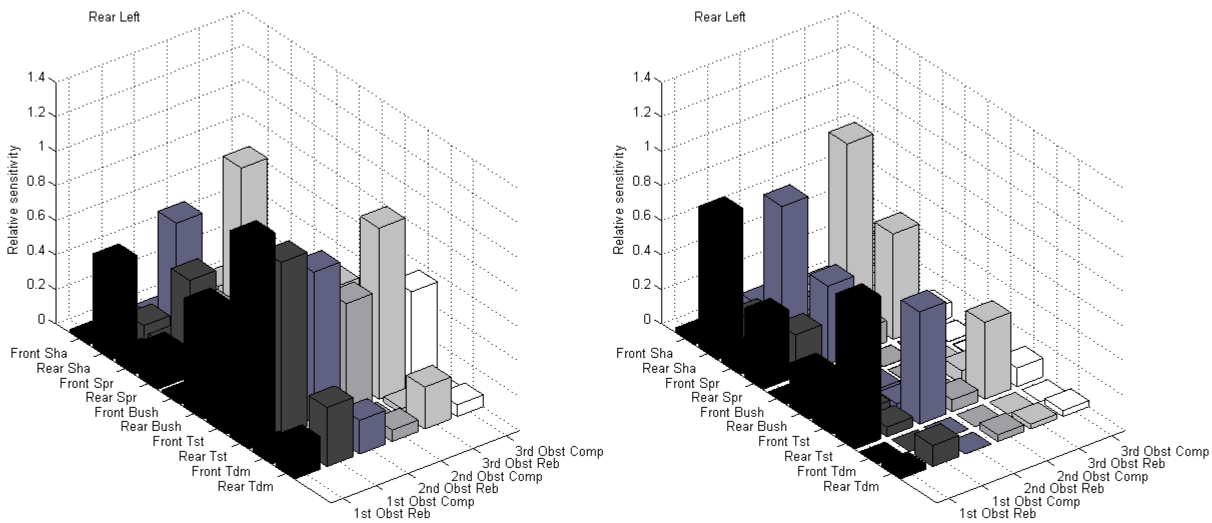

Fig. 9 Relative sensitivity of relative deflection of the left rear air springs on the change in the individual parameters: a) at the trolleybus speed 44.13 km/h and at the obstacles height 16 mm; b) at the trolleybus speed 10 km/h and at the obstacles height 60 mm (taken from Polach & Hajžman, 2008b).

In accordance with Polach & Hajžman (2006) the force-velocity characteristics of the hydraulic shock absorbers have a greater influence on the extreme values of the time histories of the air springs relative deflections than the force-deflection characteristics of the air springs. In Polach & Hajžman (2008b) the force-deflection characteristics of the front air springs have a greater influence on the extreme values of the air springs relative deflections than the force-velocity characteristics of the front hydraulic shock absorbers (see Figs. 7 and 8).

In comparison with Polach & Hajžman (2008a) the tire radial force-deformation characteristics have not a significantly greater influence on the results of driving on the virtual test track than the tire radial damping characteristics and the rear tire radial force-deformation characteristics considerably influence even the extreme values of the time histories of the front air springs relative deflections. Those findings conform with Polach & Hajžman (2008b).

The force-deformation characteristics of the shock absorbers bushings and the tire radial damping characteristics have, as it also was found in Polach & Hajžman (2006), Polach & Hajžman (2008a) and Polach & Hajžman (2008b), a minor influence on the results of the simulations.

6. Conclusion

The vertical dynamic properties of the ŠKODA 21 Tr low-floor trolleybus were investigated on the artificially created test track when simulating driving on the virtual test track with the most complex multibody model created in the **alaska 2.3** simulation tool.

The sensitivity analysis of the influence of the change of the spring-damper suspension elements characteristics and the tires radial characteristics in the trolleybus multibody model was performed.

The results of the sensitivity analysis in the course of the simulations with the multibody model of the ŠKODA 21 Tr low-floor trolleybus (at the speed 44.13 km/h and at the obstacles height 16 mm) showed, in accordance with Polach & Hajžman (2008b), that the force-deformation characteristics of the tires, the force-velocity characteristics of the hydraulic shock absorbers and the force-deflection characteristics of the air springs have the greatest influence on the results of driving on the virtual test track out of the characteristics of the decisive spring-damper structural elements. In contradiction to Polach & Hajžman (2008b) (the simulations at the trolleybus speed 10 km/h and at the obstacles height 60 mm) the force-deformation characteristics of the tires have a greater influence than force-velocity characteristics of the hydraulic shock absorbers on the result of simulations. The differences among the results of the sensitivity analyses at the different conditions of the test drives simulations are caused by strongly nonlinear character of the spring-damper structural elements used in the trolleybus multibody model.

7. Acknowledgements

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