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PARAMETER IDENTIFICATION OF A LEAF SPRING DYNAMIC MODEL

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Summary: *The problems of leaf springs behaviour are very complex due to the presence of many factors and influences. Mainly the flexibility, contacts and friction play important roles. The paper deals with the modelling and parameter identification of a leaf spring model intended for vehicle dynamics simulations. The investigated leaf spring is used in the coal wagon multibody model. The presented methodology is based on the experimental measurements of particular leaf spring, the numerical simulations with the multibody models and developed optimization procedures.*

1. Introduction

Leaf springs are common suspension elements of road and rail vehicles and therefore their dynamic models have to be developed for the simulations. The problems of leaf springs behaviour are very complex due to the presence of many factors and influences. Mainly the flexibility, contacts and friction play important roles. Many types of dynamic models can be created in order to incorporate leaf springs into the complex multibody models of the whole road or rail vehicles. The chosen type of the model is strongly dependent on the available experimental data.

In the framework of the EU Footprint EUREKA project the vertical dynamics of the MGR Coal Hopper HAA two-axle open coal wagon was experimentally investigated (Chvojan et al., 2004). This wagon is characterized by the so called UIC suspension composed of five-leaf springs (Fig. 1). The standard type of the used leaf springs is the parabolic steel five-leaf spring (Fig. 2 – left). In order to improve dynamic characteristics of the wagon and durability of the suspension system the composite GRP (glass-reinforced-plastic) two-leaf springs (Fig. 2 – right) were used instead of the steel ones. The goal of the experimental measurements was the study of the different leaf springs influence on the freight wagon vertical dynamics.

The experiments were also supplied by the numerical simulations (Polach et al., 2006). The multibody models of the MGR Coal Hopper HAA wagon characterized by various loads were created in the **alaska** simulation tool (Maisser, P. et al., 1998). The parabolic steel leaf springs and GRP leaf springs were described by their average static deformation characteristic (acting force of the leaf spring as a function of the leaf spring deformation, without the consideration of hysteresis) and by the constant damping coefficient obtained by drop tests (Chvojan et al.,

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Fig. 1 The MGR Coal Hopper HAA wagon on the test stand.

2004). However the comparison of the experimental and numerical results showed the considerable differences between the experimental and numerical approaches to the investigation of the coal wagon dynamics. From the analysis of the problem it is obvious that the main role in the inaccuracy of the numerical results is played by the mathematical model of leaf springs. This fact was the motivation for further study of the leaf spring modelling in the framework of the multibody models.

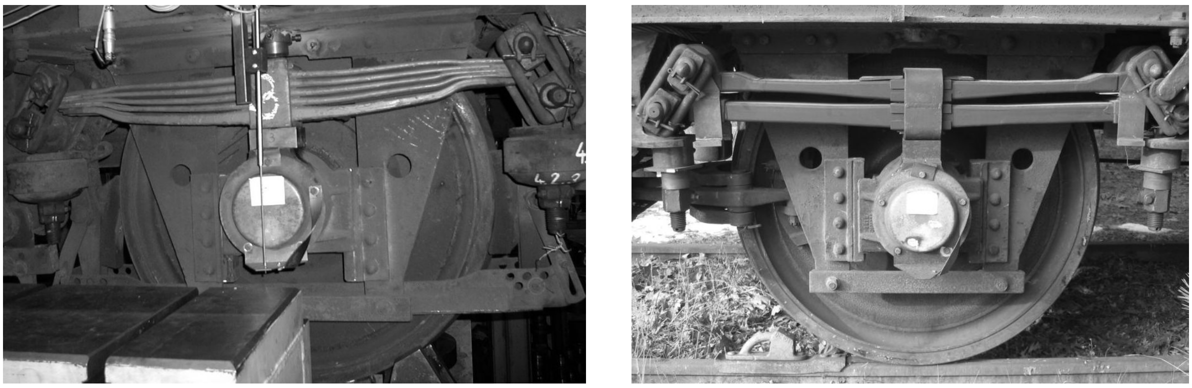

Fig. 2 The steel parabolic leaf spring and the GRP leaf spring.

The SIMPACK simulation tool (INTEC, 2008) was chosen as the main tool for further analyses of the coal wagon vertical dynamics with different types of the leaf springs. At first the force-deformation characteristic and spring damping were improved with respect to the better accordance of the numerical and experimental results of kinematic excitation tasks (Polach et al., 2007). The second improvement was based on the development of a new original leaf spring model composed of several bodies connected by joints (Polach & Hajžman, 2008). The special force elements characterizing elastic and friction forces were introduced between the bodies.

In both cases of the leaf spring model improvements the parameters of the force elements were determined by manual changes with respect to expert estimations. The contribution and the goal of the work presented in this paper is to develop the general identification methodology for the determination of the leaf spring dynamic model parameters on the basis of available experimental data. The created procedure will be verified using both GRP and steel parabolic leaf spring models. The identified leaf spring models will be used in the coal wagon multibody model. The presented methodology is based on the experimental measurements of the particular leaf spring, the numerical simulations with the multibody models and developed optimization procedures.

2. General approaches to the modelling of leaf springs

The mathematical modelling of leaf springs is a very complex task due to their complicated design and structure. The main feature of the leaf spring behaviour is the hysteresis caused by the friction between the particular leaves. Another difficulty in the leaf spring modelling is the involving of the complex dynamic elastic behaviour in all three directions of the possible force loading. The models intended mainly for the vehicle vertical dynamics will be discussed in this section.

The possible approaches to the modelling of leaf springs in the framework of multibody models can be divided (Blundell & Harty, 2004) into several groups:

- *Simple equivalent spring model*
  The elastic behaviour of a leaf spring can be described by the spring model with the calculated stiffness coefficients, see e.g. Kovanda et al. (1997). This model is the simplest and the most inaccurate.

- *Spring model using the nonlinear deformation characteristic*
  The dynamic behaviour of the leaf spring modelled by the equivalent spring can be improved by the implementation of the nonlinear deformation characteristic that has to be measured on the real leaf spring. As this characteristic shows strong hysteresis the used curves have to be obtained by the averaging of the values, e.g. Polach et al. (2006), Polach et al., (2007).

- *Spring model using the nonlinear deformation characteristic with hysteresis*
  If the chosen tools allow to use the complete deformation characteristic including the hysteresis loop it is the best choice for the leaf spring approximation by the nonlinear spring in the multibody model. The measured characteristic can be employed directly or the mathematical model composed of spring and friction elements with identified coefficients (Petersen & Hoffman, 2003) can be used.

- *Approximation by the finite segment method*
  Flexible bodies in multibody dynamics can be modelled by the finite segment method (Shabana, 1997), sometimes also called the rigid finite element method (Wittbrodt et al., 2006). This approach is based on the approximation of a deformable body by the set of

rigid bodies coupled by kinematic constraints (joints) with imposed stiffnesses and damping. The mass, stiffness and damping properties are calculated using the basic mechanical principles. This method can capture naturally the flexible behaviour in all three directions and therefore it is not limited only to the investigation of vertical dynamics. The contacts and friction can be introduced in this type of models but it is rather difficult and not so natural mainly in case of contacts.

- *Approximation by the lumped mass and beam elements*
  This approach does not use the measured deformation characteristic compared to the previous mentioned spring models based on the force elements only. The particular leaves are modelled as a combination of lumped masses and massless beams. The contacts and friction can be easily introduced between the leaves. This method may be the best choice if the available software tools do not allow to use the flexible bodies. It is similar to the finite segment method but the number of degrees of freedom is higher.

- *Detailed model using the flexible multibody approaches*
  The best method is obviously the implementation of the leaf spring as a flexible body or as a set of flexible bodies with defined contacts. It can be done by means of special modules of the commercial software tools (e.g. in SIMPACK) or an original implementation (Sugiyama et al., 2006) can be made. The flexible multibody methods can be based on the floating frame of reference formulation, which describes the deformable body in the moving body reference frame, or on the absolute nodal coordinate formulation, which uses so called global shape functions for the approximation of body deformations (Shabana, 2005).

The modelling of the leaf spring damping in the framework of multibody models is determined by the chosen leaf spring model. The dissipation of the kinetic energy in the course of a suspension vibration can be caused mainly by friction effects and by a material damping.

The approach used for the multibody modelling of the leaf springs presented in this paper is based on the finite segment method with nonlinear elastic forces and friction. It was chosen because it is the compromise between the complex behaviour and the number of degrees of freedom of the model. The nonlinear force elements allow to avoid the solution of the contact problem. The particular leaf spring models are implemented in the SIMPACK simulation tool and will be described in next sections including the parameter identification procedure.

3. Briefly about the SIMPACK simulation tool

The SIMPACK simulation tool (INTEC, 2008) is being developed in INTEC GmbH, Wessling, Germany. Similarly as other MBS software it is intended for investigating kinematic and dynamic properties of a nonlinear three-dimensional coupled mechanical system consisting of many bodies. The approach to solving the tasks in the field of mechanics using computer models, which is based on the systems of bodies, enables to solve substantially more general problems than the approach based on the finite element method because it is not dependent on the continual model of the investigated system. As this approach is more general and due to the character of the studied mechanical systems the demands for the computing time of the solution of the nonlinear equations system are growing. When creating a multibody model it is necessary to pay attention to choosing the number of bodies, the number of kinematic pairs and especially the total number of degrees of freedom in kinematic pairs of a mechanical system, i.e. to interpret optimally the physical substance of the solved problem. The total number of degrees of freedom in kinematic pairs determines the number of constructed nonlinear equations of motion, solution of which should be within a real period of time.

Multibody models are created by a finite number of bodies connected by kinematic pairs and massless force elements, which enable to model spring-damper structural parts. With respect to the multibody models creating methodology and automatic generating of the differential equations in the SIMPACK simulation tool kinematic pairs are classified into two types (two separate groups within the framework of modelling in the SIMPACK simulation tool) — joints and constraints. Exactly one joint with a given number of degrees of freedom belongs to each body, which enables a body motion considering the previous body in a kinematic chain. Constraints are utilized for the closing of kinematic chains, i.e. for creating kinematic loops, and constraining the relevant degree of freedom. This separation of couplings is caused by the multibody formalism based on the relative coordinates. Bodies can move in space in the framework of joints, constraints, force elements, the way of coupling to the reference frame and boundary conditions. Each body is defined by inertial properties (mass, centre of mass coordinates and moments of inertia). It is possible to bind different markers to the bodies. A marker is a point, in which a local coordinate system is defined. Markers can be used to locate reference frames, to define the centre of mass. Through the markers it is possible to couple bodies by joints, constraints and force elements, it is possible to act on bodies by applied forces and torques, etc. After creating a multibody model it is possible to simulate the modelled system motion. At simulating motion with multibody models in the MBS software non-linear equations of motion are generated. The equations are solved by means of numerical time integration. Generally, displacements, velocities and accelerations of the individual bodies, forces and torques acting in kinematic pairs and force elements are the monitored quantities. It is possible to obtain results in the form of time series, in the form of graphs or in the form of multibody model visualisation (static or with animation). In outputs in the form of graphs it is possible to compare e.g. influences of changes of various parameters of the multibody model on the simulations results, it means to evaluate operatively the influences of permitted design adjustment on the desired kinematic and dynamic properties of the real structure.

Besides the basic SIMPACK Kinematics & Dynamics module it is possible to buy additional SIMPACK simulation tool modules and data interfaces with other software. In ŠKODA VÝZKUM s.r.o. there are at disposal the SIMPACK Automotive+ module (support of road vehicles modelling including tire models), the SIMPACK Wheel/Rail module (support of rail vehicles modelling including wheel-rail contact models), the SIMPACK Contact module (support of contacts between bodies modelling) and the SIMPACK FEMBS module (support of flexible bodies modelling).

4. Leaf spring model identification

The real leaf springs (Fig. 2) are modelled by means of the finite segment method. The leaves are divided into three articulated rigid bodies connected by spherical joints. Other rigid bodies are used for the modelling of shackles (chain links) in order to connect the leaf spring with the car body (see Fig. 3 for the visualization and Fig. 4 for the kinematic scheme of the SIMPACK model). The force (torque) elements are introduced between the bodies in the spherical joints.

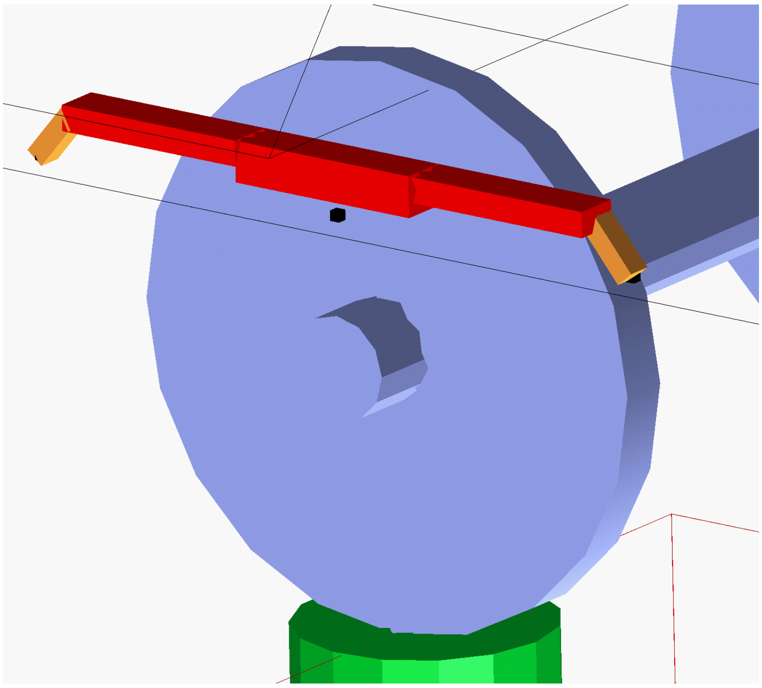

Fig. 3 Visualization of the leaf spring multibody model connected to the wheelset and car body (in the SIMPACK).

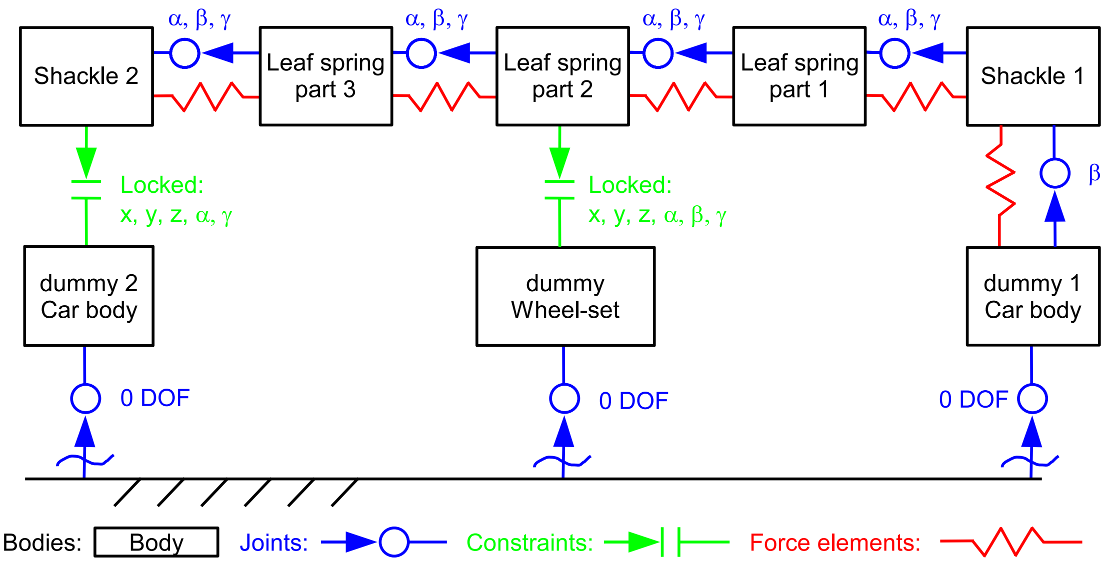

Fig. 4 Kinematic scheme of the leaf spring multibody model substructure for the SIMPACK.

Each torque has three spatial components and represents nonlinear elastic behaviour, damping and friction effects.

In order to determine the static elastic behaviour the experimental measurement was set up (Černý, 2005). The leaf springs were mounted on the special test stand (see Fig. 5) and loaded by means of determined force with the frequency 0.05 Hz. The main idea of the identification of the leaf springs multibody model parameters is characterized by following considerations:

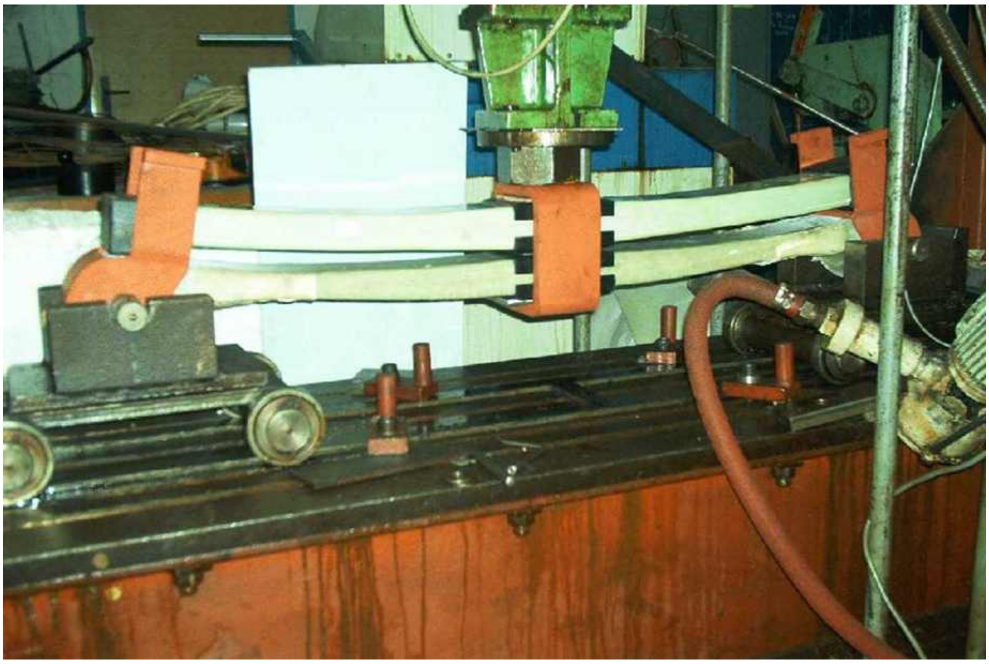

Fig. 5 The illustrative photo of the measurement of the GRP leaf spring static characteristic, taken from Černý (2005).

- The force-deformation characteristics obtained from the numerical simulations should be the same as the characteristic obtained by the experimental measurement. The transverse and longitudinal flexible properties can be estimated using the determined vertical properties.
- The tuning of the parameters connected with the elements (shackles), which are not present in the experimental set up but can influence the dynamic behaviour (friction in the connection by chain links), can be made in the second step of the identification process on the basis of numerical simulations and experimental measurements of the vertical dynamics of the coal wagon.

The visualization of the numerical model of the experimental stand in the SIMPACK simulation tool is shown in Fig. 6. The kinematic scheme of the multibody model is shown in Fig. 7. The torque M_e between two rigid body segments representing elastic properties can be expressed as

$$M_e(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{cases} k_1 q + b\dot{q} & \text{for} \quad q \leq q_1 \\ k_1 q + (k_2 - k_1)(q - q_1) + b\dot{q} & \text{for} \quad q > q_1 \end{cases}, \tag{1}$$

where q is the joint coordinate (angle), k_1 is the stiffness coefficient in the first part of the characteristic, k_2 is the stiffness coefficient in the second part of the curve, q_1 is the joint coordinate value defining the position of the curve slope change and b is the material damping coefficient. The implementation of break of the torque-deformation characteristic is made by the `step` function in order to make the course of the curve continuous. The grow of the characteristic stiffness for higher deformations of the spring is caused by the contact of the main leaves of the spring with the supplementary leaf. The solution of the contact task is avoided in this case and therefore the calculation model is faster.

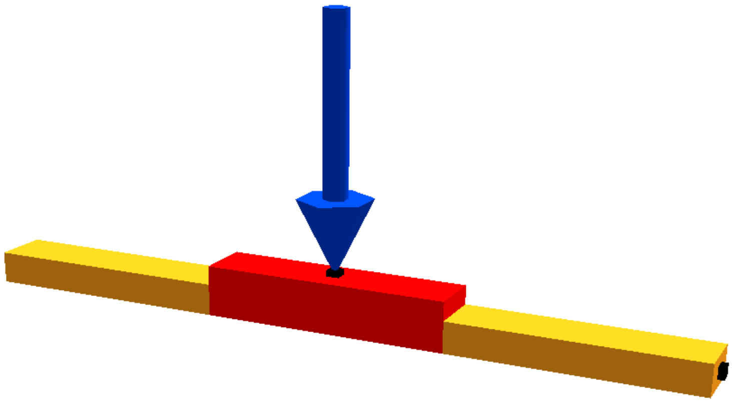

Fig. 6 The visualization of the multibody model used for the determination of the leaf spring static characteristics in the SIMPACK.

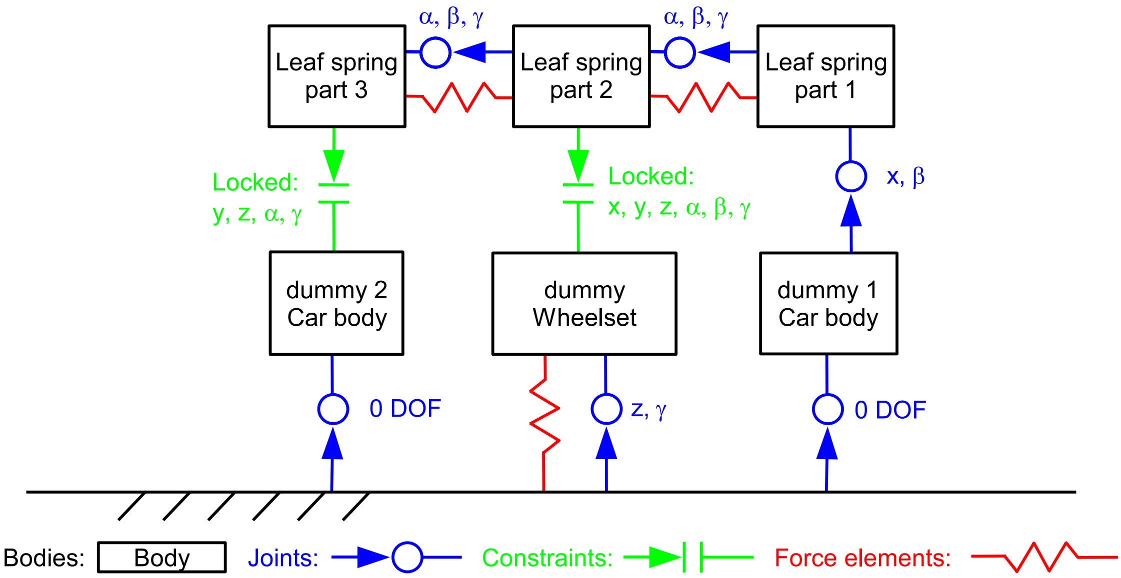

Fig. 7 The kinematic scheme of the multibody model used for the determination of the leaf spring static characteristics.

The friction effects caused by the friction between leaves are modelled by special friction force (torques) elements (INTEC, 2008) applied between the rigid body segments similarly as the previous elastic torque. The value of this torque is calculated by means of the value of the elastic torque (to catch the growing friction in the course of higher deformations and successive higher contact forces), by means of the frictioin coefficient f and so called effective moment arm r_f. The last value important for the friction moment calculation is the switch velocity v_ε denoting the range of the transition of the velocity sign (velocity direction).

The identification process can be viewed as an optimization process with the goal of the best accordance of the experimentally and numerically obtained force-deformation characteristics. The important parameters of the multibody model can be ordered into the vector of design parameters

$$\boldsymbol{p} = [\, k_1 \; k_2 \; q_1 \; f \; r_f \; v_\varepsilon \,]. \tag{2}$$

For the sake of the better numerical handling of the identification (optimization) problem the relative parameters $\bar{\boldsymbol{p}}$ are introduced as the ratio

$$\bar{\boldsymbol{p}} = \frac{\boldsymbol{p}}{\boldsymbol{p}_0} \tag{3}$$

of the real values $\boldsymbol{p}$ and initial values $\boldsymbol{p}_0$ of the particular parameters.

The next problem is the selection of the objective function, which is the best quantification of the specified goal. It will be limited on the objective function with a scalar value because of the possible optimization procedure. Various types of the objective function fulfilling this criterion can be found. For example the correlation coefficient is one the possibilities. Other types of the objective functions can be based on the evaluation of the difference between chosen point of the original and the calculated characteristics. Such objective function can be of the form

$$\psi(\bar{\boldsymbol{p}}) = \sum_{i=1}^{N} |\, d_{mi} - d_{ci}(\bar{\boldsymbol{p}})\,|, \tag{4}$$

while N is the number of points in which the characteristic is compared, d_{mi} is the deformation for the i-th chosen point of the measured characteristic and $d_{ci}(\bar{\boldsymbol{p}})$ is the deformation for the i-th chosen point of the calculated characteristic dependent on the design parameters. The real value of the objective function was calculated by means of 200 chosen points of the characteristic.

The particular optimization procedure was chosen from the possibilities of the Optimization toolbox of the MATLAB system. Since any design constraints were not imposed on the problem the gradient method implemented in `fminunc` function and the simplex Nelder-Meed procedure implemented in `fminsearch` function were the possible procedures. The simplex method was found to be the better procedure for this optimization problem.

The optimization process was thus managed by the MATLAB system. The overall method of the objective function evaluation can be summarized into several steps:

1) Modification of the source SIMPACK model files on the basis of the input design parameters (in the MATLAB). The source model files are text files.

2) Batch call of the SIMPACK simulation tool in order to perform the numerical simulation of the (quasi-)static loading of the leaf spring.

3) Batch call of the SIMPACK postprocessor in order to evaluate the force-deformation characteristic for the actual design parameters.

4) Import of the calculated characteristic to the MATLAB system, interpolation of the deformation in the chosen points and calculation of the objective function value (4).

The initial values of the design parameters were manually estimated. The results of the optimization (identification) process are shown in Figs. 8 to 11. The measured force-deformation characteristic of the GRP leaf spring is compared with the initial optimization characteristic calculated by means of the multibody model in Fig. 8. The measured and the final identified characteristics of the GRP leaf spring are compared in Fig. 9. Analogous comparisons are shown in Figs. 10 and 11 for the steel parabolic leaf spring.

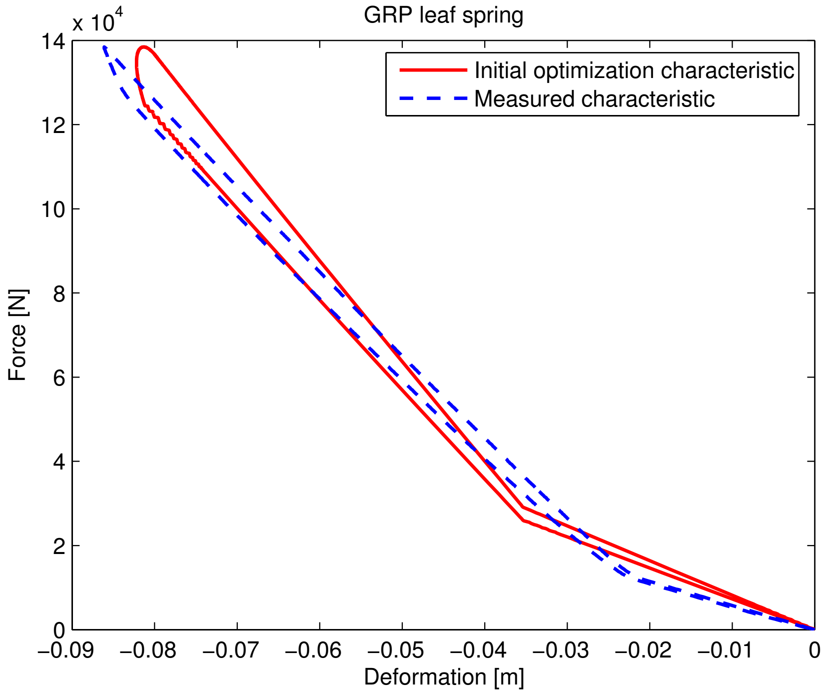

Fig. 8 The measured force-deformation characteristic and the initial optimization characteristic of the GRP leaf spring.

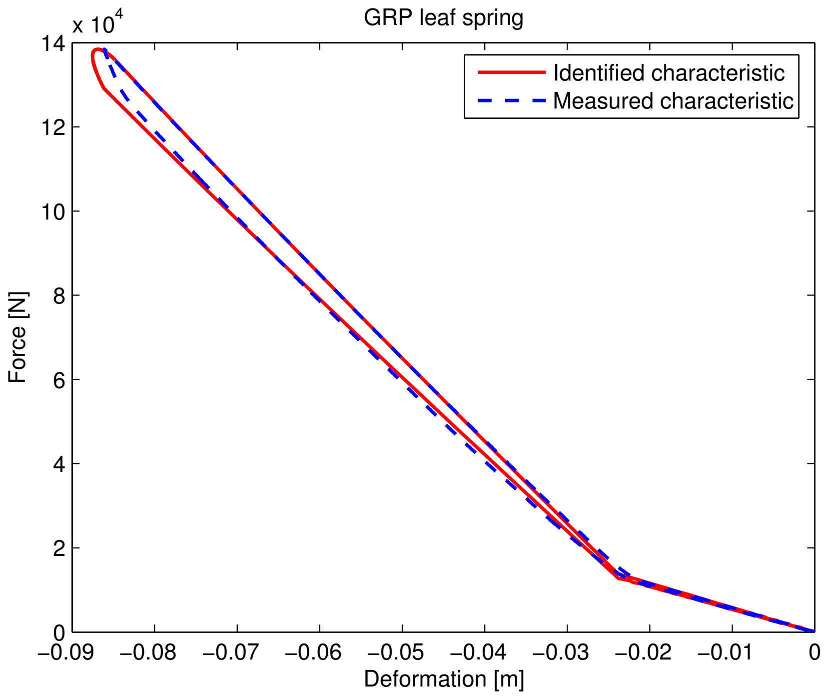

Fig. 9 The measured force-deformation characteristic and the final identified characteristic of the steel parabolic leaf spring.

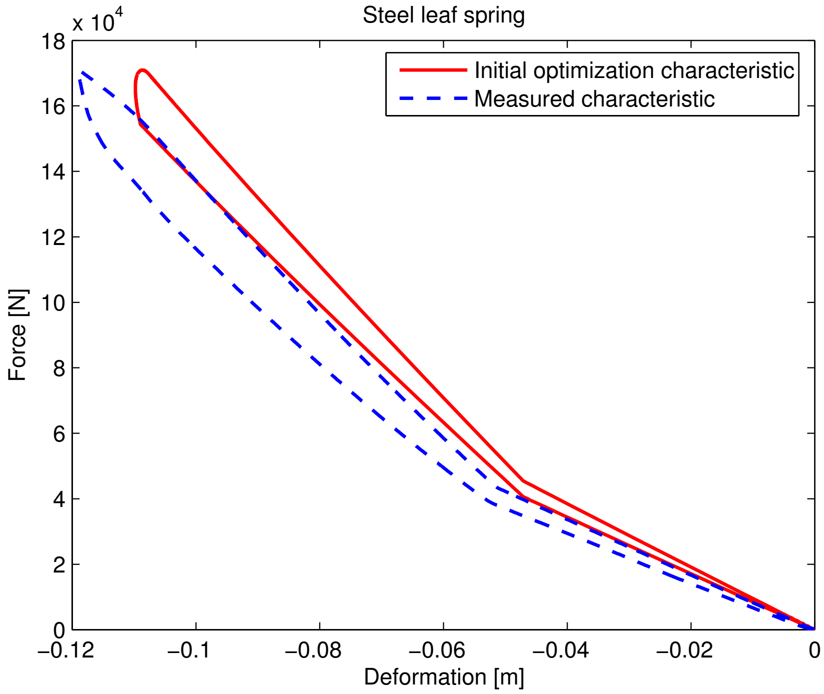

Fig. 10 The measured force-deformation characteristic and the initial optimization characteristic of the steel parabolic leaf spring.

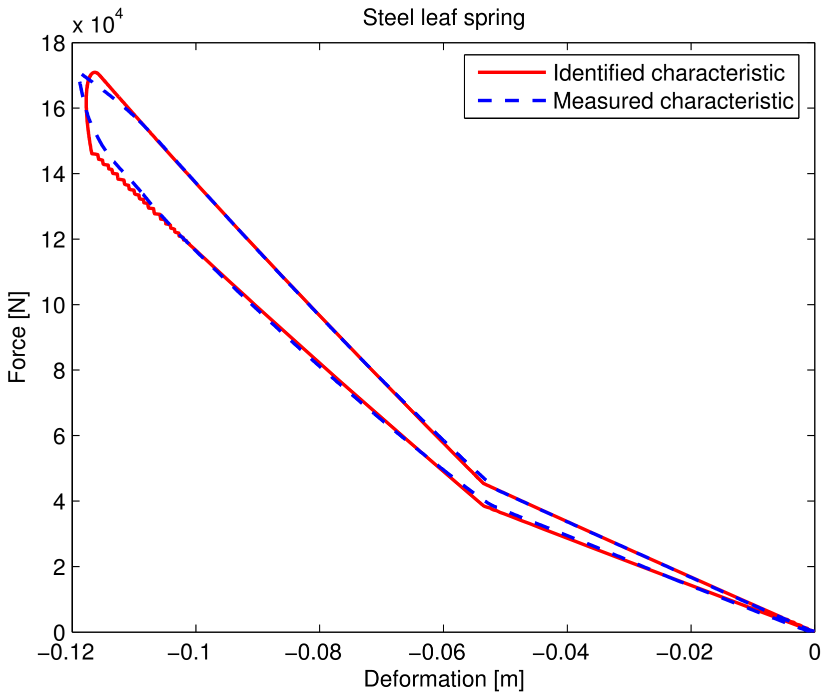

Fig. 11 The measured force-deformation characteristic and the final identified characteristic of the steel parabolic leaf spring.

The initial values of the optimization parameters in the beginning of the identification process and the optimal values of the identified parameters are shown in Tab. 1 for the GRP leaf spring and in Tab. 2 for the steel leaf spring.

Tab. 1 Initial and identified values of the optimization parameters for the GRP leaf spring.

	Relative values		Absolute values	
	Initial $\bar{\boldsymbol{p}}_0$	Optimal $\bar{\boldsymbol{p}}$	Initial $\boldsymbol{p}_0$	Optimal $\boldsymbol{p}$
k_1 [Nm/rad]	1	0.7204	$6 \cdot 10^4$	$4.3224 \cdot 10^4$
k_2 [Nm/rad]	1	0.8479	$1.7 \cdot 10^5$	$1.4415 \cdot 10^5$
q_1 [rad]	1	0.6747	0.09	0.0607
f [-]	1	0.5399	0.7	0.3779
r_f [m]	1	1.1886	0.08	0.0951
v_ε [rad/s]	1	1.7082	$0.3 \cdot 10^{-3}$	$0.5108 \cdot 10^{-3}$

Tab. 2 Initial and identified values of the optimization parameters for the steel leaf spring.

	Relative values		Absolute values	
	Initial $\bar{\boldsymbol{p}}_0$	Optimal $\bar{\boldsymbol{p}}$	Initial $\boldsymbol{p}_0$	Optimal $\boldsymbol{p}$
k_1 [Nm/rad]	1	0.8557	$7 \cdot 10^4$	$5.9897 \cdot 10^4$
k_2 [Nm/rad]	1	0.9450	$1.4 \cdot 10^5$	$1.3230 \cdot 10^5$
q_1 [rad]	1	1.1378	0.12	0.1365
f [-]	1	1.2203	0.7	0.8542
r_f [m]	1	1.1852	0.08	0.0948
v_ε [rad/s]	1	1.0566	$0.3 \cdot 10^{-3}$	$0.3170 \cdot 10^{-3}$

5. Conclusions

The methodology for the parameter identification of the multibody leaf spring model is presented in this paper. It was motivated by the effort to create the accurate model of rail (or road) vehicles with the leaf springs used as main suspension elements. The SIMPACK simulation tool was chosen as the suitable software for the creation of the multibody model of the GRP (composite, glass-reinforced-plastic) leaf spring and of the steel parabolic leaf spring. The model of a whole rail vehicle (the coal wagon in this case) was also created. The leaf spring was modelled as a system of coupled rigid bodies using the finite segment method with imposed force (torque) elements characterizing elastic properties, damping and friction.

The identification procedure was built as the optimization process with the design parameters representing the force (torque) elements and with the objective function evaluating the accordance of the measured and calculated force-deformation characteristics. The objective function was calculated by the sum of the absolute values of the curves differences in the chosen curve points. The Nelder-Meed simplex method implemented as the function of the MATLAB's Optimization toolbox was used for the solution of the optimization problem. It can be concluded that the developed identification process is correct and effective on the basis of the presented results.

6. Acknowledgement

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