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SOLUTION METHOD FOR PREDICTION OF RHEOLOGY OF CONCRETE REINFORCED BEAM STRUCTURES AFTER CRACKING

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Summary: *For the analysis of reinforced concrete it is often necessary to take into account creep, shrinkage and change of stiffness caused by cracking. The authors of this paper develop an effective method for the solution of these problems. The developed method is based on the plane beam model, and the finite element method (FEM) is used. The goal of this method is to predict creep and shrinkage of concrete structures with respect to the nonlinear stress-strain diagram of concrete.*

1. Introduction

One of the concrete features is its rheological behavior which affects the stress distribution in structure thus influencing the ability of the structure to resist the load and its serviceability. Furthermore concrete behavior is nonlinear even upon the single stress and its tensile strength is low. Longtime behavior of the reinforced concrete structures is affected by the both mentioned phenomena. In this paper both problems are aimed to be solved in the interaction by development of a usable method and computational tool.

2. FEM element

During the material nonlinear analysis of structures, the neutral axes of beam elements shift, especially after cracking. Commonly-used types of elements are not suitable for this purpose due to the difficulties with the description of the modified cross-sectional characteristics particularly in the case of the neutral axis shifts.

For this reason the authors of this paper developed a special type of element. This type of element is based on constitutive equations for a reference axis that can be different from a centroidal axis.

$$\begin{Bmatrix} N \\ V \\ M \end{Bmatrix} = \begin{bmatrix} EA & -ES & 0 \\ -ES & EI & 0 \\ 0 & 0 & GA_\kappa \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_n \\ \varepsilon_m \\ \varepsilon_v \end{Bmatrix}, \tag{1}$$

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where E and G are material characteristics, A is the cross-sectional area, S is the first moment of area and I is the second moment of area for reference axis (fig. 1), ε_n, ε_m and ε_v are longitudinal strain of reference axis, bending strain (curvature) and shear strain, respectively. The transfer matrix method was used for derivation of a stiffness matrix. The stiffness matrix was successfully created and the type of element is completely compatible with the displacement version of the finite element method. The derivated type of element fulfils the common assumptions valid for beams (Zídek & Nečásek, 2003).

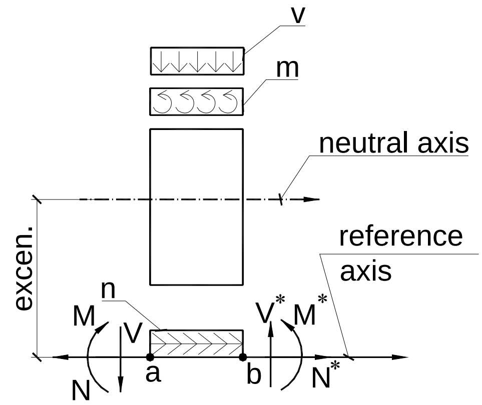

Fig. 1: Beam on eccentricity

3. Constitutive relations

For the purpose of a time dependent analysis the groups of the reinforcement and concrete part of the cross-section are considered as individual elements on eccentricity. As these elements have the same nodes they can be assigned to one cross-section. The layered concept is used for evaluation of the cross-sectional stiffness of concrete finite elements.

Stress-strain diagrams for concrete in compression and for steel are assumed according to (Eurocode2, 2005). The relation between stress and crack opening of concrete is considered to be exponential and is given by the specific fracture energy G_f. The real discrete crack is replaced into zone of one element in the developed program Asteres.

The common problem of this approach, the dependency of the result on element size, is solved using mesh adjusted softening modulus. This approach has been modified for structures in bending (Brdečko & Zídek, 2006). When the first cracks appear in the bending member of the structure, the reinforcement forestalls localization. The zone for smearing of an inelastic strain is considered equal to the distance of cracks (solved according to Eurocode 2, 2005). After yielding of steel the dominant cracks can be developed and the mesh adjusted softening modulus is used.

For this type of finite element it is considered that the cross-sectional stiffness is constant along the element. On the opposite the value of bending strain (curvature) is considered different for each node. Assuming that the crack can be found anywhere along the element and the inelastic strain is smeared into the whole element, the stiffness of the cross-section of more damaged node is used for the element.

3.1. Examples

This example was intended for testing of the material nonlinear model implemented in program Asteres without the time dependent analysis. It is based on experiments carried out on the two T-beams. The measured structures varied in the widths of the slabs and in reinforcement. The experimental data were taken from the literature (Abdel-Rahman, 1982).

3.1.1. Description of experiments

The lengths of both beams were 3.6 m and their span 3.3 m. The height of ribs was 0.21 m and of slabs 0.04 m. The width of the ribs was 0.12 m and the width of the slab was in the first case 1 m and in the second case 0.36 m. The size and location of reinforcements are obvious from fig. 3 and 4. Both beams were loaded by the increasing force in the middle of the span (fig 2).

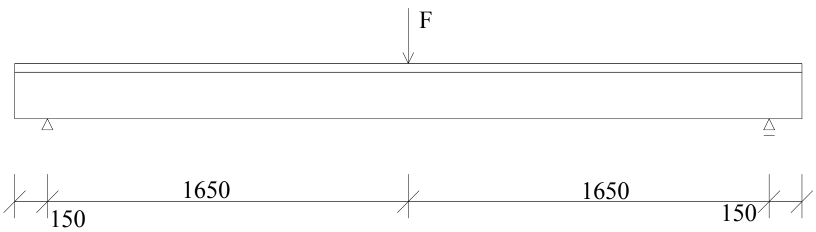

Fig.2: Beams 1 and 2 - scheme of loading and boundary condition

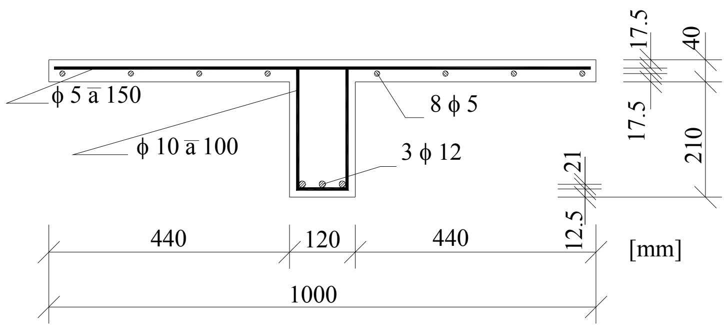

Fig. 3: Beam 1 – a scheme of reinforcement

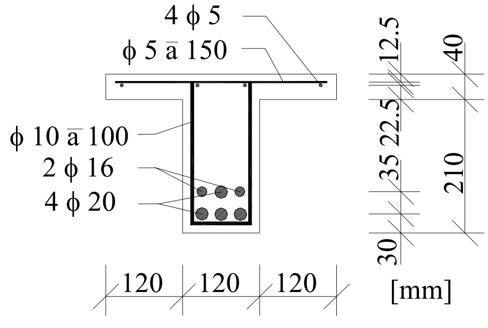

Fig. 4: Beam 2 – a scheme of reinforcement

3.1.2. Material properties of concrete and steel

The experimentally determined material properties of concrete were following: the modulus of elasticity E_o = 35 GPa; the strength in compression f_c = 48 MPa and the strength in tension f_t = 4.8 MPa. For steel were mentioned in (Abdel-Rahman 1982): the modulus of elasticity E_s = 200 GPa and the yield limit f_y = 340 MPa. The other properties were assumed according to the recommendations found in literature. It was concerned about the specific fracture energy that was assumed according to (Červenka et al., 2003) G_f = 0.000025.f_t = 60.1 N/m. The parameters of stress-strain diagram of concrete in compression were considered according to (Eurocode 2, 2005) as ε_{c1} = 0.0023 and ε_{cu1} = 0.0035. Also for the determination of a crack distance the approach given in (Eurocode 2, 2005) was used. The final distances were s_r = 0,16 m and s_r = 0.085 m for beam 1 and 2, respectively.

3.1.3 Solutions and results

The beams were statically analyzed by the program Asteres. The beams were divided into 40 elements along the length. One cross-section corresponded to one concrete element (with 8 layers for slabs and 8 layers for ribs) and two steel elements in the first case and four elements in the second case. For solution of load-displacement diagram the load force in the middle of the span was increasing in steps.

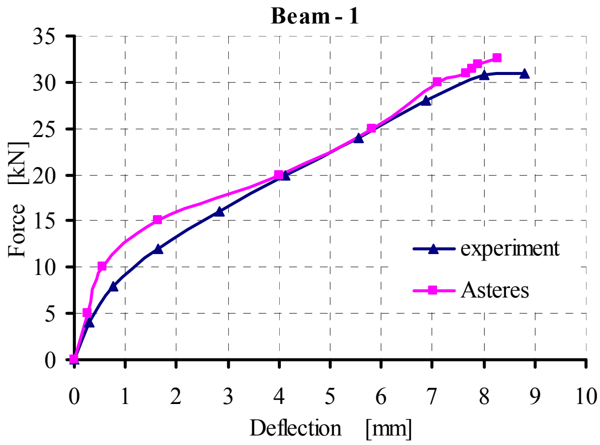

Fig. 5: Deflection in the midle of the span of beam 1

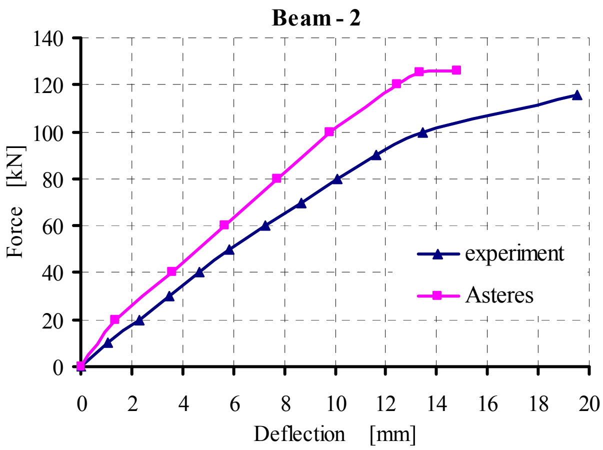

Fig. 6: Deflection in the midle of the span of beam 2

Deflections in the middle of the span in dependency on the loading force were observed. Comparison of the experiment with results of the Asteres analysis for beam 1 (2) is shown in fig. 5 (6).

4. Rheology of concrete

Rheology is considered not only as property of material but also as property of a cross-section. This approach makes possible to express creep and shrinkage of concrete using standards. The total strain of a concrete member is expressed as

$$\varepsilon(t)=\varepsilon_e(t)+\varepsilon_{ne}(t)+\varepsilon_c(t)+\varepsilon_t(t)+\varepsilon_s(t), \tag{2}$$

where ε_e is the elastic strain, ε_{ne} is the instantaneous plastic strain, ε_c is the strain caused by creep of concrete, ε_t expresses the temperature changes and ε_s is shrinkage of concrete.

A real analysis is complicated due to two problems. The former is to evaluate creep of concrete in analysis; the latter is a combination of the material nonlinear analysis and the creep analysis.

4.1. Time discretization method

For a practical solution the time discretization method has been chosen. The observed period of structural analysis is divided into the time segments. The stresses within these time segments are considered constant. The FEM analyses are carried out in time nodes, within that the rheological strains are considered as loads. Creep strain ε_c for the time interval from t_1 to t_2 can be expressed as

$$\varepsilon_c(t_2)=\frac{\sigma(t_1)}{E(t_1)}\varphi(t_2,t_1)+\int_{t1}^{t2}\frac{d\sigma(t)}{dt}\frac{\varphi(t_2,t)}{E(t)}dt\,, \tag{3}$$

where $\sigma(t_1)$ is the stress at time t_1, E is the modulus of elasticity (the first fraction in (1) is what is known as elastic strain) and $\varphi(t_1, t_2)$ is the creep coefficient for interval $\langle t_1,t_2\rangle$. An exact solution of the integral (3) is usually impossible. Integral in (3) can be solved numerically. After some arrangements the formula (3) reads

$$\varepsilon_c(t_2)=\sum_{i=1}^{n}\frac{\Delta\sigma(t_i)}{E(t_i)}\varphi(t_2,t_i), \tag{4}$$

where n is the number of intervals of observed time, $\Delta\sigma(t_i)$ is the change of stress in i-time node, $E(t_i)$ is the modulus of elasticity valid for i-time node and $\varphi(t_i, t_2)$ is the creep coefficient for interval $\langle t_i,t_2\rangle$. Stress is considered constant for each time interval.

Common procedure within a time node:

1. The evaluation of creep - solution of creep coefficients for the previous changes in strain for all concrete elements. Creep loading is expressed by means of the initial strains. Shrinkage and temperature loads are added.
2. The iteration process of material nonlinear analysis. Rheology loading is taken into account.
3. The calculation of strains for all beam elements and the saving of strains for the next time step.

4.2. Material nonlinear analysis and rheology of concrete

In the case of beams, general longitudinal strains can be described as the relative rotation and the longitudinal strain of a reference axis. On the base of these strains and material characteristics internal forces and stresses can be evaluated.

For the analysis of creep of the whole cross-section the following presumptions were accepted:

1. Creep is considered as a feature of cross-sections.
2. Concrete is considered as an aging linear visco-elastic material, i.e. it is assumed that stress in compression is less than 0.4 f_c, where f_c is the compressive strength of concrete. Due to this a linear creep can be considered.
3. Creep coefficient is valid for concrete in compression as well as in tension.

In the case of material nonlinear solution, the cross-sectional stiffness changes by implication of cracking. Loading by creep is expressed using strains ε_n and ε_m according to formulas

$$\varepsilon_n = \frac{du}{dx}, \ \varepsilon_m = \frac{d\phi}{dx}, \tag{5}$$

where u is the longitudinal translation in reference axis x and ϕ is the rotation of cross-section. Loading by rheology expressed in this way is independent of the change of the cross-sectional characteristics even the reason is cracking or aging of concrete that is expressed by the change of Young modulus. The total strain can be expressed

$$\varepsilon(t) = \varepsilon_{mech}(t) + \varepsilon_t(t) + \varepsilon_s(t), \tag{6}$$

where ε_{mech} is the mechanical strain (depends on mechanical load)

$$\varepsilon_{mech}(t) = \varepsilon_e(t) + \varepsilon_{ne}(t) + \varepsilon_c(t) \tag{7}$$

and can be expressed for the *n*th time node

$$\varepsilon_{mech}(t_n) = \sum_{i=1}^{n-1} \Delta(\varepsilon_e(t_i) + \varepsilon_{ne}(t_i))\varphi(t_n, t_i). \tag{8}$$

Fig. 2 shows a stress-strain diagram. E_{sec} is the secant modulus of elasticity. The secant cross-section characteristic EA_s, EI_s, ES_s are known at the end of an iteration material nonlinear analysis (shear strain is neglected) and are used for the evaluation of strain.

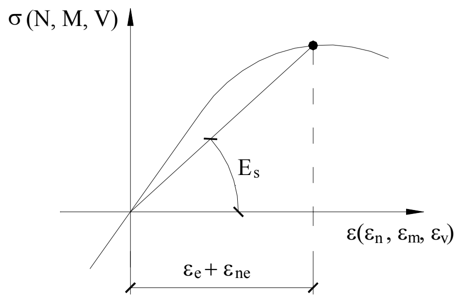

Fig. 7: Stress-strain diagram

Conclusions

The method and computer tool for the solution of creep in reinforced concrete structures after cracking is being developed. This article describes the theoretical background and presents the first results of solutions. In the future, the authors would like to use the program Asteres for solution of practical problems.

Acknowledgement

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