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PROCEDURE FOR INVESTIGATION OF THE EQUILIBRIUM POSITION STABILITY AND FOR THE NONLINEAR RESPOSE OF THE ROTOR SUPPORTED BY FLUID FILM BEARINGS AND HAVING A DISC SUBMERGED IN INVISCOUS, INWETTABLE LIQUID

J. Zapoměl *

Summary: *Lateral vibration of rotors is significantly influenced by their supports and by their interaction with the medium in the ambient space. If the disc of the rotor is submerged in a liquid and if it vibrates, the pressure field is induced and the liquid acts by a force on the wall of the disc. It is assumed that the disc performs oscillations only with small amplitudes and it enables to describe the produced pressure field by a Laplace's equation and by the relation for the boundary conditions. The liquid is inwettable and it means that it does not lean to the disc surface and therefore no tangential forces acting on the disc are produced. The resulting force is obtained by integration of the pressure distribution around the circumference and along the height of the submerged part of the disc. Its components are proportional to the disc accelerations and it implies that the negatively taken coefficients of proportionality can be considered as additional masses. As the bearing gap is very narrow, the pressure field in the oil film can be described by a Reynolds' equation. In the areas of a vapour cavitation the pressure is considered to be constant. Components of the bearing forces are obtained by integration of the pressure distribution in the oil layer around the circumference and along the length of the bearing. Lateral vibration of such rotor systems is governed by a nonlinear equation of motion. In the neighbourhood of the equilibrium position it can be linearized and this enables to judge its stability utilizing the natural frequencies of the rotor system. For solution of the equation of motion including the transient component a modified Newmark method has been chosen.*

1. Introduction

In a lot of technological applications the rotors are supported by fluid film bearings and have the discs partly or fully submerged in various liquids. Lateral vibration of such rotors is significantly influenced by their interaction with the medium in the surrounding space. A computer modelling method represents an important tool for investigation of their behaviour.

* Doc. Ing. Jaroslav Zapoměl, DrSc.: Centre of Inteligent Systems and Structures, Institute of Thermomechanics, v.v.i., branch at the VSB-Technical University of Ostrava, 17. listropadu 15, Ostrava-Poruba, 708 33 ; tel.: 59 732 3267, fax: 59 732 1287; e-mail: jaroslav.zapomel@vsb.cz

The movement of the disc submerged in a liquid or oscillation of the vessel wall produce the pressure that acts on the disc and has an influence on vibration of the rotor. In general the pressure distribution and the velocity field in a liquid are described by the Navier-Stokes equations and the equation of continuity. On certain conditions (small amplitudes of oscillations) the governing equations can be reduced to the Laplace' one (Bathe, Levy & Wilkinson, 1976). Components of the resulting force acting on the disc are then obtained by integration of the pressure distribution over the total surface of the submerged part of the disc.

The hydrodynamic bearings are usually implemented into the computational models by means of nonlinear force couplings. The determination of the bearing forces starts from solving the Reynolds' equation (Cameron, 1966, Krämer, 1993).

2. Hydraulic forces acting on the disc submerged in a liquid

The investigated system is a vessel filled with a liquid in which a disc of a vertical rotor is submerged. If the disc vibrates or if the vessel moves, the pressure field is induced and the liquid acts on the wall of the disc by inertia forces. To determine their magnitudes it is assumed that

- the disc is circular and its surface is absolutely smooth,
- the inner wall of the vessel is cylindical of general cross section (e.g. circular, elliptical, etc.),
- the surface lines of the interior surface of the vessel and of the outer surface of the disc are vertical and parallel,
- the disc and the vessel are absolutely rigid bodies,
- the disc and the vessel can move only in the radial direction and their displacements are small,
- the liquid is incompressible, inviscous, and inwettable,
- vibration of the liquid in the vessel is treated as 2D.

As the disc performs oscillations only with small amplitudes, the pressure field in the liquid can be described by a Laplace's equation (Levy & Wilkinson, 1976)

$$\frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = 0 \tag{1}$$

and by the relation for the boundary conditions

$$\frac{\partial p}{\partial n} = -\rho a_n \tag{2}$$

p - pressure,
y, z - cartesian coordinates,
n - coordinate in the direction of the outer normal to the boundary,
a_n - acceleration of the point on the boundary in the direction of its normal,
ρ - density of the liquid.

Equation (2) can be rewritten into the following form

$$\frac{\partial p}{\partial y}\cos\alpha_n + \frac{\partial p}{\partial y}\sin\alpha_n = -\rho a_n \tag{3}$$

where

$$a_n = a_y \cos\alpha_n + a_z \sin\alpha_n \tag{4}$$

a_y, a_z - acceleration of the point on the boundary in the y, z directions,
α_n - directional angle of the outer normal of the boundary (orientation into the liquid).

For solving the Laplace's equation a finite element method can be applied. As shown in Levy & Wilkinson, 1976, solution of equation (1) with the boundary condition (2) is equivalent to minimizing the functional Ψ

$$\Psi = \frac{1}{2}\int_{\Omega}\left[\left(\frac{\partial p}{\partial y}\right)^2 + \left(\frac{\partial p}{\partial z}\right)^2\right] dy\,dz - \int_{\Gamma_D} \rho\, a_n p\, ds - \int_{\Gamma_V} \rho\, a_n p\, ds \tag{5}$$

Ψ - functional,
Ω - investigated region (area filled with liquid),
Γ_D - interior boundary of the investigated region - edge of the disc,
Γ_V - outer boundary of the investigated region - edge of the vessel.

After performing manupulations described in details in Levy & Wilkinson, 1976 the functional Ψ takes the form

$$\Psi = \frac{1}{2}\mathbf{p}^T \mathbf{H}\,\mathbf{p} - \mathbf{p}^T \left(a_{Dy}\,\mathbf{g}_{Dy} + a_{Dz}\,\mathbf{g}_{Dz} + a_{Vy}\,\mathbf{g}_{Vy} + a_{Vz}\,\mathbf{g}_{Vz} \right) \tag{6}$$

$\mathbf{H}$ - coefficient matrix,
$\mathbf{g}_{Dy}$, $\mathbf{g}_{Dz}$ - coefficient vectors,
$\mathbf{g}_{Vy}$, $\mathbf{g}_{Vz}$ - coefficient vectors,
a_{Dy}, a_{Dz} - y, z components of the disc acceleration,
a_{Vy}, a_{Vz} - y, z components of the vessel acceleration.

To achieve its minimum it must hold

$$\left[\frac{\partial \Psi}{\partial \mathbf{p}}\right] = \mathbf{o} \tag{7}$$

$\mathbf{o}$ - zero vector.

Consequently calculation of the pressure results at solving a set of linear algebraic equations. The unknowns are elements of vector $\mathbf{p}$

$$\mathbf{H}\,\mathbf{p} = a_{Dy}\,\mathbf{g}_{Dy} + a_{Dz}\,\mathbf{g}_{Dz} + a_{Vy}\,\mathbf{g}_{Vy} + a_{Vz}\,\mathbf{g}_{Vz} \tag{8}$$

It is evident from (8) that the induced pressure is expressed as a linear combination of the acceleration components of the dics and of the vessel wall.

The liquid is inviscid and inwettable. Therefore it does not lean to the disc surface and it means no tangential forces are produced. The components of the total force acting on the disc are obtained by integration of the pressure distribution around the circumference and along the height (thickness) of the submerged part of the disc

$$F_{Fy} = -h_D R_D \left(a_{Dy} \int_{\Gamma_D} H^{-1} g_{Dy} \cos\alpha_n d\alpha_n + a_{Dz} \int_{\Gamma_D} H^{-1} g_{Dz} \cos\alpha_n d\alpha_n \right) -$$

$$- h_D R_D \left(a_{Vy} \int_{\Gamma_D} H^{-1} g_{Vy} \cos\alpha_n d\alpha_n + a_{Vz} \int_{\Gamma_D} H^{-1} g_{Vz} \cos\alpha_n d\alpha_n \right) \qquad (9)$$

$$F_{Fz} = -h_D R_D \left(a_{Dy} \int_{\Gamma_D} H^{-1} g_{Dy} \sin\alpha_n d\alpha_n + a_{Dz} \int_{\Gamma_D} H^{-1} g_{Dz} \sin\alpha_n d\alpha_n \right) -$$

$$- h_D R_D \left(a_{Vy} \int_{\Gamma_D} H^{-1} g_{Vy} \sin\alpha_n d\alpha_n + a_{Vz} \int_{\Gamma_D} H^{-1} g_{Vz} \sin\alpha_n d\alpha_n \right) \qquad (10)$$

3. The equation of motion

The hydrodynamic bearings are usually implemented into the mathematical models of rotor systems by means of force couplings. Lateral vibration of such rotors is then governed by a nonlinear equation of motion

$$\mathbf{M}\,\ddot{\mathbf{x}} + (\mathbf{B} + \eta_V\,\mathbf{K}_{SH} + \omega\,\mathbf{G})\,\dot{\mathbf{x}} + (\mathbf{K} + \omega\,\mathbf{K}_C)\,\mathbf{x} = \mathbf{f}_A + \mathbf{f}_V + \mathbf{f}_B(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{f}_F(\ddot{\mathbf{x}}) \qquad (11)$$

$\mathbf{M}$, $\mathbf{B}$, $\mathbf{K}$, $\mathbf{G}$, $\mathbf{K}_C$ - mass, damping, stiffness, gyroscopic, circulation matrices,
$\mathbf{K}_{SH}$ - stiffness matrix of the shaft,
$\mathbf{f}_A$, $\mathbf{f}_V$, $\mathbf{f}_B$, $\mathbf{f}_F$ - vectors of applied, constrained, bearing, fluid induced forces,
$\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$ - vectors of general displacements, velocities, accelerations,
η_V - coefficient of viscous damping in the shaft material

and by relations for the boundary conditions.

Taking into consideration the relationships (9) and (10) the force by which the liquid acts on the disc can be expressed in the following manner

$$\mathbf{f}_F(\ddot{\mathbf{x}}) = -\mathbf{M}_F\,\ddot{\mathbf{x}} + \mathbf{f}_{FV} \qquad (12)$$

$\mathbf{M}_F$ - matrix of additional mass,
$\mathbf{f}_{FV}$ - vector of the fluid induced forces acting on the disc due to the movement of the vessel.

$\mathbf{M}_F$ can be considered as an additional mass matrix of the disc. It expresses inertia properties of the liquid that influence vibration of the disc. In general matrix $\mathbf{M}_F$ is real and not symmetric. Its elements depend on the mutual position of the disc relative to the wall of the vessel.

After a simple manipulation the equation of motion (11) can be modified into this form

$$(\mathbf{M} + \mathbf{M}_F)\,\ddot{\mathbf{x}} + (\mathbf{B} + \eta_V\mathbf{K}_{SH} + \omega\,\mathbf{G})\,\dot{\mathbf{x}} + (\mathbf{K} + \omega\,\mathbf{K}_C)\,\mathbf{x} = \mathbf{f}_A + \mathbf{f}_V + \mathbf{f}_B(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{f}_{FV} \qquad (13)$$

To determine components of the bearing force it is necessary to know a pressure distribution in the oil film. As the bearing gap is very narrow a classical theory of lubrication may be applied for this purpose. The pressure distribution in the oil layer is then described by the Reynolds' equation (Cameron, 1966, Krämer, 1993)

$$\frac{1}{R^2}\frac{\partial}{\partial\varphi}\left(h^3\frac{\partial p}{\partial\varphi}\right)+\frac{\partial}{\partial Z}\left(h^3\frac{\partial p}{\partial Z}\right)=\frac{6\eta}{R}\left(u_1+u_2\right)\frac{\partial h}{\partial\varphi}+\frac{6\eta}{R}h\left(\frac{\partial u_1}{\partial\varphi}+\frac{\partial u_2}{\partial\varphi}\right)+12\eta\frac{\partial h}{\partial t} \tag{14}$$

where

$$h = c - e_H \cos(\varphi - \gamma) \tag{15}$$

u_1 - circumferential velocity component of the points on the liner interior surface,
u_2 - circumferential velocity component of the points on the journal surface,
p - pressure,
c - width of the bearing gap,
R - journal radius,
h - thickness of the oil film ,
e_H - eccentricity of the journal centre,
γ - position angle of the line of centres,
φ, Z - circumferential, radial, and axial coordinates,
η - oil dynamical viscosity,
t - time.

The Reynolds' equation holds only in the areas where the hydrodynamic effect occurs. If the pressure drops to a critical level, a vapour cavitation takes place. The experiments (Zeidan & Vance, 1990) showed that pressure of the medium in such areas remained approximately constant. Therefore from the simplest distinguishing level the pressure distuibution in the oil film can be described

$$p_d = p \qquad \text{for} \qquad p \geq p_{CAV} \tag{16}$$

$$p_d = p_{CAV} \qquad \text{for} \qquad p < p_{CAV} \tag{17}$$

p_d - pressure distribution in the bearing gap,
p_{CAV} - pressure of the medium in the cavitated area.

Components of the bearing force by which the oil film acts on the rotor journal are calculated by integration of the pressure distribution p_d around the circumference and along the length of the bearing

$$F_{By} = -R\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi} p_d \cos\varphi \, d\varphi \qquad , \qquad F_{Bz} = -R\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi} p_d \sin\varphi \, d\varphi \tag{18}$$

F_{By}, F_{Bz} - y, z components of the bearing force.

4. Stability analysis of the equilibrium position and transient response of the rotor system

Stability analysis of the equilibrium position of the rotor is performed according to the real parts of the system eigenvalues. Their determination requires to solve a quadratic eigenvalue problem

$$\det\left[\,\lambda^2(\mathbf{M}+\mathbf{M}_F)+\lambda(\mathbf{B}+\eta_V\mathbf{K}_{SH}+\omega\,\mathbf{G})+(\mathbf{K}+\omega\,\mathbf{K}_C)\,\right]=0 \tag{19}$$

The transient response of the rotor system is obtained by solving the equation of motion. For this pupose a Newmark method has been chosen. This one is implicit and it implies its algorithm starts from the equation of motion related to time t+Δt. At each integration step the solution arrives at solving a set on algebraic equations that are nonlinear in this case due to the bearing forces. To avoid this manipulation vector $\mathbf{f}_B$ related to the point of time t+Δt can be approximately expressed by means of its expansion into a Taylor series in the neighbourhood of time t

$$\mathbf{f}_{B,t+\Delta t} = \mathbf{f}_{B,t} + \mathbf{D}_{B,t}\,(\dot{\mathbf{x}}_{t+\Delta t} - \dot{\mathbf{x}}_t) + \mathbf{D}_{K,t}\,(\mathbf{x}_{t+\Delta t} - \mathbf{x}_t) + \dots \tag{20}$$

where

$$\mathbf{D}_{K,t} = \left[\frac{\partial \mathbf{f}_B(\mathbf{x},\dot{\mathbf{x}})}{\partial \mathbf{x}}\right]_{\mathbf{x}=\mathbf{x}_t,\dot{\mathbf{x}}=\dot{\mathbf{x}}_t} \quad , \qquad \mathbf{D}_{B,t} = \left[\frac{\partial \mathbf{f}_B(\mathbf{x},\dot{\mathbf{x}})}{\partial \dot{\mathbf{x}}}\right]_{\mathbf{x}=\mathbf{x}_t,\dot{\mathbf{x}}=\dot{\mathbf{x}}_t} \tag{21}$$

Then substitution of only the linear portion of the Taylor series (20) and carrying out several simple operations give the equation of motion related to time t+Δt that has the form which requires to solve only a set of linear algebraic equations at the current integration step

$$(\mathbf{M} + \mathbf{M}_F)\,\ddot{\mathbf{x}}_{t+\Delta t} + (\mathbf{B} + \eta_V\,\mathbf{K}_{SH} + \omega\,\mathbf{G} - \mathbf{D}_{B,t})\,\dot{\mathbf{x}}_{t+\Delta t} + (\mathbf{K} + \omega\,\mathbf{K}_C - \mathbf{D}_{K,t})\,\mathbf{x}_{t+\Delta t} =$$

$$= \mathbf{f}_{\mathbf{A},t+\Delta t} + \mathbf{f}_{V,t+\Delta t} + \mathbf{f}_{B,t} - \mathbf{D}_{B,t}\dot{\mathbf{x}}_t - \mathbf{D}_{K,t}\mathbf{x}_t + \mathbf{f}_{FV,t+\Delta t} \tag{22}$$

More details on application of this approach can be found e.g. in Zapoměl & Malenovský, 2000.

5. Example

Rotor of the investigated rotor system (Fig.1) consists of a shaft (SH) and of two discs (D1, D2). The shaft is coupled with a rigid frame (FR) by two hydrodynamic bearings (B1, B2). Disc D1 mounted on the overhung end of the shaft is placed in a vessel (VS) filled with liquid and is totally submerged. Disc D2 is coupled with a rigid rotor of an electric motor by a prestressed flexible belt. The rotor rotates at constant angular speed (300 rad/s) and is loaded by the centrifugal forces caused by unbalances of both discs.

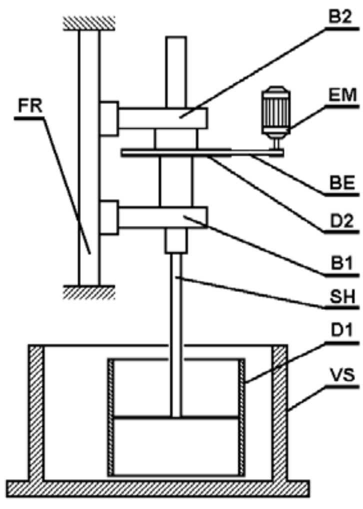

Fig. 1: Scheme of the investigated rotor system

The task was to analyze stability of the equilibrium position and motion of the rotor after dying out the initial transient component of its vibration.

In the computational model the shaft was represented by a beam like body, both discs were considered as absolutely rigid, and the liquid as incompressible, inviscid, and inwettable. For the purpose of the calculation the shaft was discretized into finite elements.

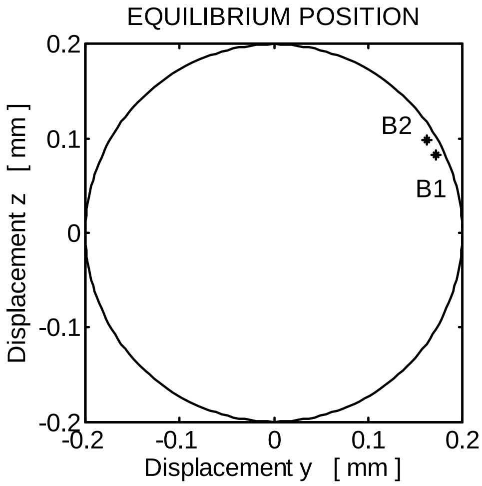

Fig. 2: Journal centres equilibrium positions in bearings B1, B2

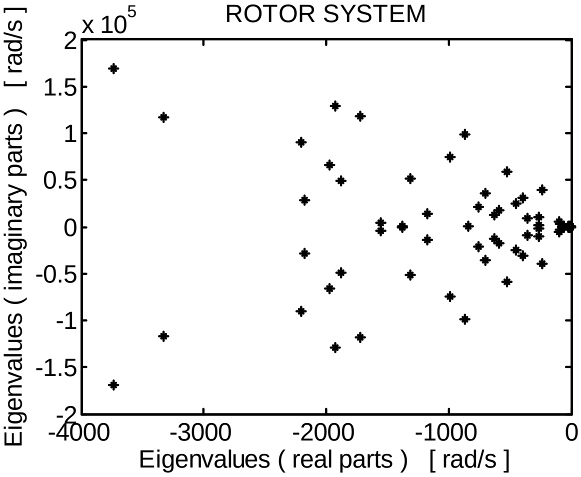

Fig. 3: Rotor system eigenvalues

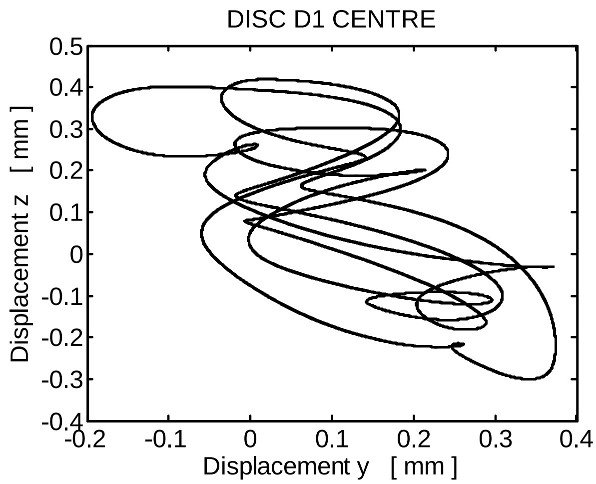

Fig. 4: Steady state trajectory of the disc centre

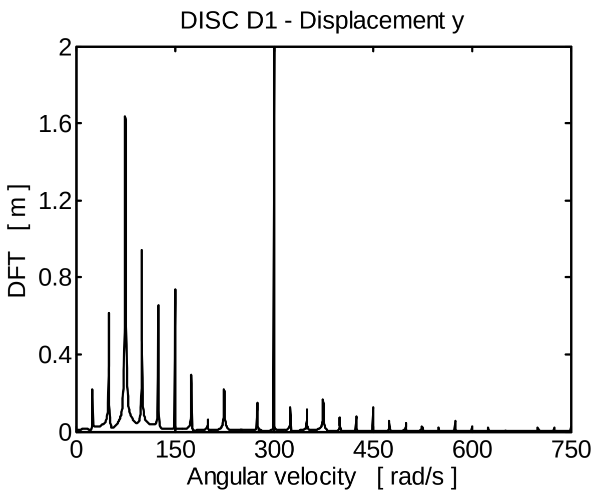

Fig. 5: Fourier spectrum

The equilibrium positions of the rotor journal centres in bearings B1 and B2 are evident from Fig.2. Distribution of eigenvalues of the rotor system whose parameters are linearized in the neighbourhood of the equilibrium position in a Gaus plane are drawn in Fig.3. Real parts of all of them are negative and it implies the equilibrium position is stable. Trajectory of the disc D1 centre after dying out the initial transient component of the vibration can be seen in Fig.4. Even if its shape is rather complicated, the vibration remains periodic. Image of the Fourier transform of y displacement of the disc D1 centre is drawn in Fig.5. Its evident that the resulting vibration is composed of a number of subharmonic and ultraharmonic partial motions.

5. Conclusions

The described method represents a new and complex approach to investigation of a mutual intraction of a rotor, of the hydrodynamic bearings, and of the liquid in which the disc of the rotor is submerged. Results of the computer simulations showed that the developed method behaved numerically stable including the cases when the vibration became irregular.

Acknowledgement

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