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THERMO-MECHANICAL INSTABILITIES OF FRICTION SYSTEMS - MEASUREMENTS AND COMPUTATIONS

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Summary: *Formation of hot spots as well as non-uniform distribution of the contact pressure aredifficulties, troubles imerging in important friction systems, e.g. disc brakes or transmission clutches. If the sliding velocity is high enough, this effect can arise and can cause vibrations, enhanced wear and material damage. Barber (1969) has called the thermoelastic instability a cause of such effects. Authors have paid attention to the field of contact mechanics both from the angle of experimental research and mathematical modelling as well.*

1. *Introduction*

Hot spots and non-uniform distribution of the contact pressure are inconveniences emerging in important friction systems, e.g. in disc brakes or transmission clutches. If the sliding velocity is high enough, this unwanted effect can become unstable and can result in frictional vibration, material damage, excessive wear and brake fading. Authors have paid attention to the field of contact mechanics for several years both from the angle of experimental research and mathematical modelling as well. Their effort has been focused especially on disc brakes.

2. Experimental techniques

The following part of the paper introduces the measuring system that has been developed at the University of West Bohemia in Pilsen for the experimental research of thermo-mechanical instabilities of disc brakes. The non-contact temperature measuring system consists of two-colour infrared detectors in combination with optical fibers and data acquisition system. Concerning the application to measure temperature field on the brake disc under rotation five infrared detectors measuring intensity of thermal radiation at five different radiuses and an inductive sensor for the measurement of the disc rotational speed are applied.

2.1 Non-contact temperature measuring system

Two-colour infrared detectors cooled by liquid nitrogen are used for the fast non-contact measurement of temperature. The detector itself consists of two layers. The upper one, InSb detector, is sensitive to short infrared wavelengths while the bottom layer, HgCdTe detector, is sensitive to longer wavelengths. The two-colour detectors are designed such a way, because

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one infrared input is needed. The InSb detector acts as a radiation filter for the bottom HgCdTe detector and hence the response curves are not overlaped. Specification of these detectors is summarized in Tab.1.

Tab. 1: Specification of InSb/HgCdTe infrared detectors.

Detector	TOP: 1 mm^2 dia InSb (Photovoltaic)	BOTTOM: 1 mm^2 HgCdTe (Photoconductive)
Spectral Range	2 – 5.5 µm, peak ≈ 5.0 µm	5.5 – 14 µm, peak ≈ 12.0 µm
Responsivity in Peak	2.5×10^5 V/W	3×10^5 V/W
Bandwidth	DC – 50 kHz	5 Hz – 50 kHz
Normalized Detectivity in Peak	1×10^{11} $cmHz^{1/2}$/W	3×10^{11} $cmHz^{1/2}$/W
Dewar Hold Time	12+ hours	12+ hours
Power Requirements	± 9 – ±15 V DC	± 9 – ±15 V DC

Infrared radiation is transmitted to the detectors by IR optical fibers. Sensitive end of the fiber is located close to the surface whose temperature is being measured. If the surface temperature is high the fiber termination is protected by thermal barrier. Spectral transmission of optical fiber has to correspond with the infrared detector relative spectral response to achieve minimal losses of optical signal. Therefore chalcogenide glass optical fibers have been choosen. The spectral transmission of the fiber and relative spectral response curves of InSb and HgCdTe detectors are shown in Fig. 1.

One-colour method (radiation intensity) or two-colour method (ratio of radiation intensities of two adjacent wavelengths or wavebands) together with a calibration system can be used for the quantitative evaluation of surface temperatures.

The electrical signal produced by the photon detector can be obtained as

$$f_{1-C}(T) = A_{\det} \cdot RVF \cdot R_{peak} \cdot \int_{\lambda_1}^{\lambda_2} \varepsilon(\lambda,T) \cdot R(\lambda) \cdot U(\lambda) \cdot M_{\lambda}(\lambda,T) d\lambda, \quad (1)$$

where λ_1, λ_2 [µm] are the minimal and maximal detectable wavelength, $A_{\det}$ [cm^2] is active detector area, RVF [-] radiation view factor, $\varepsilon(\lambda,T)$ [-] emissivity of the measured surface, R_{peak} [V/W] responsivity in peak of the spectral response curve, $R(\lambda)$ [-] spectral response function of the infrared photon detector, $U(\lambda)$ [-] transfer function of the optical system, $M_{\lambda}(\lambda,T)$ [W/cm^2.µm] spectral radiant emittance.

The major advantage of the two-colour method is based on the reduction of the emissivity influence on the temperature measurement process by means of two different infrared signals bands detection. The ratio of these two signals is proportional only to the surface temperature in the case of a grey body (emissivity in both bands is equal). Then the surface temperature can be expressed as:

$$f_{2-C}(T) = \frac{\varepsilon \cdot A_{\det 1} \cdot RVF \cdot R_{1peak} \cdot \int_{\lambda_1}^{\lambda_2} R_1(\lambda) \cdot U(\lambda) \cdot M_\lambda(\lambda, T) d\lambda}{\varepsilon \cdot A_{\det 2} \cdot RVF \cdot R_{2peak} \int_{\lambda_2}^{\lambda_3} R_2(\lambda) \cdot U(\lambda) \cdot M_\lambda(\lambda, T) d\lambda}, \quad (2)$$

where ε is the emissivity of the grey body.

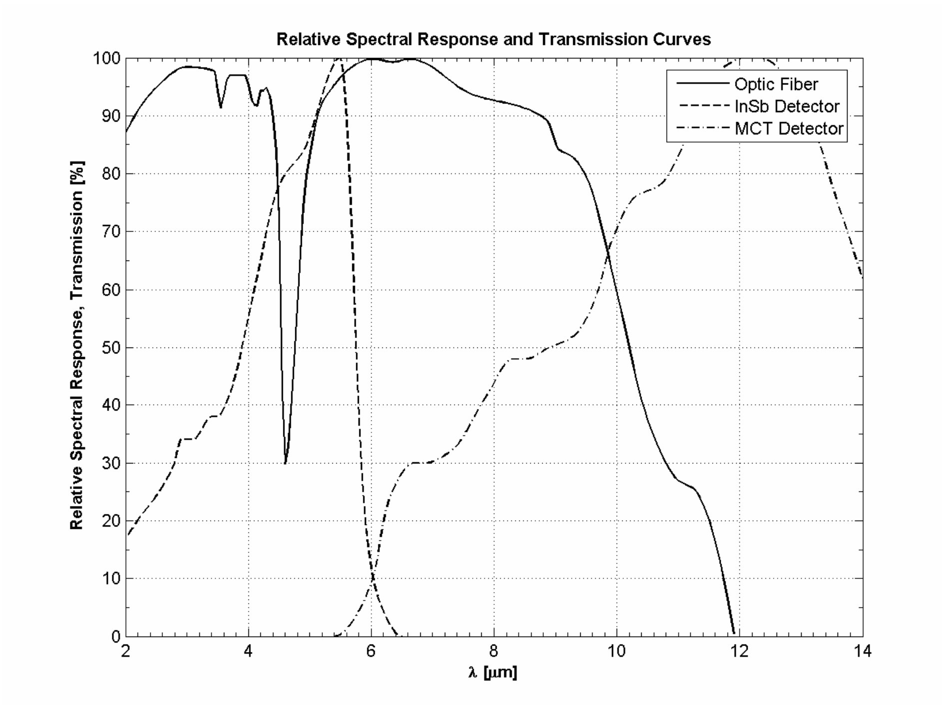

Fig. 1: Relative spectral response curves for InSb and HgCdTe detector and transmission curve for chalcogenide glass infrared fiber.

2.2 Application on the brake disc temperature field measurement

The main part of the measuring system is a fast sub-system for the measurement of brake disc temperature field. The above mentioned infrared detectors with optical fibers are applied to measure infrared radiation at five different radiuses of the disc. Measuring the disc rotational speed by an inductive sensor, the temperature field of the disc surface can be reconstructed. The experimental set-up is shown in Fig. 2.

The application of the measuring system on the brake testing bench is demonstrated in Fig. 3. Rotation of the brake disc is driven by electric engine using the car axle shaft. Braking pressure is induced by a computer controlled hydraulic piston. The experimental set-up utilizes personal car front wheel suspension. The scheme of the experimental set-up and also the photo showing installation of the measuring system are shown on Fig. 3.

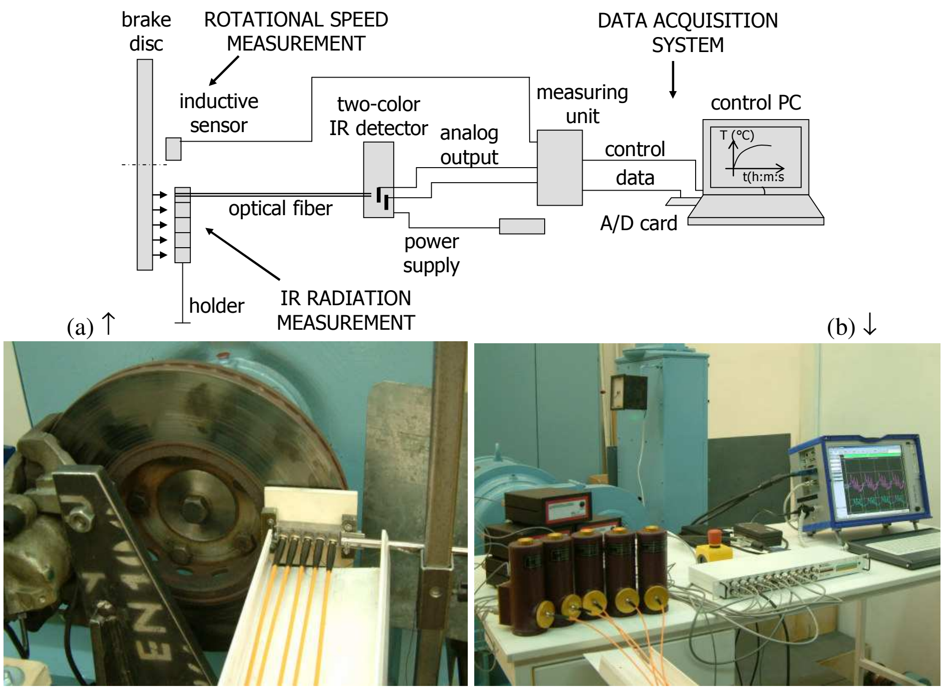

Fig. 2: System for the brake disc temperature field measurement – (a) simplified scheme, (b) experimental set-up photo.

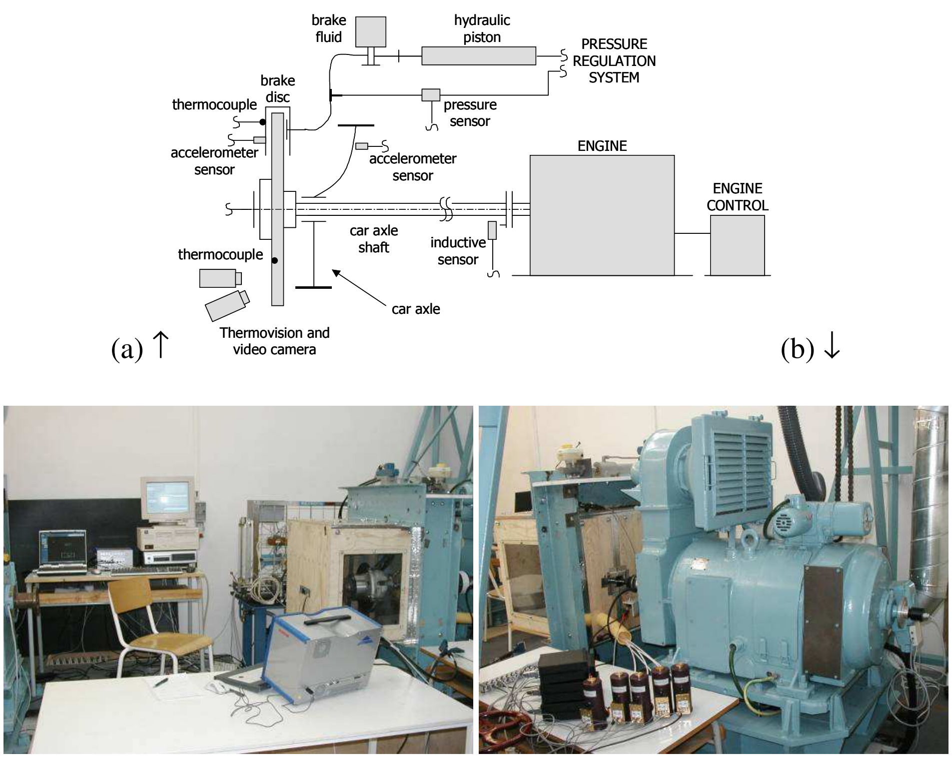

Fig. 3: Experimental set-up of the brake dynamometer rig – (a) simplified scheme, (b) experimental set-up photo.

3. Mathematical modelling

Barber (1969) pointed out for the first time the relationship between the onset of hot spots and the thermoelastic instability (TEI). Dow & Burton (1972) and Burton et al. (1973) introduced a mathematical model to establish critical sliding velocity for instability, where two thermoelastic half-space are considered in contact along their common interface. That enabled understanding the core of the effect in more detail, because material parameters, sliding velocity and friction coefficient were incorporated into their model. Consequently, Lee & Barber (1993) accomplished the next essential step as late as 20 years after. They brought an analytical periodical solution that involves the disc with a finite thickness clamped between half-spaces. Thus their model finally included a geometrical dimension. Decuzzi et al. (2000) contributed to further development of analytical approach in connection with transmission clutches. Some difficulties have appeared in the application of developed methods to disc brakes where an intermittent contact (the friction pads occupy only the part of the disc circuit) occurs and therefore the mathematical solution is no more periodic. The modified method proposed in a recent paper (Voldřich, 2007a) seems an appropriate approach to avoid the difficulty.

Analytical methods can elucidate only the propensity of friction systems to the thermoelastic instability and the initial stage of an instability rise only. The assumption of the full contact regime between the pads and the disc used by the analytical solution is the major limitation. Besides it, it is impossible to consider the material parameters and the friction coefficient as time-dependent functions during braking. Further, the intermittent contact cannot be appreciated exactly by analytical methods. Another approach is connected with FEM and the Petrov-Galerkin method (PGM) (omitted in common commercial softwares). Zagrodski et al. (2001) were the first who suggested a numerical approach that would cover both the above-stated limitations and the transition regimes connected with the TEI as well. They used the system ABAQUS, where the PGM is implemented. Voldřich (2004) created original software with the PGM for the puprpose to solve disc brake problems.

3.1 Analytical approach

Let us consider the friction system from the Fig. 4. The system consists of layers with finite thickness and finite length. The central layer moves with the velocity V and we can consider periodical boundary conditions on lateral sides of all layers. The first problem is to solve the heat transfer balance

$$\frac{\partial T_i}{\partial t} + \zeta V \frac{\partial T_i}{\partial x} = k_i \, \Delta T_i \;, \; \text{i} = 1,2,3, \tag{3}$$

$\zeta = 0$ for i = 1,3 and $\zeta = 1$ for i = 2. Here $k_i = K_i / c_i \rho_i$ denotes the thermal diffusivity, K_i is the thermal conductivity, ρ_i density and c_i is the specific heat. The temperature contact between layers i and j is described by the condition $T_i = T_j$ on the contact surfaces. Simultaneously, the heat flow generated by friction is

$$q_{ij} = f\, V\, p_{ij} \,, \tag{4}$$

where p_{ij} denotes a corresponding contact pressure and f is a friction coefficient, whereas

$$q_{12} = -K_1 \frac{\partial T_1}{\partial y} + K_2 \frac{\partial T_2}{\partial y} \qquad \text{and} \qquad q_{23} = -K_2 \frac{\partial T_2}{\partial y} + K_3 \frac{\partial T_3}{\partial y} \;. \tag{5}$$

The second requirement is to solve the contact elasticity problem with the strain plain deformations. The loading applied is partly a pressure p_o acting on the outside horizontal boundaries of the friction layers and partly temperatures fields T_1, T_2 and T_3.

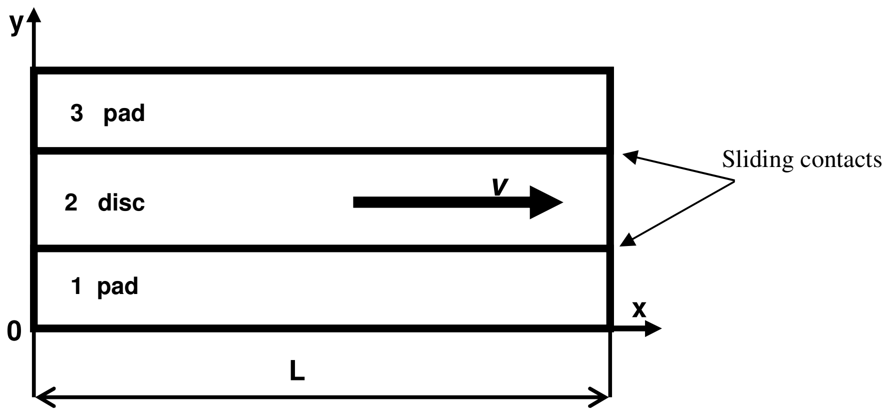

Fig. 4: Scheme of the model with one pair of sliding contacts.

Let us suppose steady state of the friction system, i.e. that both the outside preassure p_o acting on the friction layers and the sliding velocity V are time-independent. Let us suppose, as well, that material parameters and the friction coefficient are independent on temperature. Let us further consider an initial perturbation that can be caused by an initial disc corrugation or by some other reason. As a result, this perturbation disturbes the constant contact pressure p and the temperature field in the direction of the x axis. The matematical solution of the problem described above can be expressed in the form (see e.g. Voldřich, 2007a)

$$p(x,t) = p_o + p_{am} \exp(bt + jmx) , \tag{6}$$

$$T_i (x,y,t)= T_o(y,t) + T_{am} \exp(bt + jmx)\, \Psi_i(y) ,$$

where t is the time, $m = 2\pi/L$ (L is wave length of the perturbation), $j = \sqrt{-1}$ is the imaginary unit and p_{am}, T_{am} denote the amplitudes of the initial perturbation. Here $b = b(V,m)$ is a so-called growth rate that characterizes the instability. Some high velocity V is enough tobrings on the thermoelastic instability, when $b > 0$. The function Ψ_i describes an elementary perturbation mode depending also on parameters b and m. Further, $\Psi_i(y_{ij}) = 1$ for values y_{ij} respond to the contact between layers i and j.

The situation with the real disc brake is complicated by the intermittent contact. There isn't possible to find any exact analytical solution of the mathematical problem enunciated and it is necessary to fall in additional modelling. The issue consists in the convenient averaging of physical quantities alongside the disc circuit. The averaged physical quantities are the heat flow from the contact surface and the heat capacity of the adjacent layers. For that reason, we replace the real coefficient k_2 with the value k_2/N and we take NK_2 instead of K_2 in (5), where $N = 2\pi/(angle\ of\ pad)$.

We should also consider more perturbations with different wave lengths in the formula (6). Nevertheless, as Voldřich (2007a) and the other showed, the contribution of the longest wavelength is dominant, because the corresponding growth rate b is the biggest here. And it is obvious, that the longest wave length L can not be longer than the width of pads.

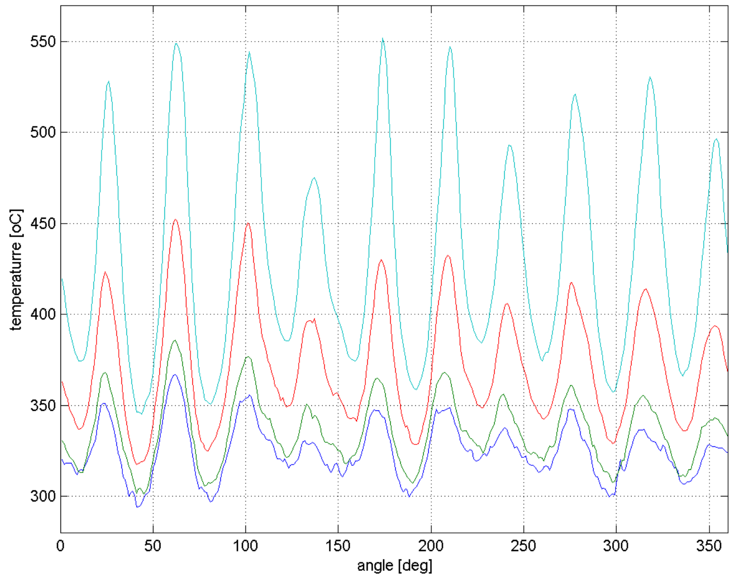

Fig. 5: Temperature development of the disc surface measured at the middle disc radius. Curves are graded by the step 5s.

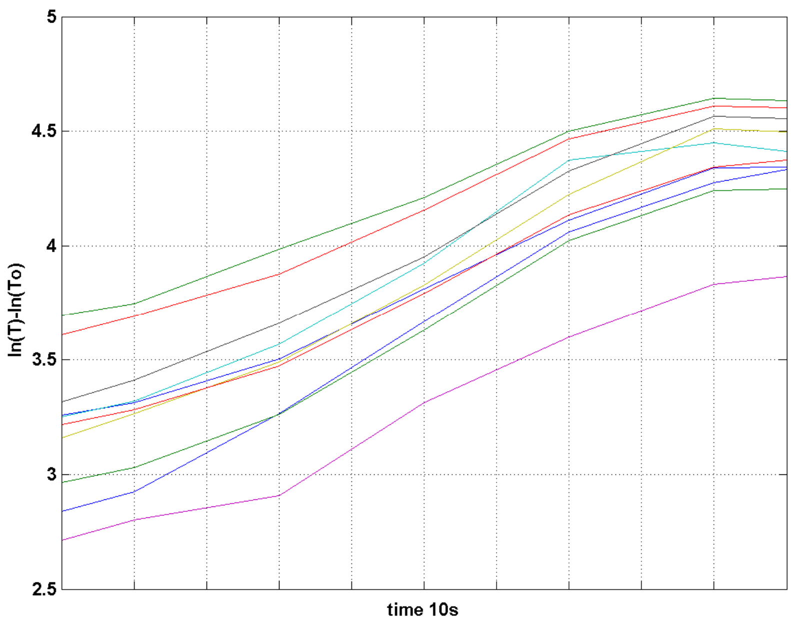

Fig. 6: Dependences of the natural logarithm of temperature amplitudes on time for the individual hot spots.

Formulae (6) result from the theoretical mathematical model and they are necessary to be verified by measurement, if they are applicable. Convenient measurements were carried out on our institut by the experimental technique and the methodology, which we described in the second paragraph. Above we present (see Fig. 5) an example of experimental results for the following configuration of the disc brake. The outer diameter of the disc was 312 mm and the width of pads were 68 mm. The material of the disc is cast iron, pads are from material usually used in automobile industry. The constant sliding velocity of 20 rev/s were kept throughout the braking process. We can see in Fig. 5 an example of the disc surface temperature development measured by the sensor placed on the middle disc radius. Temperature curves corresponding to the central angle interval from 0 to 360^o are graded by 5s step. The rise of hot spots amplitudes is evident. The next Fig. 6 can be utilized for its quantification. The dependences of the natural logarithm of the hot spot amplitude on time are plotted here; for each hot spot just one curve. It is possible to note, that the dependences $t \rightarrow \ln([T_{max}-T_{min}]/2)$ are approximately linear and have more or less the same angular coefficient for all hot spots. The advisability of the exponential function in (6) is confirmed by the fact above. We obtain the value $b \approx 0.13\ s^{-1}$ evaluating the experimental data. The shift of individual curves in Fig. 6 is due to different initial fluctuations of individual hot spots. The disparity of the initial amplitudes is related to the initial disc non-flatness and surface waveness, which may not be periodic.

It is possible to evaluate the growth factor b dependence on design of disk brake parameters and on the braking mode by using computations based on analytical approach as well. For example, we calculate $b = 0.12\ s^{-1}$ for the situation described above. Numerical results agree satisfactorily with results obtained by experiment on experimental device, or they are confirmed qualitative by car driver riding opinions.

We can able to specify the facts leading to b factor rising – first, friction segments width increasing, or decreasing its thickness, sliding velocity V increasing, stiffness of friction segments material increasing, friction coefficient increasing, thermal capacity of pads decreasing, thermal expansivity of disk material increasing, or finally, increasing of its elasticity modulus.

To ensure the stable braking mode is quite difficult, even in the case of a laboratory experimental device, so the measuring can be approximate only. That is to say, the material parameters of sliding segments and disc are temperature depended whilst the temperature is growing at breaking process. That is why the curves in Fig. 5, 6 are limited to short time window and why we have to look over the certain difference between the analytical and experimental results.

Next, it is needed to explain the fact, the exponential growth of hot-spots amplitude stops early. We can trace three different mechanisms of exponential growth interruption from experimental results:

1) The friction coeficient f declines with increasing temperature and so the friction heat source (4) declines as well. It is impossible to keep steady state braking even for the constant sliding velocity V. Further, a material rigidity is lower at higher temperature. Consequently, the growth rate b diminishes throughout braking.

2) The wear of pads accompanies every real braking and it depends on the local contact pressure p. The pressure is not uniform under pads, among others owing to the thermoelastic instability. Our experimantal results show that hot spots move periodically back and forth from the inner disc radius to the outer one.

3) The contact pressure amplitude $p_{am} \exp(bt)$ from (6) exceeds the initial uniform pressure p_o anyway, if the thermoelastic instability goes on arise.

In such a case, the loss of contact under a part of pads occurs. Thus the thermoelastic instability comes into a second, strong nonlinear, mode of its behaviour.

3.2 Numerical simulations

It is possible to evaluate approximately even transient brake modes by analytical approach, see Voldřich (2007a), although it doesn't follows from previous section 3.1. But, it can be done more precisely by numerical modelling based on finite elements method. The main reason to treat with numerical methods is given by the second instability stage being connected with the contact localization mentioned above, see point 3, section 3.1.

A difficulty of a suitable numerical method proposing lies in the fact that the appropriate Peclet number of the thermal equation (3) is well high for the sliding velocity V under consideration. The Petrov-Galerkin method (PGM), Heinrick et al. (1977), appears suitable for this case. This method consists in efficient usage of the relation

$$\int_0^t \left\{ \int_\Omega \left[M\, c\rho \left(\frac{\partial T}{\partial t} + \zeta V \frac{\partial T}{\partial x} \right) + K\, \nabla M_\Omega . \nabla T \right] dx\, dy + \int_{\Gamma_{ij}} M\; q_{ij}\, d\Gamma \right\} dt \quad +$$

$$+ \zeta \int_0^t \left\{ \sum_i \int_{e_i} P \left[c\rho \left(\frac{\partial T}{\partial t} + V \frac{\partial T}{\partial x} \right) - K\, \Delta T \right] dx dy \right\} dt = 0$$

for the discretization of a weak solution T, where $W = M+P$ is a test weight function. In the case of classic finite element method approach $P = 0$. The best choice is now

$$P(t,x,y) = \frac{\Delta x}{2} \left(\alpha + \beta \frac{\Delta t}{2} \frac{\partial}{\partial t} \right) \frac{\partial M(t,x,y)}{\partial x} \ ,$$

where Δx and Δt mean time and space discretization step, and coefficients α and β are computed (they depend on the Peclet number and Δx, Δt). It is $M = \sum_e M_e$, where functions M_e are base functions of time-space elements e. They are linear in space coordinates and quadratic in time. The factual implementation of the PGM is described in Voldřich (2004).

To illustrate contact localization let's consider the friction system from Fig. 4 with disc thickness 4 mm, segment thickness 6 mm and wavelength 40 mm. Further, usual materials (except the Poisson ratio) are considered for disc brakes design. To verify the method and program being developed, let us consider aggravated boundary situation (in compare to the common disk brake) and let's choose material Poisson ratio 0.495. Next, let p_0 = 2 MPa, friction coefficient f = 0.4 and friction velocity V = 5.5 m/s. We obtain $b \approx 24$ s^{-1} in comparison with $b \approx 21$ s^{-1} given by the analytical approach, even for such small sliding velocity. The second instability stage and contact discontinuity rising we can see on Fig. 7.

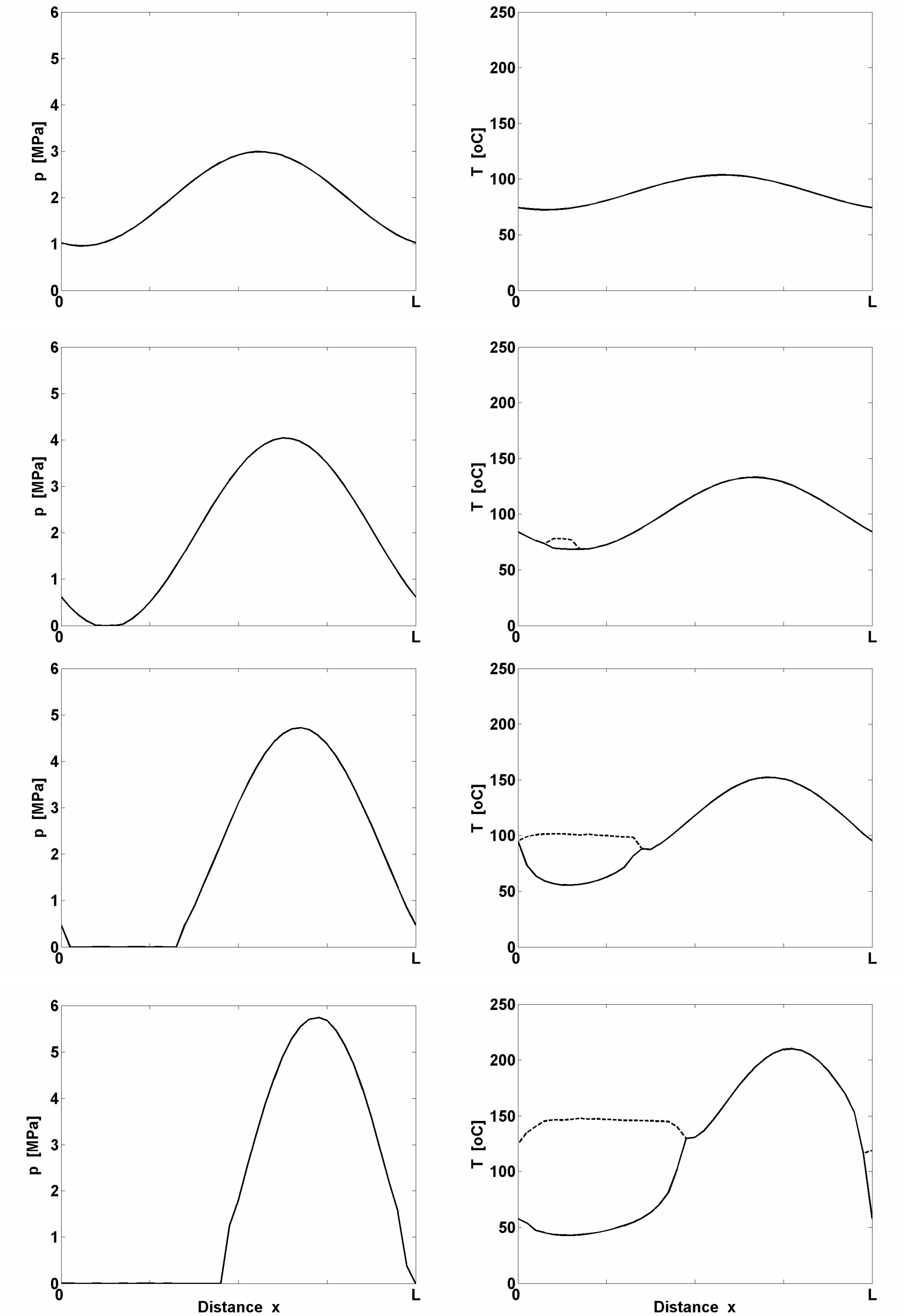

Fig. 7: The developments of contact pressure and surface temperatures for the time points 0.10 s, 0.13 s., 0.15 s and 0.20 s in the course of thermoelastic instability in the described task (*full line* plots the disk surface temperature while *dashed line* is that of friction pad).

4. Conclusions

The comparison of experimental data and computational simulations shows their qualitative correspondence to the consistent effect of material and geometric brake parameters on susceptibility to the instability. Although (semi) analytical assessments are sufficient to apply for the comparison of two brakes, the computationally demanding numerical techniques mentioned are more fitting to simulate real braking regimes. The paper's authors are also confident that dominant initial perturbations are connected with the non-planeness and waviness of the brake disc in the motionless state.

Acknowledgement

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References

Barber, J.R. (1969) Thermoelastic instabilities in the sliding of conforming solids. Proc. Roy. Soc., Series A 312, pp. 381-394.

Burton, R.A., Nerlikar, V. & Kilaparti, S.R. (1973) Thermoelastic instability in a seal-like configuration. Wear, 24, pp. 177-188.

Decuzzi, P., Ciavarella, M. & Monno, G. (2000) Frictionally excited thermoelastic instability in multi-disk clutches and brakes. ASME Journal of Tribology, 123, pp. 136-146.

Dow, T.A. & Burton, R.A. (1972) Thermoelastic instability of sliding contact in the absence of wear. Wear, 19, pp. 315-328.

Heinrich, J.C., Huyakorn, P.S. & Zienkiewicz, O.C. (1977) An 'upwind' finite element scheme for two-dimensional convective transport eqation. International Journal for Numerical Methods in Engineering, 11, pp. 131-143.

Lee, K. & Barber, J.R. (1993) Frictionally-excited thermoelastic instability in automotive disk brakes. ASME Journal of Tribology, 115, pp. 607-614.

Litoš P., Honner M., Lang V., Bártík J. & Hynek M. (2006) The measuring system for the experimental research of thermo-mechanical instabilities of disc brakes. /In/ Braking 2006, Leeds: University of Leeds, pp. 208-217.

Voldřich, J. (2004) Modelování nerovnoměrností ohřevu brzdového disku během brzdění. Proceedings of the 20th conference Computational Mechanics, pp. 547-554.

Voldřich, J. (2007a) Frictionally excited thermoelasic instability in disc brakes – Transient problem in the full contact regime. International Journal of Mechanical Sciences, 49, pp.129-137.

Voldřich, J. (2007b) Separation of contact in a sliding system with frictionally excited thermoelastic instability. Applied and Computational Mechanics, 1, pp. 357-362.

Zagrodzki, P., Lam, K.B., Al Bahkali, E. & J.R. Barber, J.R. (2001) Nonlinear transient behavior of a sliding system with frictionally excited thermoelastic instability. ASME Journal of Tribology, 123, pp. 699-708.