ENGINEERING MECHANICS 2009
National Conference with International Participation
Svratka, Czech Republic, May 11 – 14, 2009

pp. 207–217
Paper **#214**

ACTIVE SUSPENSION SYSTEM FROM THE LATERAL DYNAMICS POINT OF VIEW

V. Drobný [1], M. Valášek [2], O. Vaculín [3]

Summary: *This paper presents the investigation of active suspension control with the obejctive to stabilize the lateral vehicle motion in similar way like ESP does. The vehicle with at least four wheels gives one or more redundant vertical forces. This effect can be used for the different distribution of vertical suspension forces in such a way that resulting lateral and longitudinal forces create the required correction torque for vehicle lateral dynamics stabilization. The amount of the generated stabilization torque was measured and compared to a passive vehicle using the so-called* $\beta - method$*, which is based on the evaluation of the vehicle steady-state cornering. Proposed stabilisation control algorithm was verified on dynamic avoidance manoeuvres.*

1. Introduction

The lateral dynamics of the vehicle describes the vehicle behaviour during manoeuvres, in which together with the longitudinal trajectory the lateral trajectory is affected. The vehicle movement could be influenced by the stabilization system intervention in order to assist and ensure the driveability and safety of the vehicle. The stabilization intervention could be managed by some active systems. One of them is the Electronic Stability Programme (ESP), which uses the brake forces for generation of the stabilization torque. The basis of this control of vehicle lateral stability is the nonlinear characteristics of the tyres. The lateral and longitudinal tyre force properties describes in simplified form the Kamm's circle (Genta 2006), Fig. 1. The sum of the components of the longitudinal and lateral forces is limited by this circle. If the longitudinal brake or drive force is applied, the side force is reduced. The stabilization moment is produced as lateral stiffness change on the front and rear axle in addition to moment caused by brake force on appropriate wheel.

The vehicle state estimation by the driver is limited. As presented in (van Zanten, Erhardt & Pfaff 1995) the human driver is not able to recognize the vehicle limits in relationship to the actual coefficient of adhesion. That is the reason why ESP-like systems play significant role in vehicle stabilization.

[1] Ing. Vladislav Drobný: TÜV SÜD Auto CZ s.r.o., Novodvorská 994/138, 142 21 Praha 4, tel.: +420 239 046 977, e-mail: vladislav.drobny@tuv-sud.cz

[2] Prof. Ing. Michael Valášek, DrSc.: Department of Mechanics, Biomechanics and Mechatronics, Faculty of Mechanical Engineering, Czech Technical University in Prague; Karlovo nám. 13, 121 35 Praha 2

[3] Ing. Ondřej Vaculín, Ph.D.: TÜV SÜD Auto CZ s.r.o., Novodvorská 994/138, 142 21 Praha 4, tel.: +420 239 046 980, e-mail: ondrej.vaculin@tuv-sud.cz

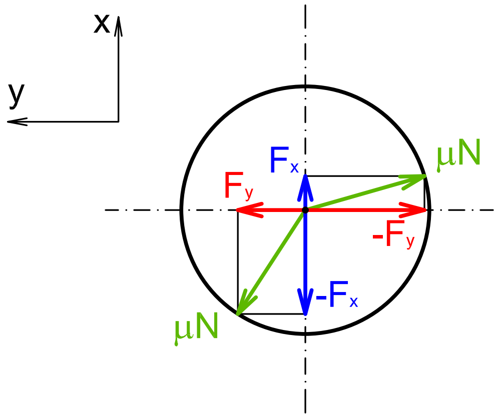

Figure 1: Kamm's circle

The devices are partially connected to each other by networks. The communication between earlier stand-alone devices will bring added value not only in operational comfort of the vehicles, but particularly in active and passive safety. The Global Chassis Control covers particular stand alone systems into one unit. The new generation of ESP II (Rieth & Schwarz 2004) integrates standards of the ESP system with the active front steering system. The steer wheel angle superposition to the driver's wish could improve the operational comfort as well as the active safety of the vehicle.

Brake based control systems are very effective at vehicle stability limits but they can affect negatively the longitudinal dynamics of vehicle in the driveability range. The objective of the vehicle control systems developed for the future will be to preserve vehicle driveability while ensuring the stability of the vehicle. The lateral stabilization of the vehicle, which uses braking forces in ESP system, can be alternatively provided by the different setting of suspension forces. The vehicle has at least four wheels and it gives one or more redundant vertical forces in order to comply with balance of gravity and vertical force. This freedom can be used for the different distribution of vertical suspension forces in such a way that resulting lateral and longitudinal forces create the required correction moment for lateral dynamic vehicle stabilization.

This problems have been already studied on magneto-rheological dampers in semi-active suspension control (Bodie & Hac 2000), in which short response time of the maximal damper force activation together with relative high forces by low damper velocities of magneto-rheological damper offer expected stabilization effect. The study also describes the axle lateral stiffness change through the vertical forces variation. As shows Fig. 2, the side forces that could be transferred on one axle are maximal, if left and right wheel vertical forces are equal. With the same sum of vertical forces, but laterally redistributed, the side axle force transfer is reduced. This principle of changing the lateral axle stiffness in the front and rear axle give us the opportunity for stabilizing the vehicle movement using active suspension.

Based on the similar principle, the active anti-roll bars become very popular during the last years. Mostly in the SUV vehicle category, in which beside vehicle driveability, the roll angle is to be controlled. The tuning of control of the active anti-roll bars in order to improve agility and safety was published e.g. in (Danesin, Krief, Sorniotti & Velardocchia 2003). Generally, for the roll angle reduction the relation is used, in which the active force of the anti-roll bar respectively roll stiffness of the vehicle is increased with lateral acceleration increment.

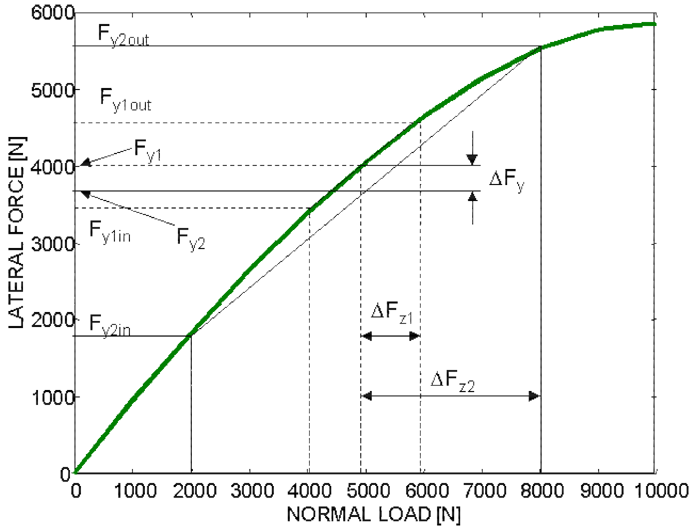

Figure 2: The axle lateral force change

2. Vehicle Model

The vehicle model consists of several subsystems. The first subsystem is the reference model, which describes the ideal lateral vehicle behaviour. This model fulfils the difficult task of the estimation of the driver manoeuvre intention. It is based on the single track vehicle model and was comprehensively described in (Drobný 2007). In the next the controlled vehicle model was created, which is designed to be more complex including lateral dynamics, vertical dynamics, longitudinal dynamics with driveline, differential as well as the complex tyre model.

The model of the vehicle is modelled as a two track model as shown in Fig. 3. Its equations of motion are following:

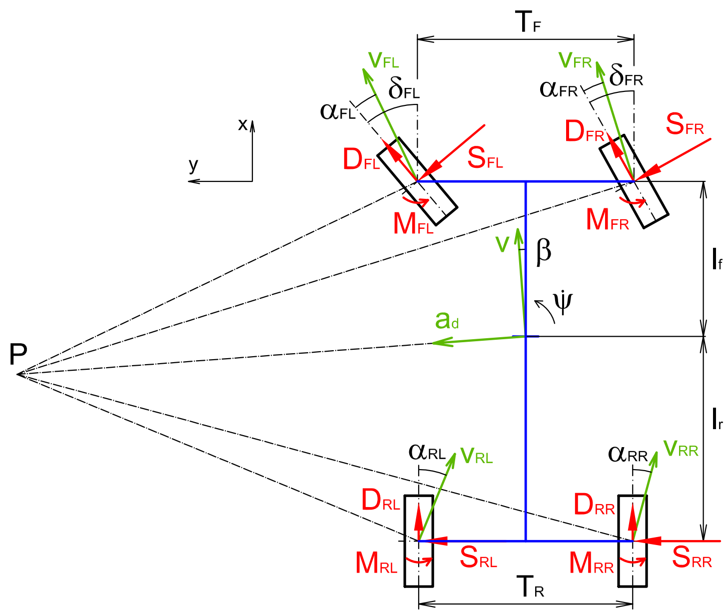

Figure 3: Scheme of the lateral mathematical model of the vehicle

$$
\begin{aligned}
\dot{v} = & \ \tfrac{1}{m\cos\beta}\left(mv(\dot{\psi}+\dot{\beta})\sin\beta - S_{FL}\sin\delta_{FL} - S_{FR}\sin\delta_{FR}\right) \\
+ & \ \tfrac{1}{m\cos\beta}\left(+D_{FL}\cos\delta_{FL} + D_{FR}\cos\delta_{FR} + D_{RL} + D_{RR}\right) ,
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
\dot{\beta} = & \ \tfrac{1}{mv\cos\beta}\left(-m\dot{v}\sin\beta + S_{FL}\cos\delta_{FL} + S_{FR}\cos\delta_{FR} + S_{RL} + S_{RR}\right) \\
+ & \ \tfrac{1}{mv\cos\beta}\left(+D_{FL}\sin\delta_{FL} + D_{FR}\sin\delta_{FR}\right) - \dot{\psi} ,
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
\ddot{\psi} = & \tfrac{1}{J_z}\left[S_{FL}\left(\cos\delta_{FL}l_f + \sin\delta_{FL}\tfrac{T_f}{2}\right) + S_{FR}\left(\cos\delta_{FR}l_f - \sin\delta_{FR}\tfrac{T_f}{2}\right)\right] \\
& + \tfrac{1}{J_z}\left[-S_{RL}l_r - S_{RR}l_r\right] \\
& + \tfrac{1}{J_z}\left[-D_{FL}\left(\cos\delta_{FL}\tfrac{T_f}{2} - \sin\delta_{FL}l_f\right) + D_{FR}\left(\cos\delta_{FR}\tfrac{T_f}{2} + \sin\delta_{FR}l_f\right)\right] \\
& + \tfrac{1}{J_z}\left[-D_{RL}\tfrac{T_r}{2} + D_{RR}\tfrac{T_r}{2} + M_{FL} + M_{FR} + M_{RL} + M_{RR}\right] .
\end{aligned} \tag{3}
$$

Parameter δ_f represents the average value of the front wheels steering angles δ_{FL}, δ_{FR}. Variables S_{FL}, S_{FR}, S_{RL}, S_{RR} are the tyre lateral forces, analogously D_{FL} drive forces and M_{FL} aligning torques. Parametres l_f and l_r are the CG (centre of gravity) distances from front and rear axle. J_z means the moment of inertia around the z-axis of the vehicle. As for the parameter T, the simplification which is based on the average value of the front and rear track T_F, T_R was used. The tyre lateral forces are computed from the Pacejka magic formula. As a result of the equations (2) and (3), there are evaluated two parameters: the sideslip angle of the vehicle β and the yaw rate $\dot{\psi}$.

2.1. Vertical Forces Model

The vertical suspension model is added into the 3DOF lateral dynamic model. This model contains next 3DOF, the whole vehicle model has then 6DOF. The added degrees of freedom are vertical displacement of the chassis z, roll angle γ and pitch angle ε. The final system of three equations (4), (5), (6) was derived using Lagrange's equations and uses among the parameters explained before the parameters for suspension dynamic description. The spring rates front/rear k_f/ k_r together with the damping ratios front/rear c_f/ c_r affect the dynamic behaviour and also the final values of the tyre forces compared to the previous studies provided with 3DOF models. There is together with the roll of the chassis also induced the pitch of the chassis. It is caused by x axis component of the side forces acting as a decelerating forces on front steered tyres at cornering.

The used parameter h_2 means the distance between the centre of gravity CG and the roll centre P. Parameters J_x and J_y are the moments of inertia around x and y axis.

$$
\begin{aligned}
\ddot{z}_p = h_2\left(\dot{\gamma}^2 + \gamma\ddot{\gamma} + \dot{\varepsilon}^2 + \varepsilon\ddot{\varepsilon}\right) - 2\frac{k_f}{m}\left(z_p - l_f\varepsilon\right) - 2\frac{k_r}{m}\left(z_p + l_r\varepsilon\right) \\
- 2\frac{c_f}{m}\left(\dot{z}_p - l_f\dot{\varepsilon}\right) - 2\frac{c_r}{m}\left(\dot{z}_p + l_r\dot{\varepsilon}\right) + \frac{1}{m}\left(dN_{FL} + dN_{FR} + dN_{RL} + dN_{RR}\right) ,
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\ddot{\gamma} = {} & \tfrac{1}{\left(J_x+mh_2^2\right)}\left[-mh_2^2\gamma\left(\dot{\varepsilon}^2+\varepsilon\ddot{\varepsilon}\right)+mh_2\gamma\ddot{z}_p+mgh_2\gamma\right] \\
& + \tfrac{1}{\left(J_x+mh_2^2\right)}\left[-\tfrac{T^2\gamma}{2}\left(k_f+k_r\right)-\tfrac{T^2}{2}\dot{\gamma}\left(c_f+c_r\right)\right] \\
& + \tfrac{1}{\left(J_x+mh_2^2\right)}\left[ma_yh_2+\tfrac{T}{2}\left(dN_{FL}-dN_{FR}+dN_{RL}-dN_{RR}\right)\right] ,
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\ddot{\varepsilon} = {} & \tfrac{1}{\left(J_y+mh_2^2\right)}\left[-mh_2^2\varepsilon\left(\dot{\gamma}^2+\gamma\ddot{\gamma}\right)+mh_2\varepsilon\ddot{z}_p+mgh_2\varepsilon+2z_p\left(k_fl_f-k_rl_r\right)\right] \\
& + \tfrac{1}{\left(J_y+mh_2^2\right)}\left[-2\varepsilon\left(k_fl_f^2+k_rl_r^2\right)+2\dot{z}_p\left(c_fl_f-c_rl_r\right)-2\dot{\varepsilon}\left(c_fl_f^2+c_rl_r^2\right)\right] \\
& + \tfrac{1}{\left(J_y+mh_2^2\right)}\left[+ma_xh_2-l_f\left(dN_{FL}+dN_{FR}\right)+l_r\left(dN_{RL}+dN_{RR}\right)\right] .
\end{aligned} \tag{6}
$$

2.2. Control of Vertical Forces

The Control Law of the designed stability control system presented in (Drobný & Valášek 2008) is based on the controlling of the vertical tyre forces and their redistribution among the four corners in the vehicle. The tyre vertical load affects longitudinal and lateral forces, which are acting on the tyre. By increasing the vertical tyre force, the lateral force increases nonlinearly with apparent degradation at high vertical forces. The lateral force dependency generated using Pacejka Magic Formula is represented in Fig. 4.

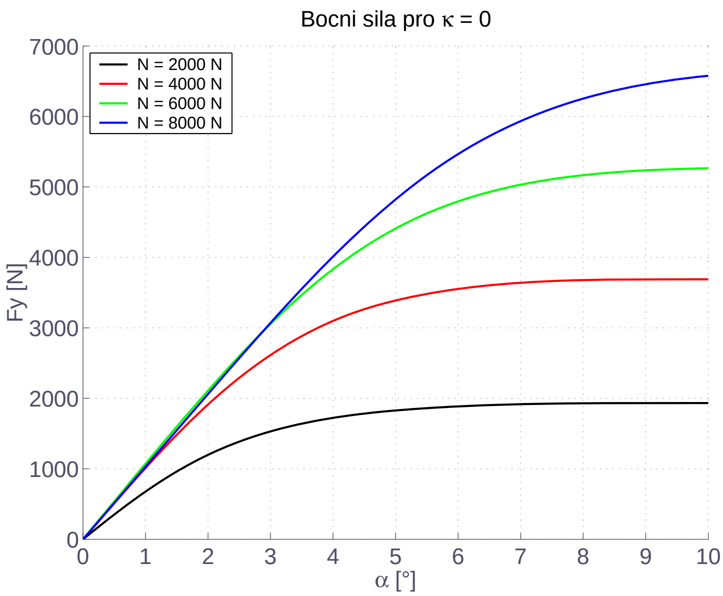

Figure 4: Lateral tyre force F_y over wheel load F_z at different slip angles

With the usage of the lateral force degradation on the appropriate tyre, it is possible to generate the yaw stabilization torque of the vehicle. Described situation occurs at the vertical force lowering. On the other side, by increasing the tyre vertical force, the response leads to enlarging the tyre lateral force. The resulted dynamic effect of the 4-wheel vertical force variations is sufficient to be described by parameter M_{corr}. It characterizes the stabilization demand and among others it allows to make the comparison to the passive vehicle.

By the formulation of this problem there was expected that the weight distribution among the 4 wheels did not cause any distinctive movement in vehicle roll and pitch. Development of the expressed problematic led to the system of four equations. The first equation is the moment equilibrium written with respect to right side, the second equation is the moment equilibrium written with respect to rear axle, the third is the sum of vertical forces, the fourth equation expresses the equality of the change of vertical forces on front and rear axles.

In the matrix form

$$\mathbf{Ax} = \mathbf{b}\,, \tag{7}$$

was the system of equations published in (Drobný & Valášek 2008)

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} N_{FL} \\ N_{FR} \\ N_{RL} \\ N_{RR} \end{bmatrix} = \begin{bmatrix} -\frac{ma_y h_{CG}}{T} + \frac{mg}{2} \\ \frac{mgl_r}{l_f + l_r} \\ mg \\ \left(\frac{ma_y h_{CG}}{T} + \frac{mg}{2}\right) \cdot \left(1 - \frac{l_r}{l_f + l_r}\right) + dN \end{bmatrix}. \tag{8}$$

The variable dN represents one input parameter. Solving the equation for specific dN results to the actual values of the vertical forces N_{FL}, N_{FR}, N_{RL}, N_{RR}. Defined formula was integrated to the vehicle control system as the actuator and was connected as a part of the feedback control loop, which compares the movement of the vehicle with the movement of the reference model.

2.2.1. Stabilization Process

The correlation between dN and M_{corr} was determined in (Drobný & Valášek 2008, Valášek, Drobný & Vaculín 2008). The corrective torque is generated in positive direction, if the vertical load at inner front and outer rear wheel increases and at the same time the outer front and inner rear tyre vertical force decreases. The vertical force shift dN has the same absolute value for all wheels. The difference consists in the force direction. The final conclusions for positive and for negative corrective torque generation are shown in Fig. 5. Plus sign on the appropriate wheel means the increase of the vertical tyre force and the minus sign means the decrease.

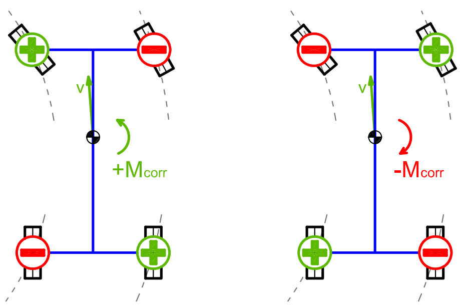

Figure 5: Vertical tyre force shift (dN) and corrective torque (M_{corr}) correlation

3. Steady-State Simulations

For the vehicle steady-state behaviour evaluation, the so called $\beta - method$ described in (Shimada & Shibahata 1994, van Zanten et al. 1995) is applicable. It collects graphical display of steady-state cornering moments in dependency on the sideslip angle β and on the front wheels steering angle δ_f. The all combinations of the input parameters are evaluated. The method can be provided by skidding the vehicle in the direction of the observed sideslip angle β. The lateral tyre forces are generated. The expected stabilization moment M_s results as a reaction force in the locked rotation joint along the vertical axis in the centre of gravity.

3.1. Limits of Passive Vehicle

The passive vehicle was set as a basis for the stabilization effect evaluation. The limits for the passive vehicle were defined by the $\beta - method$ as indicated in Fig. 6.

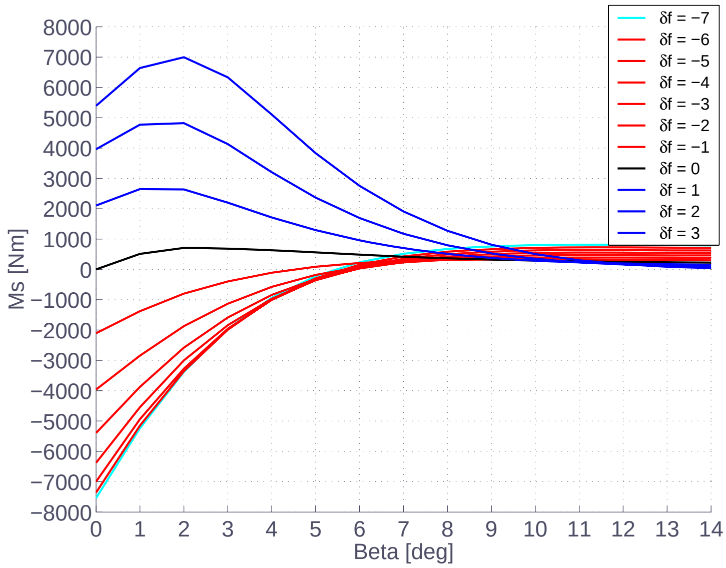

Figure 6: $\beta - method$ of passive vehicle on $\mu = 1.0$

3.2. Evaluation of Stabilization Effect

In the next step the effect of vertical force redistribution was observed. The stabilization torque M_s increment compared to the passive vehicle has been looked for. It is defined as

$$-\Delta M_s = M_{sctrl} - M_{spas} \,. \tag{9}$$

The changes caused by the variation of the verical force dN on dry surface with $\mu = 1.0$ and for vertical forces $dN = (-500, -1000, -1500)\ N$ for the understeer behaviour compensation were evaluated by the $\beta - method$. Similarly for the oversteer behaviour compensation, the vertical forces $dN = (+500, +1000, +1500)\ N$ were applied.

The understeer behaviour compensation outputs are shown in Fig 7 - Fig 9. The oversteer behaviour compensation outputs are shown in Fig 10 - Fig 12.

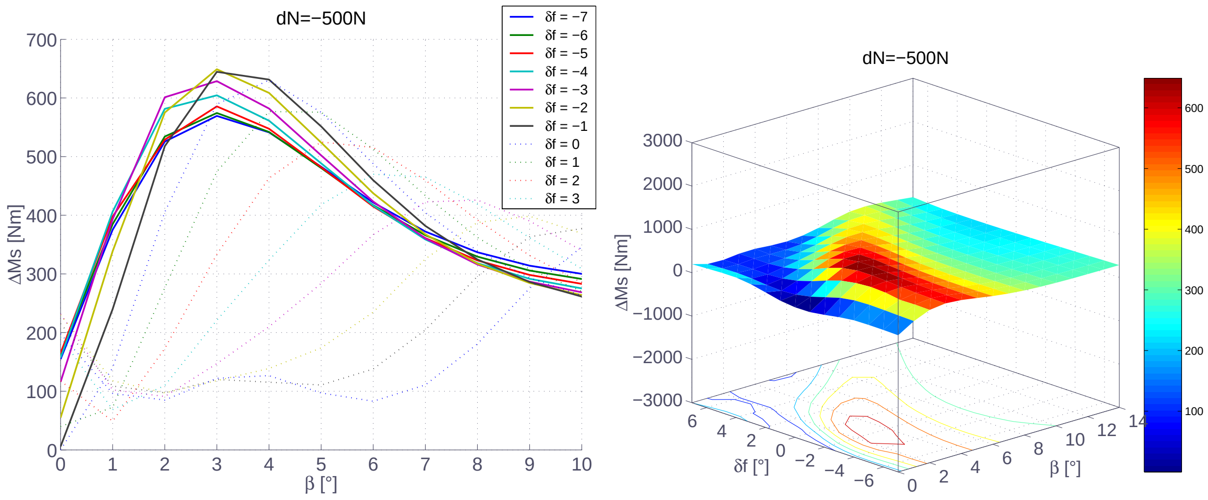

Figure 7: $\beta - method$ on $\mu = 1$ for $dN = -500N$

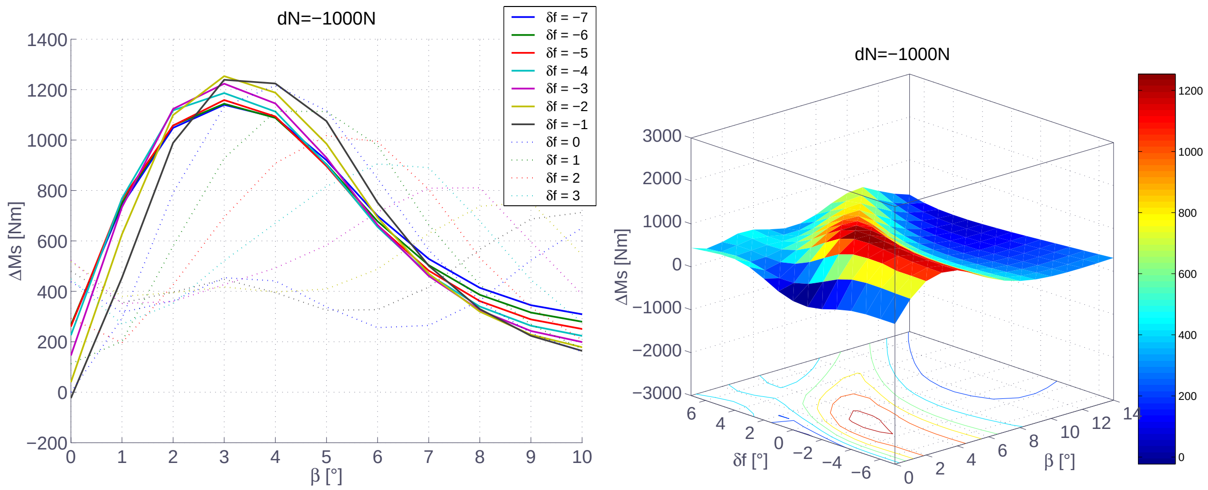

Figure 8: $\beta - method$ on $\mu = 1$ for $dN = -1000N$

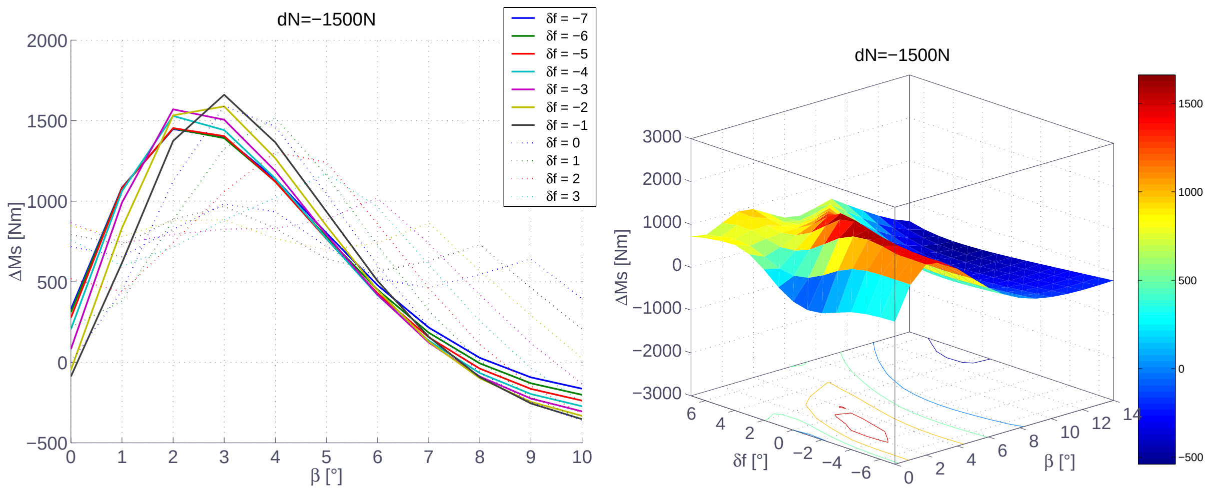

Figure 9: $\beta - method$ on $\mu = 1$ for $dN = -1500N$

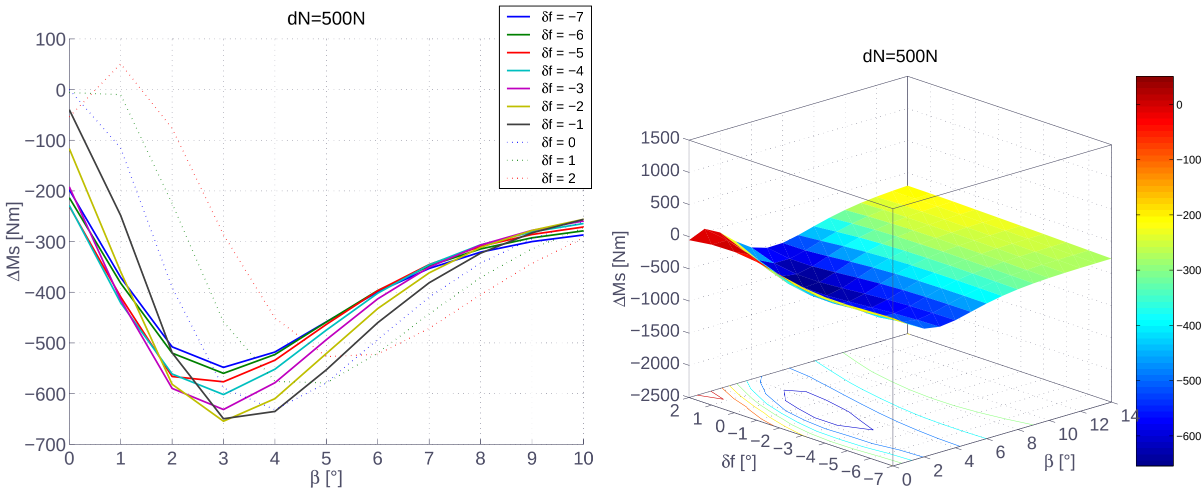

Figure 10: $\beta - method$ on $\mu = 1$ for $dN = 500N$

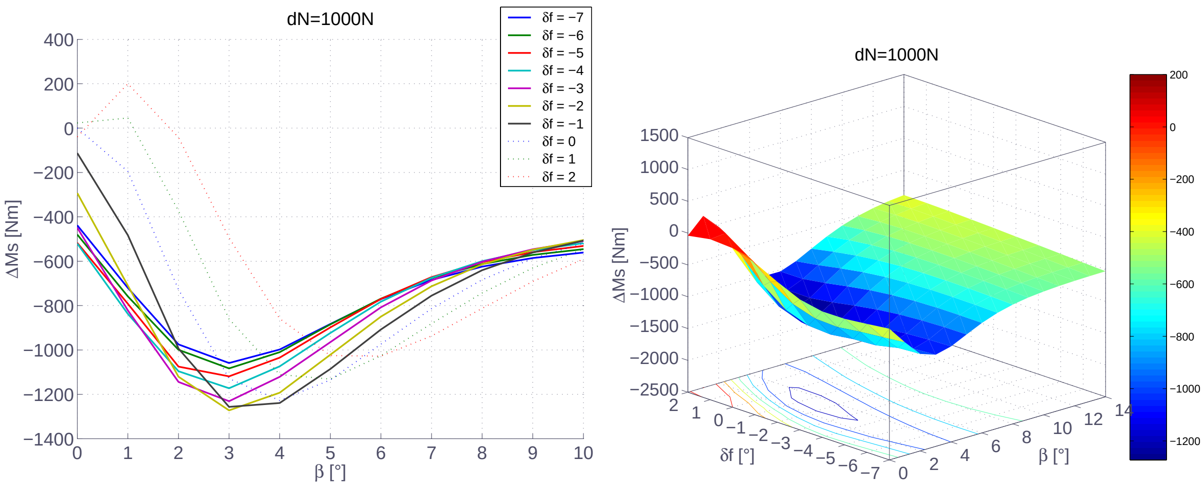

Figure 11: $\beta - method$ on $\mu = 1$ for $dN = 1000N$

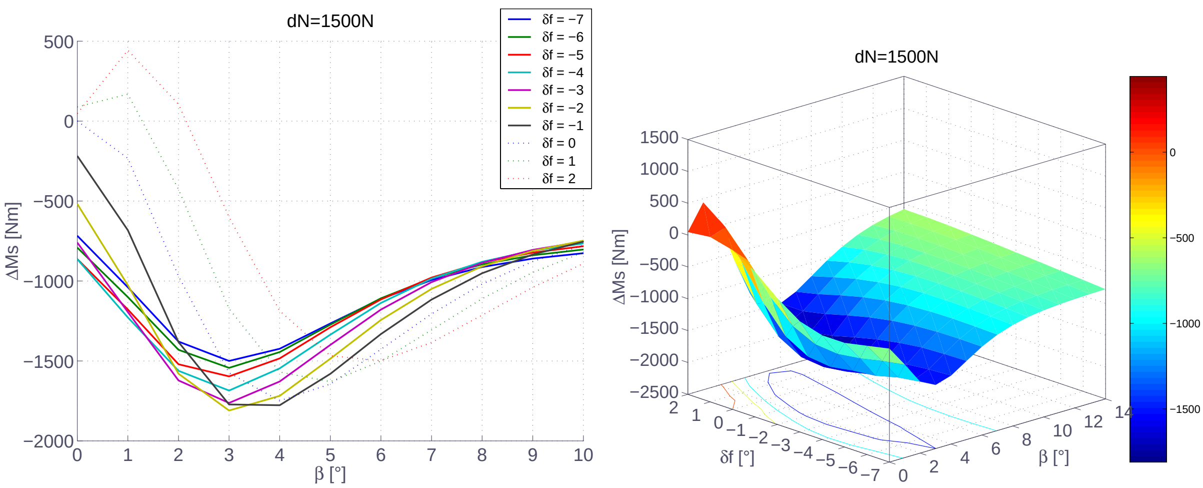

Figure 12: $\beta - method$ on $\mu = 1$ for $dN = 1500N$

3.2.1. Analysis of Results

Analysis of the results of the stabilization torques ΔM_s leads to the fact that the best area for vehicle stabilization is in the case when the side slip angle β grows from $0°$ to the value $\beta = 3 - 4°$. The given stabilization effect rises in this area with β growing. The definition is valid for the understeer behaviour compensation related to all adhesion coefficients as well as for all dN increments, but in case of oversteer behaviour compensation is the effect slightly reduced for values higher then $dN = 2500N$ and for lower adhesion coefficients. The maximal ΔM_s gain is generated on dry surface, where for understeer behaviour compensation reaches $1660Nm$ and for oversteer behaviour compensation $-2220Nm$.

The maximal ΔM_s gains for steering angles $\delta_f = -1°$ and $\delta_f = -5°$ and for adhesion coefficients of $\mu = (0.5, 0.7, 1.0)$ are shown on the Fig. 13. In case of understeer behaviour compensation in range from $dN = 0$ to $dN = -1500N$ grows the resulted stabilization moment with increasing dN linearly. The change in the linearity is caused by inner rear wheel lift, where the stabilization potential is being lost.

In case of oversteer behaviour compensation the system efficiency is rapidly decreased with adhesion coefficients lowering. For $\mu = 0.5$ and $dN = 3000N$ the gained stabilization moment results into $\Delta M_s = 500Nm$ only. Nevertheless the maximal absolute value of the stabilization effect is in case of oversteer behaviour compensation by $560Nm$ higher.

The limitation of the control system is given by the value of static load of the appropriate tyre. The controlled lateral dynamics movement of the vehicle keeps stable as long as the potential to transfer the vertical forces is sufficient. By the wheel lift is the contribution of the controlled system reduced. Such a limitation factor for vehicle with front wheel drive and front placed engine represents the value of inner rear tyre vertical force. This wheel is during cornering lifted. The understeer behaviour compensation using change of dN leads to even higher off-load of this wheel.

In the situations where the sideslip angle $\beta > 5°$ the efficiency of proposed stabilization system is descending. To stabilize the vehicle in this range near the vehicle limit, the vehicle speed lowering through the brake activation is required. It could be managed by other brake based stabilization system intervention.

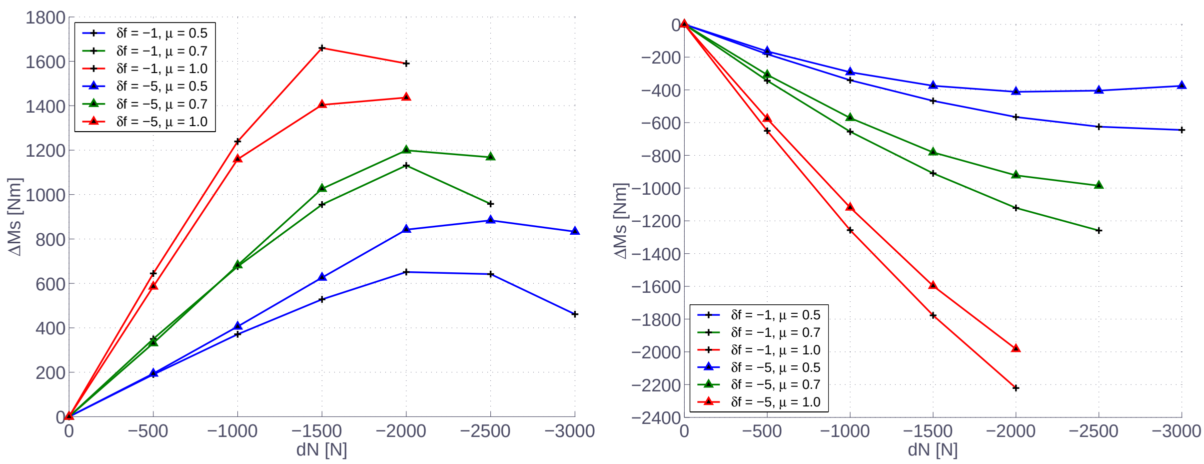

Figure 13: Stabilization moments for udersteer behaviour compensation (left) and for oversteer behaviour compensation (right)

4. Conclusions

The clear dependency between the corrective torque of the vehicle and the tyre vertical forces redistribution among four corners of the vehicle was found. The results give to the dynamic body control system the opportunity to control the lateral dynamics of the vehicle. The limitation of the system is caused by the contact of the tyre. The maximal actuated force dN should be less then the tyre static load force.

The side slip angle β in range from $0°$ to the value $\beta = 3 - 4°$ defines the best area for vertical load redistribution where the active suspension system gives the best response. The same area could be described as the range of vehicle driveability. For higher β angles the vehicle approaches its stability margin, where the efficiency of active suspension system is limited. In such case the ESP system could help. The synergetic effects are expected in the combination of active suspension and ESP system. It is an objective for the future work.

5. Acknowledgements

The authors acknowledge the kind financial support of the Ministry of Education, Youth and Sports of the Czech Republic by the grant 1M0568.

6. References

Bodie, M. O. & Hac, A. (2000), Closed loop yaw control of vehicles using magneto-rheological dampers, *in* 'SAE Paper 2000-01-0107'.

Danesin, D., Krief, P., Sorniotti, A. & Velardocchia, M. (2003), Active roll control to increase handling and comfort, *in* 'SAE Paper 2003-01-0962'.

Drobný, V. (2007), Reference model for vehicle dynamics control, *in* 'Engineering Mechanics 2007', pp. 1–7.

Drobný, V. & Valášek, M. (2008), Wheel vertical forces distribution as the source of vehicle stabilization, *in* 'Engineering Mechanics 2008', pp. 168–172.

Genta, G. (2006), *Motor Vehicle Dynamics, Modeling and Simulation*, World Scientific.

Rieth, P. E. & Schwarz, R. (2004), Esc ii - esc with active steering intervention, *in* 'SAE Paper 2004-01-0260'.

Shimada, K. & Shibahata, Y. (1994), Comparison of three active chassis control methods for stabilizing yaw moments, *in* 'SAE Paper 940870'.

Valášek, M., Drobný, V. & Vaculín, O. (2008), Vehicle lateral dynamic stability by limited-active nonlinear suspension control, *in* 'AVEC'08, 9th International Symposium on Advanced Vehicle Control', JSAE, pp. 104–109.

van Zanten, A., Erhardt, R. & Pfaff, G. (1995), Vdc, the vehicle dynamics control system of bosch, *in* 'SAE Paper 950759'.