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PNEUMATIC SUSPENSION SYSTEM WITH OPTIMAL CONTROL OF DAMPING, STIFFNESS AND RIDE-HEIGHT

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Summary: *The application to improved vehicle dynamics requires the stiffness and dumping coefficient change to be instantaneous. However many other authors say that the duple-volume spring can change the stiffness with no power input, it is not so precisely true. The electromagnetic actuators, which are used as controllers have power input and also own dynamics. We can see that the dynamics of the controllers' actuators has an important influence on the whole controlled semi-active system. The behavior of the system which includes valves dynamics is far worse than we would expect when we neglected the valves dynamics.*

1. Introduction

The motivation for an automotive suspension system with independent control of stiffness, damping and ride-height comes from the trade-offs involved for the conflicting requirements of comfort and handling. The application to improved vehicle dynamics requires the stiffness change to be instantaneous and no change in ride-height during stiffness change. The duple-volume system can change the stiffness with no power input, and no ride-height change due to stiffness change. In this paper, is shown a design of pneumatic automotive suspension system with independent control of stiffness and damping. There is used a primary volume as an air spring and an auxiliary volume which is connected thought electromagnetic valve to the primary volume. Opening and closing of the valve, changes the effective volume of the air springs and thereby changes the stiffness of the air spring. In the suspension system is also used a viscous one jacket dumper with a fast electromagnetically controlled valve which changes damping force. In this paper, we made efforts to comparison between a passive, active and two different types of semi-active suspension systems. One of the semi-active suspension models is created without the dynamics models of electromagnetic valves. The actuators can work only in on/off state, what will provide that this semi-active system is the worst possibility of the semi-active systems which can by created, with the some control strategy. In this simulations results we used the LQR controller.

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2. Dynamics models of suspensions

Let ~~as~~ consider the passive, completely active and semi-active suspension system with two degrees of freedom schematized in Figure 1. We used the following notation:

- m_1 is the sprung mass;

- m_2 is the unsprung mass consisting of wheel and its moving parts;

- k_1, k_{1a}, k_{1sa} are the elastic constants between the sprung and unsprung mass;

- k_2 is the elastic constant of the tire;

- b_1, b_{1a}, b_{1sa} are the damping coefficients of the damper between the sprung and unsprung mass;

- b_2 is the damping coefficient of the tire;

- F_a is the control force produced by the actuator and suspension parts;

- F_{sa} is the controlled force obtained by the controlled semi-active suspension parts, dumper and spring;

- $x_1(t), \dot{x}_1(t)$ are the state variable of the sprung mass;

- $x_2(t), \dot{x}_2(t)$ are the state variable of the unsprun mass;

- $w(t), \dot{w}(t)$ are the function representing the disturbance;

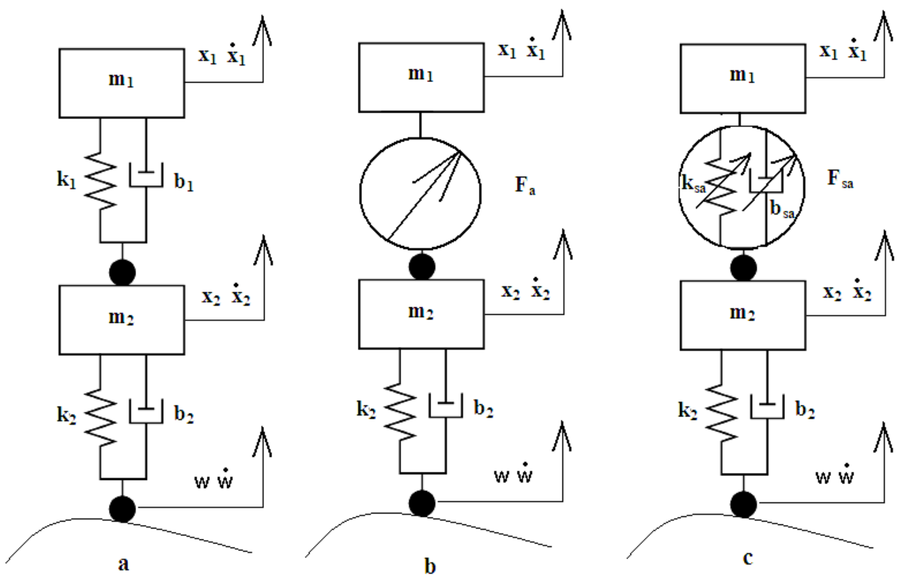

Fig. 1: Model of a considered suspensions systems (a. passive, b. active, c. semi-active)

The equations of motion for the passive system can by simply written using matrix equation for the state space (1) or (2).

$$\dot{\mathbf{x}}_{\mathbf{s}}(t)=\mathbf{A}_{\mathbf{s}}\mathbf{x}_{\mathbf{s}}(t)+\mathbf{B}_{s}\mathbf{w}_{s}(t) \tag{1}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \ddot{x}_1\\ \ddot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\frac{k_1}{m_1} & \frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{b_1}{m_1}\\ \frac{k_1+k_2}{m_2} & -\frac{k_1}{m_2} & \frac{b_1+b_2}{m_2} & -\frac{b_1}{m_2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \dot{x}_1\\ \dot{x}_2\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ \frac{k_2}{m_2} & \frac{b_2}{m_2}\end{bmatrix}\begin{bmatrix}w\\ \dot{w}\end{bmatrix} \tag{2}$$

The active and semi-active systems can by described whit the some matrix equation for both of the considered systems (3), or (4). The differences in the state space equation between the active and semi-active system is only in the controlled force, which is written in the second input vector $\mathbf{f}_{\mathbf{s}}(t)$.

$$\dot{\mathbf{x}}_{\mathbf{s}}(t)=\mathbf{A}_{\mathbf{s}}\mathbf{x}_{\mathbf{s}}(t)+\mathbf{B}_{s}\mathbf{w}_{s}(t)+\mathbf{F}_{\mathbf{s}}\mathbf{f}_{\mathbf{s}}\left(t\right) \tag{3}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \ddot{x}_1\\ \ddot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ \frac{k_2}{m_2} & 0 & \frac{b_2}{m_2} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \dot{x}_1\\ \dot{x}_2\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ \frac{k_2}{m_2} & \frac{b_2}{m_2}\end{bmatrix}\begin{bmatrix}w\\ \dot{w}\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \frac{1}{m_1}\\ \frac{-1}{m_2}\end{bmatrix}\begin{bmatrix}f_{(a),(sa)}\end{bmatrix} \tag{4}$$

3. Mathematical models of suspension parts.

In the case of the active system, we have assumed that the force between the sprung and unsprung mass is exactly the same as the force required from the LQ controller. In case of semi-active system it is necessary to create exacts mathematical models of controlled suspensions parts and they are the semi-active spring and semi-active viscous dumper.

3.1 Semi-active viscous dumper

The supplied force of a viscose dumper Fig. 2, can by calculated as the force caused by the resistance which is between the moving piston and the cylinder wall (5) and as the flow resistance trough the controlled valve (6). The sum of these both forces represented the controlled damper force (7). We used the following notation:

- R_t is the radius of the dumper cylinder
- r_t is the radius of the dumper piston
- l_p is the length of the piston

- x_{vt} is the displacement of dumper control valve
- y_{vt} is the bright of the valve
- v_t is the piston velocity
- η is the dynamics viscosity of dumper oil
- b_{tp} is the dumping coefficient cost by piston motion in the cylinder
- b_{tQ} is the dumping coefficient cost by flow resistance trough the valve
- k_h is the correlation coefficient between the valves bright and its height, Fig.2

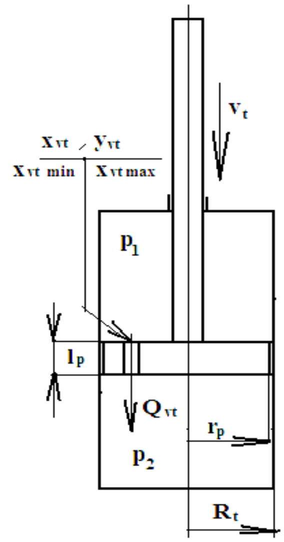

Fig. 2: Viscous dumper

$$F_{tp} = b_{tp} v_t = \frac{2\pi\, \eta\, l_p}{\ln\left(\frac{r_p}{R_t}\right)}\; v_t \tag{5}$$

$$F_{tQ} = b_{tq} v_t = \frac{12\pi^2 \eta\, l_p R^4}{y_{vt} k_h} \frac{1}{x_{vt}^3} v_t = K_{btQ} \frac{1}{x_{vt}^3} v_t \tag{6}$$

$$F_t = b_t\, v_t = F_{tp} + F_{tQ} = \left(b_{tp} + b_{tq}\right) v_t = \left(b_{tp} + K_{btQ} \frac{1}{x_{vt}^3}\right) v_t \tag{7}$$

The characteristic of the dumper we have used in our suspensions models is shown in Fig. 2.

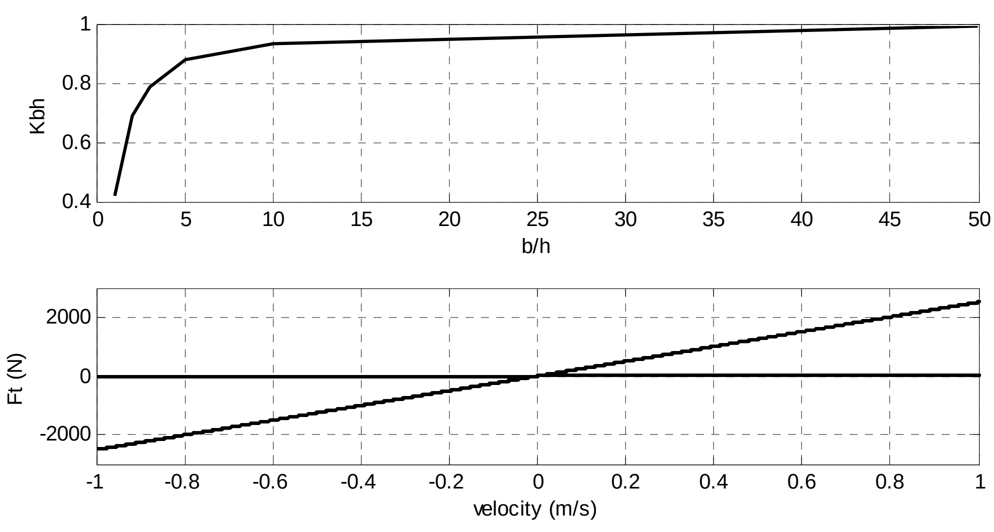

Fig. 3: The valves correlation coefficient and dumper characteristic

3.2 Semi-active pneumatic spring whit auxiliary volume.

The characteristic of the spring is nonlinear with hysteresis. The force produced by the spring depends on several parameters. They are: the ratio between the primary and auxiliary volume, pressure in the primary volume, rate of opening of the controlled on-off electromagnetic valve, displacement of the piston and frequency of the piston motion. When we consider that the temperature in bought volumes is still almost the some and equal to the temperature of surroundings, than the force can by calculated by equations (8) - (18). We used the following notation:

- p_1, V_1, M_1, ρ_1 are the parameter in the primary volume, pressure, volume, mass and density;
- p_2, V_2, M_2, ρ_2 are the parameter in the auxiliary volume: pressure, volume, mass, density;
- V_{10}, M_{10} are the parameter at the static state in the primary volume: volume, mass;
- M_{20} are the parameter at the static state in the auxiliary volume: mass, density;
- p_0, ρ_0 are the pressure and the density at the static state in bought of the volumes;
- m_1 is the sprung mass;
- $\dot{m}$ is the mass flow trough the controlled valve;

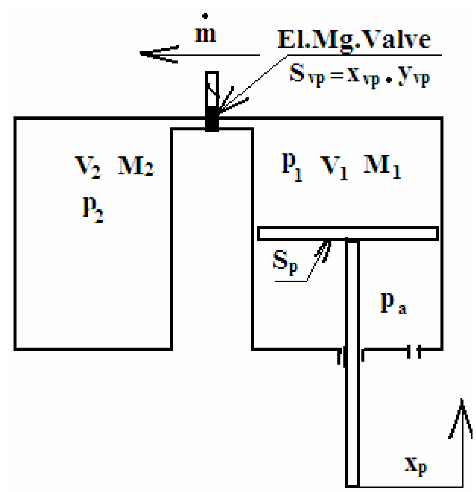

Fig. 4: Semi-active spring

- v is the air flow velocity;
- v_{krit} is the maximal flow velocity;
- S_{vp}, x_{vp}, y_{vp} are the flow area in the valve, the bright of the gab and the height of the gap;
- S_p is the piston area;
- x_p is the motion of the piston;
- $g,\ \mu,\ \kappa$ are the constants which represented the gravity, flow resistance and the exponent of the adiabatic variation.

$$p_1 = p_0 \left(\frac{V_{10}^{\kappa} \left(M_{10} - m \right)^{\kappa}}{M_{10}^{\kappa} \left(V_{10} - \Delta V \right)^{\kappa}} \right) \tag{8}$$

$$p_2 = p_0 \left(\frac{\left(M_{20} - m \right)^{\kappa}}{M_{20}^{\kappa}} \right) \tag{9}$$

$$\dot{m} = \mu \ \rho_2 \ S_{vp} \ v \qquad (10)$$

$$v = \sqrt{\frac{2\kappa}{\kappa - 1}\left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right)} \qquad (11)$$

$$v_{krit} = \sqrt{\kappa \frac{p_1}{\rho_1}} \qquad (12)$$

$$p_0 = \frac{m_1 \ g}{S_p} \qquad (13)$$

$$\rho_0 = \left(\frac{p_0}{p_a}\right)^{\frac{1}{\kappa}} \rho_a \qquad (14)$$

$$M_{10,20} = V_{10,20}\rho_0 \qquad (15)$$

$$V_1 = V_{10} + \Delta V = V_{10} + \left(S_p \ x_p\right) \qquad (16)$$

$$\rho_1 = \frac{M_{10} - m}{V_1} \qquad (17)$$

$$\rho_2 = \frac{M_{20} + m}{V_2} \qquad (18)$$

The considered spring is shown in Fig. 4. and here characteristic is shown in Fig. 5.

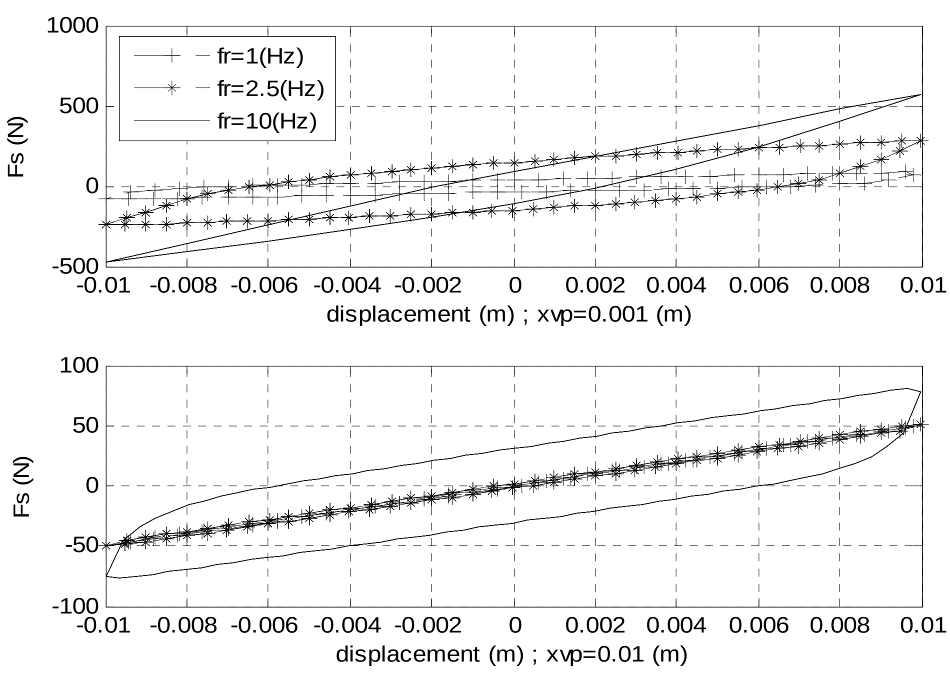

Fig. 5: The characteristics of semi-active pneumatic spring whit auxiliary volume

3.3 Electromagnetic valve (solenoid actuator)

The electromagnetic valve is an electro-mechanics system where the moving plunger (piston) is moved (when is opening) by the force of the solenoid actuator and (when is closing) by the spring. The solenoid, which is used as an actuator in the spring and dumper valves, has a very simple construction, consists of a plunger or iron piston which is moved in by the field generated by current in the coil. A spring is used in order to bring the plunger back to its original position. In Fig. 6 a simple schematic of the actuator is shown. The actuator may be expressed as an equivalent consisting of two inductors in series. The first inductor would be representative of the inductance caused due to the coil containing the plunger (iron core), while the other would be the one without the plunger (air core). These two inductors appear in series. The equivalent of the actuator is shown below in Fig. 7.

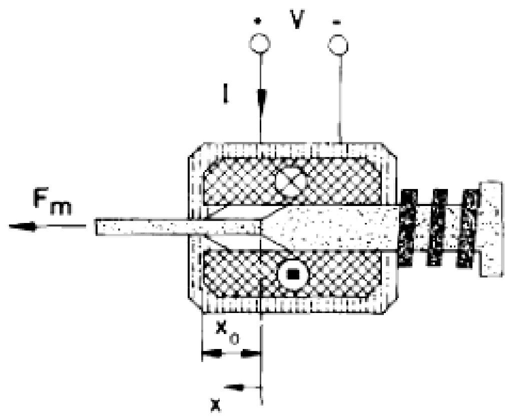

Fig. 6: Electromagnetic actuator

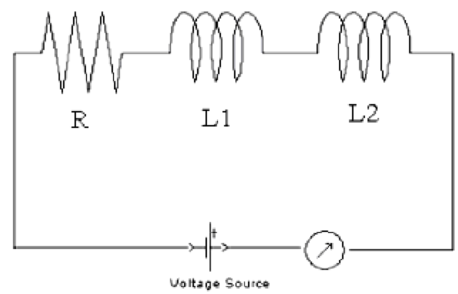

Fig. 7: Equivalent of the actuator

Using the basics physics knowledge of electromagnetic field we can calculate the effective inductance (20). Because we need to calculate the force of the actuator (22), first we have to calculate the variation of inductance with respect to position (21) and the current in the circuit (19). After we know the actuator force we can establish a differential equation of motion for the plunger (23). We used the following notation:

- R_e is the equivalent resistance of the whole coil and connecting wires.

- I, V, Φ, B, H are the current in the circuit, input voltage, magnetic flux, magnetic induction and the intensity of magnetic field;

- $L,\ l,\ z,\ R,$ are the inductance, length, number of windings and the middle diameter of the whole coil;

- $L_{1,} l_1, z_1$ are the inductance, length, number of windings of the coil with iron plunger;

- $L_{2,} l_2, z_2$ are the inductance, length, number of windings of the coil with air;

- x_{v0}, x_v are the static plunger position, current plunger position;

- μ_0, μ_2 are the permeability of free space and the relative permeability of air;

- μ_1 is the relative permeability of iron. It is a heavy nonlinear function. $\mu_1 = B_1 / H_1$, where $H_1 = \frac{z_1 I}{l_1}$ is the intensity of magnetic field in coil with iron plunger.

- m_v are the mass of the plunger and moving parts connect with him;

- b_v is the dumping coefficient of the plunger calculated the some way as is shown in (5);

- k_v is the elastic constant of the return spring;

The nonlinearity is shown in Fig. 9. The Dynamics of a solenoid actuator is shown in Fig. 8.

$$V = R_e I + \frac{d}{dt}(\Phi) = R_e I + \frac{d}{dt}\left(L(x_v)\ I\right) = R_e I + I\frac{d}{dt}L(x_v)\frac{dx_v}{dt} + L(x_v)\frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{1}{L(x_v)}\left(V - R_e I - I\frac{d}{dt}L(x_v)\frac{dx_v}{dt}\right) \tag{19}$$

$$B_{1,2} = \frac{\mu_0 \mu_{r1,2} I\ z}{l\sqrt{1+\left(\frac{2R}{l_{1.2}}\right)^2}}$$

$$L_{1,2} = \frac{z_{1,2} B_{1,2} S}{I}, \quad \text{where } S = \pi\ R^2, \quad z_1 = \left(\frac{l_1}{l}\right)z, \quad z_2 = \left(\frac{l_2}{l}\right)z, \quad l_1 = x_{v0} - x_v, \quad l_2 = l - x_{v0} + x_v$$

$$L(x_v) = L_1(x_v) + L_2(x_v) = \frac{\mu_0 \mu_1 S\ z^2}{l^2}\frac{(x_{v0} - x_v)}{\sqrt{1+\left(\frac{2R}{(x_{v0}-x_v)}\right)^2}} + \frac{\mu_0 \mu_2 S\ z^2}{l^2}\frac{(l - x_{v0} + x_v)}{\sqrt{1+\left(\frac{2R}{(l-x_{v0}+x_v)}\right)^2}} \tag{20}$$

$$\frac{dL(x_v)}{dx_v} = \frac{\mu_0 \mu_1 S\ z^2}{l^2}\left(\frac{1}{\sqrt{1+\left(\frac{2R}{(x_{v0}-x_v)}\right)^2}} + \frac{4}{\sqrt{1+\left(\frac{2R}{(x_{v0}-x_v)}\right)^{\frac{3}{2}}}}\left(\frac{R}{(x_{v0}-x_v)}\right)^2\right) - \frac{\mu_0 \mu_2 S\ z^2}{l^2}\left(\frac{1}{\sqrt{1+\left(\frac{2R}{(l-x_{v0}+x_v)}\right)^2}} + \frac{4}{\sqrt{1+\left(\frac{2R}{(l-x_{v0}+x_v)}\right)^{\frac{3}{2}}}}\left(\frac{R}{(l-x_{v0}+x_v)}\right)^2\right) \tag{21}$$

$$F_m = \frac{1}{2}\frac{dL(x_v)}{dx_v} I^2 \qquad (22)$$

$$m_v \ddot{x}_v + b_v \dot{x}_v + k_v x_v = F_m \qquad (23)$$

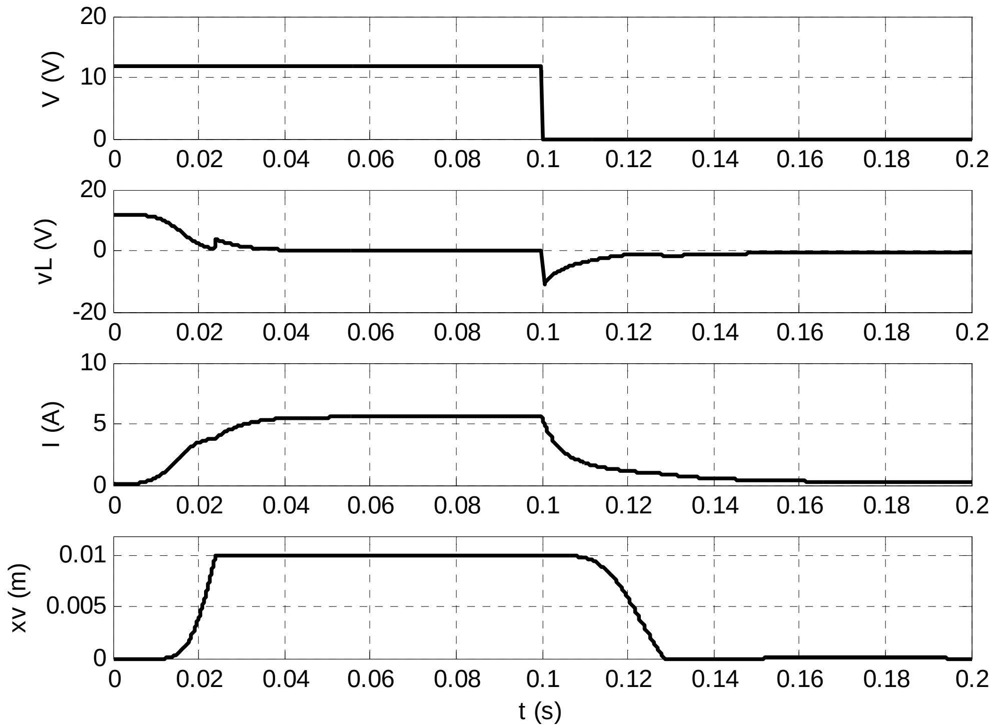

Fig. 8: Dynamics of solenoid actuator used for control of pneumatic spring.

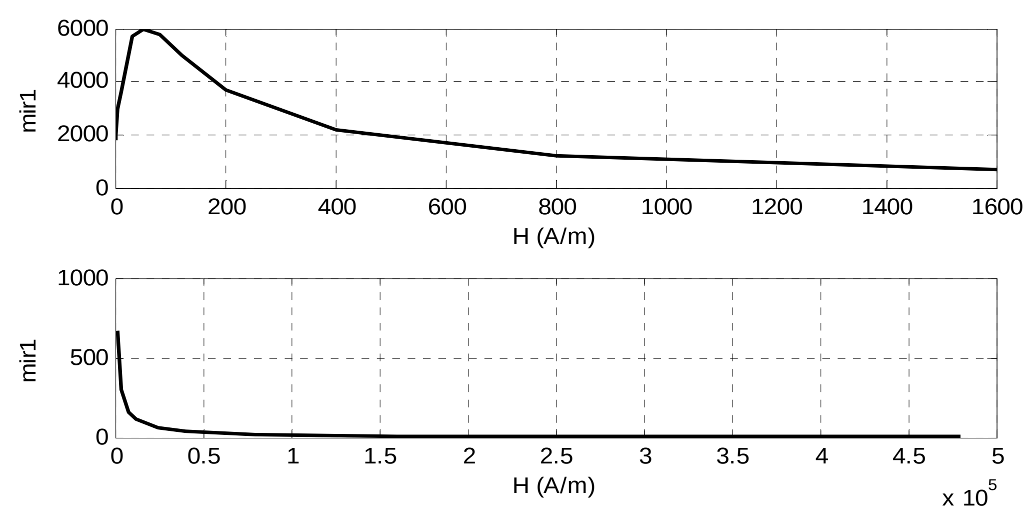

Fig. 9: The relation between intensity of magnetic field and relative permeability of iron.

4. LQR Controller.

The LQR controller is based on minimizing of parameters chosen to by optimized. We have chosen the acceleration of the sprung mass, the relative displacement between the sprung and unsprung mass and the relative displacement between the tire and the road (24), (25). The LQ algorithm is well known procedure. First we have to establish the quadratic criteria function (26) with penalizations matrices Q and R. In our case the R is only one value and we sad it equal zero. For the diagonal values in the matrix Q we used the values $q_1=10^4$, $q_2=10^5$. To obtain the optimal constants of the controller and than calculated the optimal force between the sprung and unsprung mass (29), we need to solve the Riccati equation (28).

$$\mathbf{y} = \mathbf{C}_{LQ}\mathbf{x}_{\mathbf{s}} + \mathbf{B}_{LQ}\mathbf{w} + \mathbf{F}_{LQ}\mathbf{u} \tag{24}$$

$$\begin{bmatrix} \ddot{x}_1 \\ x_1 - x_2 \\ x_2 - w \end{bmatrix} = \begin{bmatrix} -\frac{k_1}{m_1} & \frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{b_1}{m_1} \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} [w] + \begin{bmatrix} \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} [u] \tag{25}$$

$$J = M\left[\mathbf{y}^{\mathbf{T}}\mathbf{Q}\mathbf{y} + \mathbf{u}^{T}\mathbf{R}\mathbf{u}\right] \tag{26}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & q_1 & 0 \\ 0 & 0 & q_2 \end{bmatrix}, \quad \mathbf{u}^{T}\mathbf{R}\mathbf{u} = ru^2 \tag{27}$$

$$\mathbf{Q}_0 = \mathbf{C}_{LQ}{}^{\mathbf{T}}\mathbf{Q}\ \mathbf{C}_{LQ}\,, \qquad \mathbf{n} = \mathbf{C}_{LQ}{}^{T}\mathbf{Q}\ \mathbf{F}_{LQ}\,, \qquad \mathbf{r}_0 = (\mathbf{F}_{LQ}{}^{T}\mathbf{Q}\ \mathbf{F}_{LQ} + \mathbf{R})$$

$$\mathbf{A}_{\mathbf{n}} = \mathbf{A}_s - \mathbf{F}_s\mathbf{r}_0^{-1}\mathbf{n}^{\mathbf{T}}, \qquad \mathbf{B}_n = \mathbf{F}_s\mathbf{r}_0^{-1}\mathbf{F}_s^{\ T}\ , \qquad \mathbf{C}_n = \mathbf{Q}_0 - \mathbf{n}\mathbf{r}_0^{-1}\mathbf{n}^{\mathbf{T}}$$

$$\mathbf{A}_n{}^{T}\mathbf{P} + \mathbf{P}\mathbf{A}_n - \mathbf{P}\mathbf{B}_n\mathbf{P} + \mathbf{C}_n = \mathbf{0} \Rightarrow \mathbf{P} \tag{28}$$

$$\mathbf{u} = -\mathbf{K}_0\mathbf{x}(t) = -\mathbf{r}^{-1}\ (\mathbf{n}^{T} + \mathbf{F}_s{}^{T}\mathbf{P})\mathbf{x}_s(t) \tag{29}$$

5. Simulation model

For the numerical simulation we have created four different types of suspension models shown in Fig. 10. A passive suspension model, fully active suspension model, semi-active suspension model where the dynamics of the controllers is not considered, and a semi-active model where the spring and the dumper are controlled directly by means of electromagnetic valves which are working in on-off mode. For control of the semi-active suspension parts is used in both cases only one electromagnetic actuator, with same construction, bud different dimensions for the dumper valve and for the pneumatic spring valve.

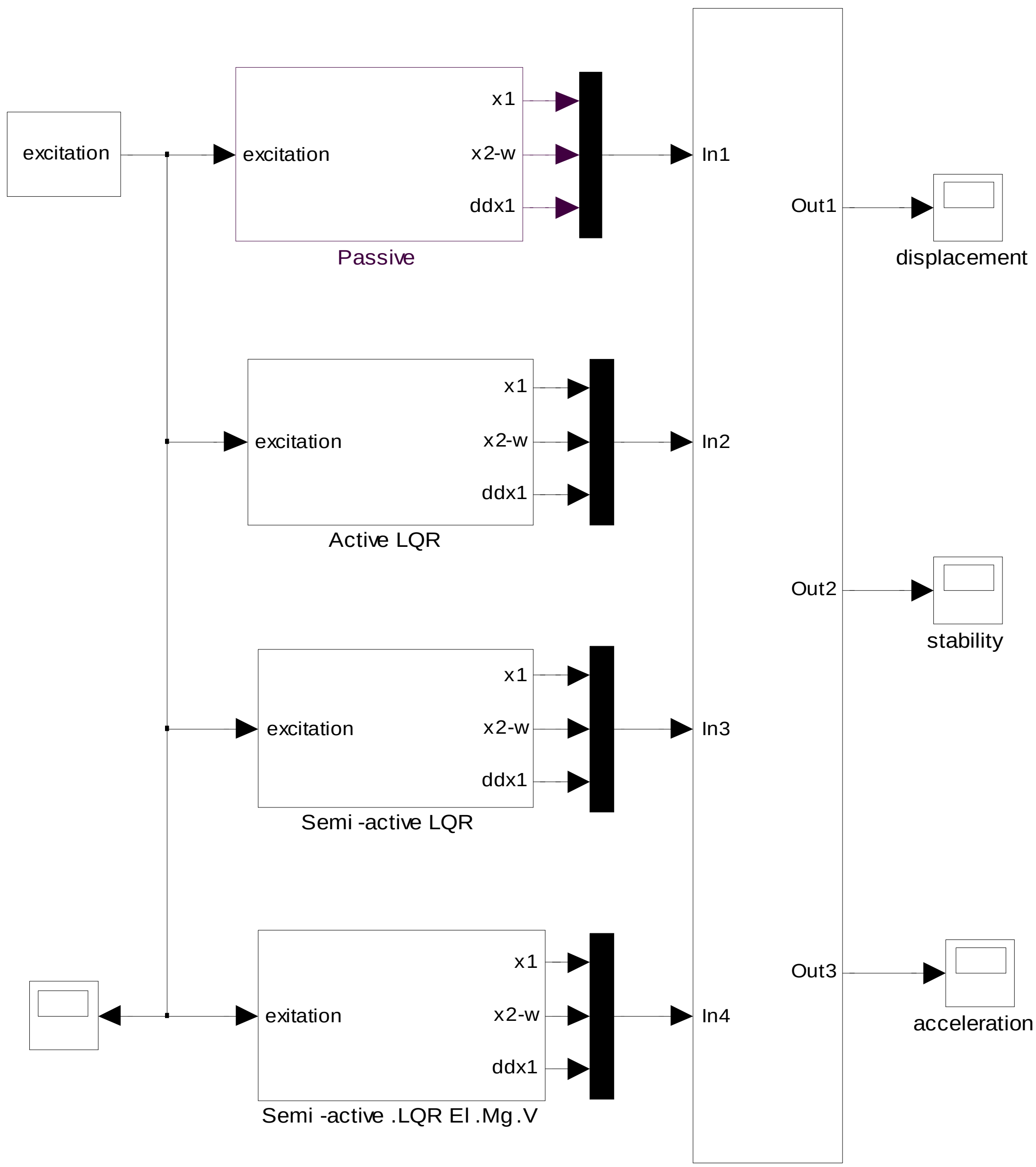

Fig. 10: Simulation model.

6. Simulation results with bump excitation.

Below we have simulated a few results which provide an insight about the working and behavior of the considered suspension models. These simulations results shows the responses of the considered models when the excitation bump is still the same dimensions which are shown in Fig. 11.a, bud the velocity of the car is changing Fig. 11 - 13. In Fig. 14 are shown the performance indexes of all four types of suspensions in each window. In each row are the performances indexes on response by different velocity. In the first column are the displacements of sprung mass. In second column are considerate the stabilities and in the third are the performance indexes of sprung mass accelerations.

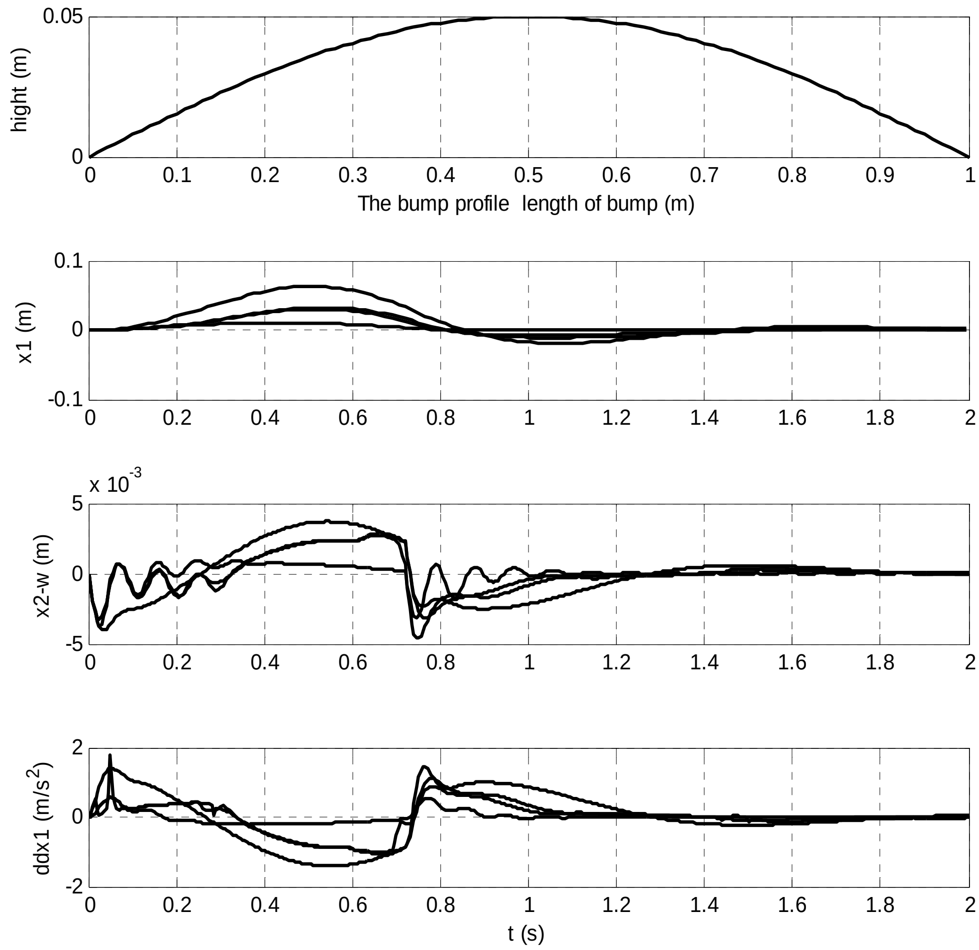

Fig. 11: The excitation response on crossing bump by 5 km/h.

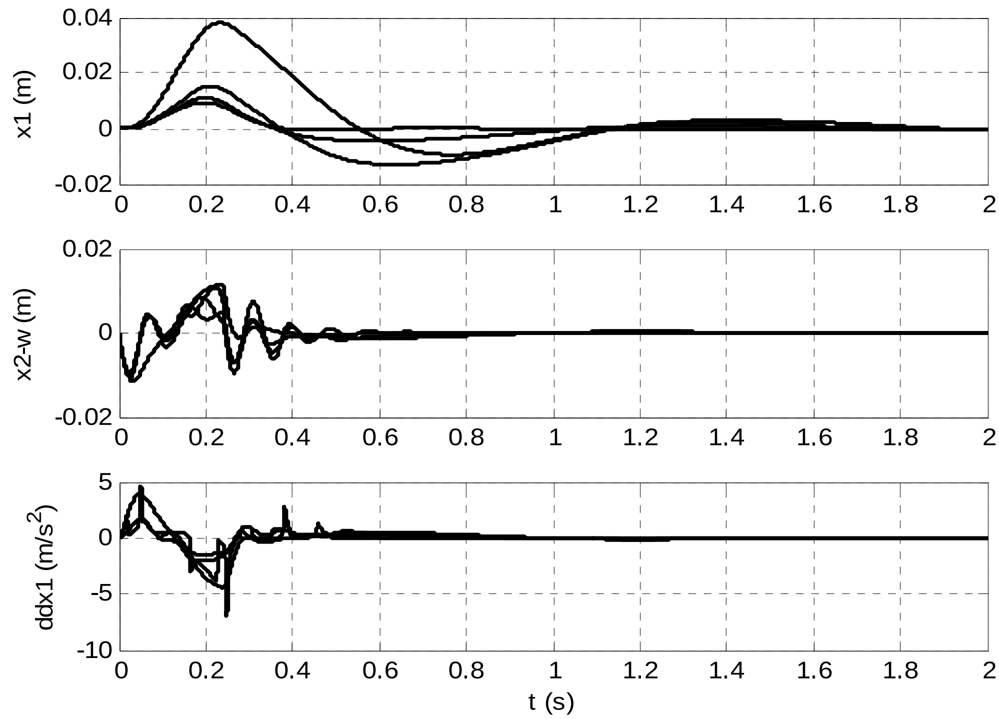

Fig. 12: The excitation response on crossing bump by 15 km/h.

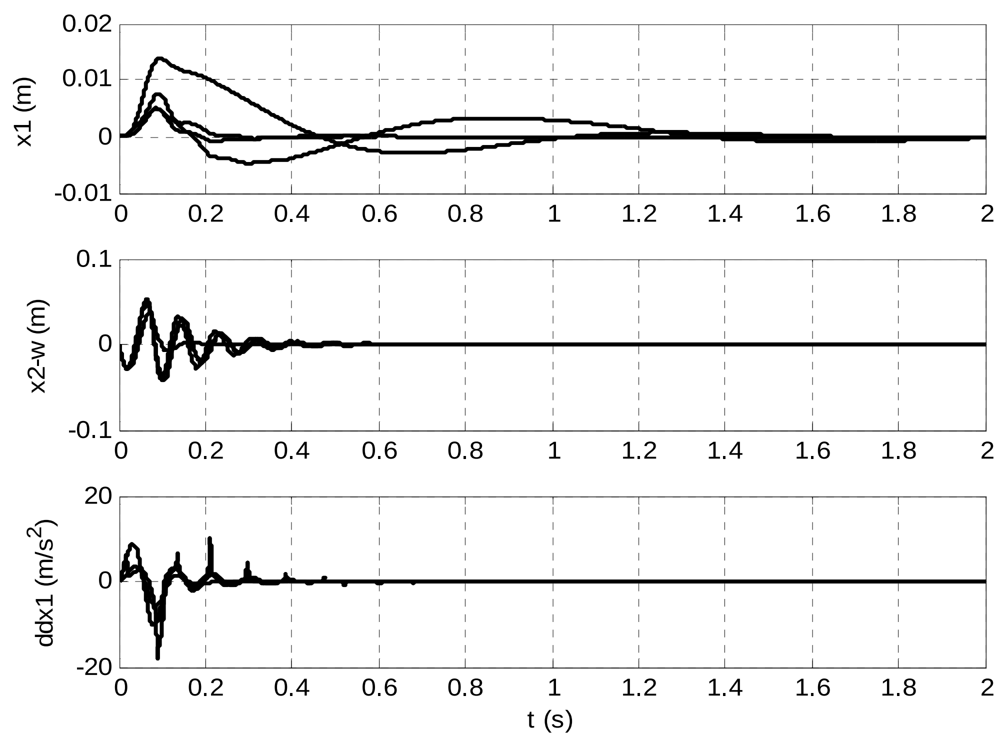

Fig. 13: The excitation response on crossing bump by 50 km/h.

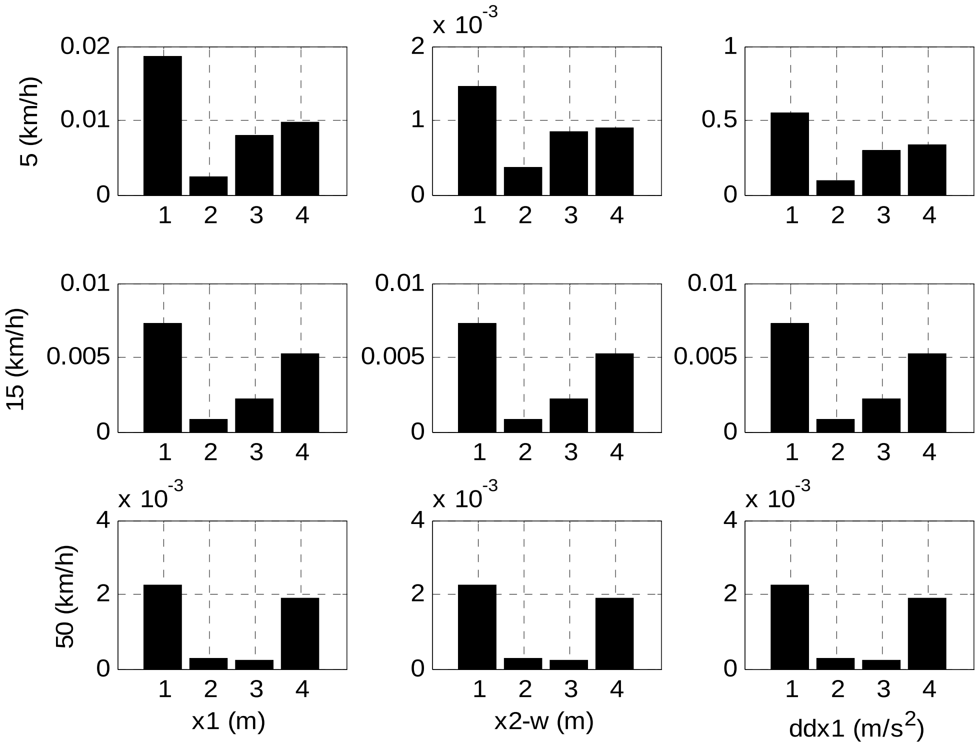

Fig. 14: The performance index's calculated from the response by crossing the bump:
1. Passive suspension, 2. Active suspension, 3. Semi-active suspension without inherent control valves dynamics, 4. Semi-active suspension with electromagnetic valve.

7. Conclusions

In this paper, we made efforts to make comparison between four different types of suspension systems. There were compared a passive, active and two different types of semi-active suspension. One of the semi-active suspension models was made without the models of electromagnetic valves. The other one is created with detail models of electromagnetic valves which are used to control the semi-active spring and dumper. The actuators can work only in on/off state, what will provide that this semi-active system is the worst possibility of the semi-active systems which can by created, with the some control strategy. In this case we used the LQR controller. On the simulation result it is clearly to see that the active system is the best and the passive the worst. Bud we also can see that the dynamics of the controllers' actuators has an important influence on the whole controlled semi-active system. The behaviour of the system with includes valves dynamics is far worse than we would expect when we neglected the valves dynamics. Also we can made a conclusion that more of smaller valves will provided a better result as we obtain, using only one valve to control the stiffens of the spring and the dumping coefficient of the viscous damper. If parametric models of all presented components ware created, it would be possible to make a multidisciplinary optimization and so enhance the results of the semi-active suspension system dynamics.

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