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# INFLUENCE OF NOTCH WIDTH ON FRACTURE RESPONSE OF BENDED CONCRETE SPECIMENS

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**Summary:** *An influence of the test specimens preparation – shape and width of a stress concentrator – on the values of parameters of nonlinear fracture models of quasi-brittle cement-based composites is analyses in the paper presented. The parameters – e.g. effective crack extension, effective fracture toughness, and specific fracture energy – can be calculated from results of loading tests performed on notched three point bend specimens. The effects are investigated by means of a comparison of numerically simulated fracture process in the cracked specimen with the specimens with the double V-notch. The numerical simulations of the fracture according to the linear elastic fracture mechanics were performed by using finite element method programs (ANSYS, and FRANC2D). The influence on the load-carrying capacity and the initial compliance of the cracked specimen are introduced and discussed.*

## 1. Introduction

This paper analyzes a potential influence of the test specimens preparation on the values of parameters of nonlinear fracture models of quasi-brittle materials. The parameters are calculated from results of loading tests performed on three point bend specimens with a notch. Primary attention is paid to the shape and width of the notch, which is usually cut by saw into the test specimens of the materials mentioned. The preparation procedure for fracture testing specimens made of cementitious composites, which are typical and very frequent representatives of quasi-brittle materials in building industry, differs from the preparation of specimens used for typical tests (compressive strength, Young's modulus etc.) in the use of a stress concentrator. The records from the performed tests are then processed by using failure models based on nonlinear fracture mechanics (Bažant & Planas, 1998, Karihaloo, 1995, Shah et al., 1995).

The usual way to process results from fracture tests is to assume the stress concentrator to be a crack. Then, fracture mechanics for a crack as a stress concentrator with the exponent of

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singularity $p = 1/2$ is employed. Therefore, the stress concentrator in the testing specimen is formed into a shape which is as close to the shape of a crack as possible. It can be performed either by inserting a sharp wedge into the mould during cementitious composite specimen casting and removing it after the specimen hardens, or by cutting the concentrator into the hardened plain specimen using a diamond saw. In reality, of course, none of the procedures creates a crack from the classical fracture mechanical point of view (a sharp one). Both procedures provide a notched specimen, the stability of which should be evaluated by using fracture mechanics of general singular stress concentrators (Klusák et al., 2007; Seitl et al., 2007). This theory, however, is correct for specimens made of brittle materials. In the case of silicate-based composites, the difference in shape between the stress concentrator in the real testing specimen (a narrow V-notch or a right-angled double V-notch, respectively) and the stress concentrator assumed by the fracture models (a crack) is not usually considered. This simplification is based on the quasi-brittle nature of tensile failure of these materials in contrast with the brittle fracture mechanism. Quasi-brittle materials are characterized by the development (growth) of a large non-elastic zone (the fracture process zone – FPZ) at the tip of the stress concentrator during loading up to a critical stage when the zone separates from the concentrator tip and moves through the body leaving a stress-free crack behind it. In contrast to the brittle fracture, the FPZ is large in comparison to the specimen dimensions and therefore it is usually larger than the width of the stress concentrator.

## 2. Problem specification

The paper tries to answer whether the non-consideration of the shape and width of the stress concentrator present in quasi-brittle testing specimens can influence the values of the parameters of non-linear fracture models determined from the test records. Two classes of non-linear fracture models are considered: the equivalent elastic crack models and the cohesive crack models. Particularly, there is an investigation into the effect of the notch width on the effective crack length, effective fracture toughness, and the progress of specific energy dissipated within the FPZ. We can expect that the parameter's determination is influenced by the notch effect in the two following areas: (i) the initial compliance of the cracked/notched testing specimen, (ii) the load carried by the specimen during the test in the early stages of the crack propagation from the notch tip.

### The influence on the initial compliance of the specimen

The equivalent elastic crack approach exploits a technique determining the effective crack length at the current stage of the fracture process by considering the change between the initial specimen compliance and the secant/unloading compliance at this stage. Both compliances (initial and current) are considered as compliances of the cracked body; however, the former is not. This effect influences

- the critical length of the equivalent elastic crack determined from the load peak,
- the effective fracture toughness due to incorrect evaluation of the critical equivalent elastic crack length,
- the current (local) fracture energy calculated from the load–displacement diagram due to incorrect evaluation of the current equivalent elastic crack length.

### The influence on the load-carrying capacity

The presence of the right-angled double V-notch instead of a crack increases the load carried by the body during the stages of the fracture process, where the path of the crack growing from the notch corner is deflected from that of the initial crack. This fact can influence

- the effective fracture toughness due to the increase of the load peak,
- the specific parts of the dependence of the local fracture energy due to an enlargement of the appropriate area under the load–displacement curve resulting from the increase of the load component.

## 3. Methodological and conceptual approach

The effects described above are investigated by means of a comparison of numerically simulated fracture process in the cracked specimen with the specimens with the double V-notch of several widths. The numerical simulations of the fracture according to the linear elastic fracture mechanics (LEFM) and LEFM of general singular stress concentrators were performed by using ANSYS (Ansys, 2005) and FRANC2D (Franc2D), finite element method (FEM) programs. The FEM simulations were performed under plane strain conditions for static loading. All stresses are assumed to remain in the elastic range and the assumptions of LEFM are considered. Details are mentioned e.g. Seitl et al. (2007).

The explanation of reasons for application of LEFM (LEFM of general singular stress concentrators) within these analyses consists in the fact, that the techniques of determination of fracture-mechanical properties of quasi-brittle materials (based on classical non-linear fracture models mentioned above) employ the approach of equivalent elastic crack, which essentially is the concept of LEFM supplemented with additional assumptions. The computational framework of LEFM is used both within the determination of parameters of effective crack models (effective crack length or its extension, effective fracture toughness or effective toughness, i.e. fracture energy) and cohesive crack models (specific fracture energy, current – local – specific fracture energy). As these techniques work with a presumption that a crack (equivalent elastic, i.e. effective, but definitely no notch) is propagating in the loaded body, it is important to know how much the conditions (stresses, displacements) in the body differ in the cases when the initial stress concentrator was the crack or the notch. Since the length of the imaginary effective crack (or the crack extension) propagating from the concentrator tip is than calculated with no regard to its shape (possibly together with the other fracture parameters appropriate to the model used models, for which the effective crack length serves as an input) the values of such parameters can be substantially affected by this simplification.

### Numerical study

The approach can be illustrated in the following example. The configuration of the three point bending test of a beam with a central stress concentrator is considered for the fracture test. Nominal dimensions of the beam $W \times B \times L$ are equal to $100 \times 100 \times 400$ mm with span $S = 300$ mm, initial length of the stress concentrator is 33 mm, i.e. approx. 1/3 of the depth $W$. As the stress concentrator both the crack and the double V-notch of widths $b_n = 0.6$, 1, 2, 3, 4, 5, 6, and 8 mm is assumed. Numerical analyses were performed in computational programs employing tools of LEFM (see previous section). Material of the beam is described by elastic constants: Young's modulus $E = 42$ GPa and Poisson ratio $\nu = 0.3$, and the fracture toughness $K_{Ic}$ whose values are assumed to be equal to 1, 1.5, 2, 3, 4, and 5 MPa·m$^{1/2}$.

During simulated fracture process a dependence of loading force on the mid-span deflection (load–displacement diagram, *l*–*d* diagram) was recorded for each combination of the fracture toughness $K_{Ic}$ and notch widths $b_n$. The typical numerical result of the *l*–*d* diagrams is plotted in Fig. 1 and Fig. 2.

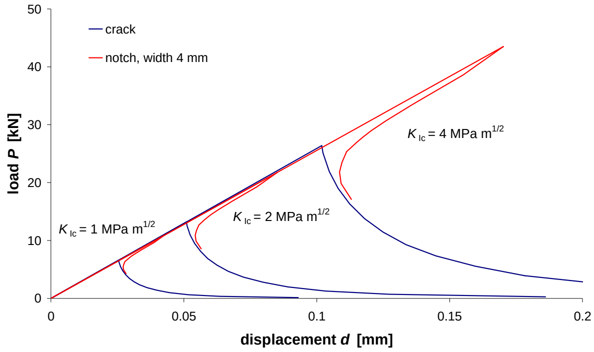


Fig. 1 *l*–*d* diagrams for a cracked beam and a beam with the notch of the width $b_n = 4$ mm for $K_{Ic} = 1$, 2, and 4 MPa·m$^{1/2}$

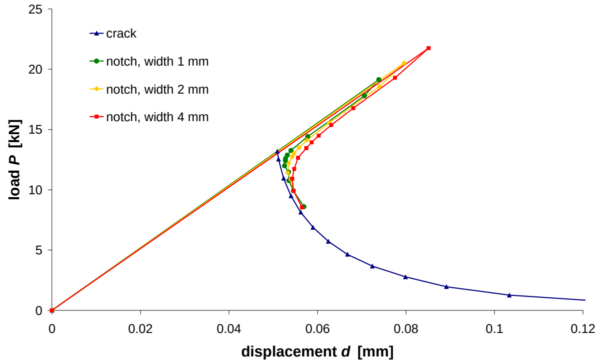


Fig. 2 *l*–*d* diagrams for a cracked beam and a beam with the notch of the width $b_n = 1$, 2, and 4 mm for $K_{Ic} = 2$ MPa·m$^{1/2}$

The analyses regarding the two basic influences mentioned in the section 2 are performed based on these results.

### As to the influence on the load-carrying capacity

As it is evident from the Figs. 1 and 2, the load carried by the notched specimen is considerably higher than that of the cracked one in certain parts of the *l*–*d* diagram; particularly at the load peak and in limited region thereafter. The following stages of the fracture process up to the end of initial ligament remain almost intact. This fact holds true for each input value of fracture toughness $K_{Ic}$ and the intensity of the phenomenon escalates with the increasing notch width $b_n$.

The effect on the fracture parameters can be demonstrated e.g. by means of the resistance curve or the curve of the local (current) fracture energy that are constructed from the simulated *l*–*d* diagrams using the equivalent elastic crack approach (for example using the effective crack model, Nallathambi & Karihaloo, 1986). In Fig. 3 $K_R$–curves (the effective fracture toughness versus the effective crack length) are plotted for selected cases of the stress concentrator shapes/notch widths and for the value of fracture toughness $K_{Ic} = 2$ MPa·m$^{1/2}$. Fig. 4 shows curves of $K_R$–$\Delta a_e$ (the dependence of the effective fracture toughness on the effective crack extension), that correspond to Fig. 3, in emphasizing scale.

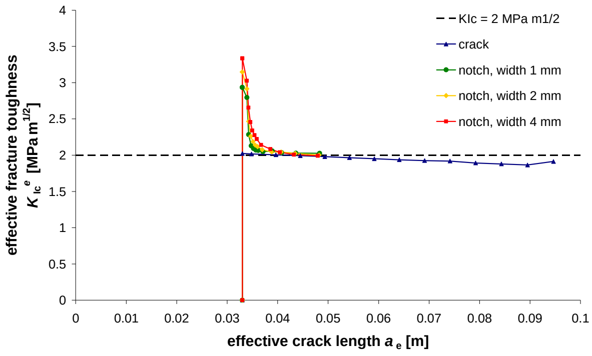


Fig. 3 $K_R$–$a_e$ curve for a cracked beam and a beam with the notch of the width $b_n$ = 1, 2 and 4 mm for $K_{Ic} = 2$ MPa·m$^{1/2}$

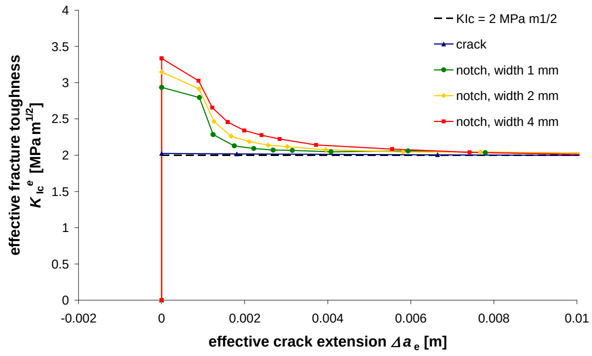


Fig. 4 $K_R$–$\Delta a_e$ curve for a cracked beam and a beam with the notch of the width $b_n$ = 1, 2 and 4 mm for $K_{Ic} = 2$ MPa·m$^{1/2}$

## As to the influence on the initial compliance of the specimen

The initial compliance of the modelled specimens is higher for the notched ones than for the cracked specimen. The effect, similarly to the previously described influence, becomes more pronounced with the increase of the notch width. Despite the difference between the initial compliance of the cracked and notched specimen is hardly visible in the *l*–*d* diagram, the consequences which follow from this difference for the fracture parameters determination are not so negligible.

An example is illustrated in Figs. 5 and 6. If the difference between the initial compliance of the cracked and notched specimen (see Fig. 5) is neglected, i.e. if the notched specimen is regarded to be a cracked one in the calculation of effective crack extension, the influence of increased load only affects the $K_R$–$\Delta a_e$ curve, as it was shown Fig. 4. However, if the compliance increase is taken into account, then following phenomenon arises: the extension of the effective crack length $\Delta a_e$ for the initial notch length is not equal zero and emerges from the difference between the cracked and notched body compliance. Consequently also current effective fracture toughness $K_{Ic}^e$ at the tip of the propagating effective crack $a_e$ is increased, which results from higher value of $a_e$ in comparison to the same quantity in the case of assuming the cracked body compliance for this specimen (see Fig. 6).

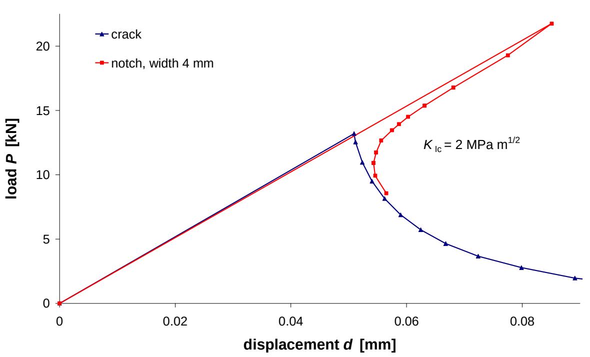


Fig. 5 $K_R$–$a$ curve for a cracked beam and a beam with the notch of the width $b_n$ = 4 mm for $K_{Ic} = 2$ MPa·m$^{1/2}$

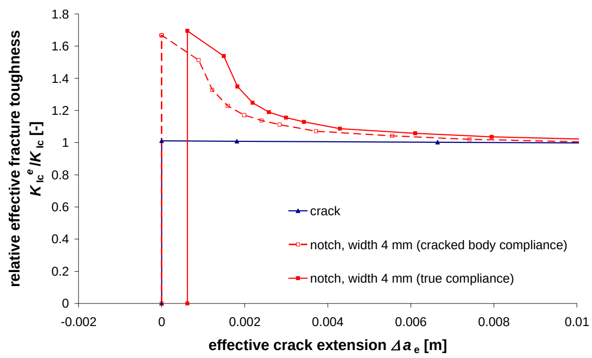


Fig. 6 The relative effective fracture toughness $K_{Ic}^e$ as a function of effective crack extension $\Delta a_e$ for a cracked beam and a beam with the notch of the width $b_n$ = 4 mm ($K_{Ic} = 2$ MPa·m$^{1/2}$)

The significance of the consideration of the real notched-body compliance in the procedure of calculating of the current length of the effective crack is also shown in Fig. 7. It is evident, that the effect decreases when the notch approaches to the crack.

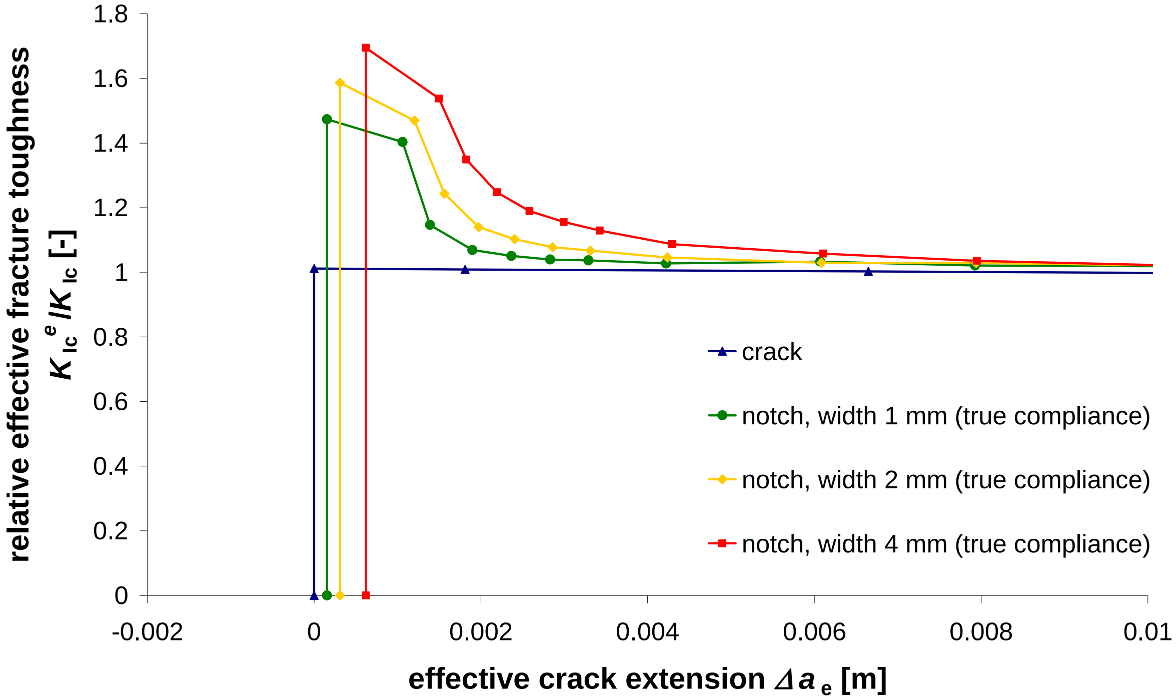


Fig. 7 The relative effective fracture toughness $K_{Ic}^e$ as a function of effective crack extension $\Delta a_e$ for a cracked beam and a beam with the notch of the width $b_n$ = 1, 2, and 4 mm calculated considering the true initial compliance of the cracked/notched specimen ($K_{Ic}$ = 2 MPa m$^{1/2}$)

## 4. Conclusions

Main conclusions following from the performed numerical study can be explained with the help of Fig. 8. Font and back boundaries of the specimen, together with the indication of the stress concentrator, are schematically drawn to the $K_R$–$\Delta a_e$ graph ($K_R$ in relative coordinates with regard to $K_{Ic}$ used as the input in the simulations). From this picture it is evident, that the stress concentrator in the form of double V-notch with relative length 0.33 and width 4 mm present in three point bending beam of dimensions 100 × 100 × 400 mm with span $S$ = 300 mm considerably influences the fracture process through the specimen ligament only at its initial stages, approximately up to 15% of the remaining ligament (i.e. up to effective crack extension 10 mm, effective crack length 43 mm). At the subsequent stages of the fracture its influence is negligible.

This result may have substantial consequences on the procedures of determination of fracture parameters of nonlinear fracture models. In cases, where the internal (characteristic) length of the material does not exceed significantly the notch width, the fracture behaviour of such notched body can be influenced by the notch width and/or notch tip shape, and therefore also fracture parameters determined from the appropriate test record can be affected. An example concerning e.g. effective crack model parameters can be given in Fig. 8. If a *l*–*d* diagram for specimen of mentioned shape and dimensions was recorded during the test, so that the calculated critical effective crack extension was about 3 mm, the effective fracture toughness would be overestimated at about 20%.

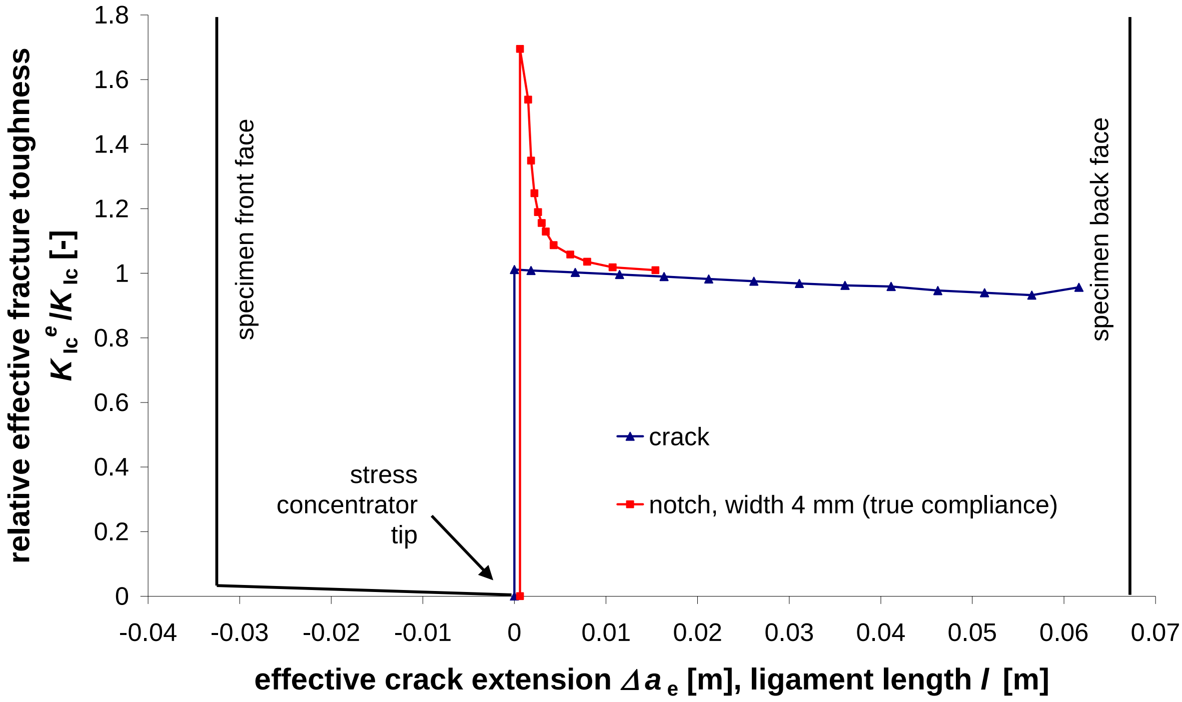


Fig. 8 Implication of the studied phenomenon on the procedure of determination of effective fracture parameters depicted in the $K_{Ic}^{e}$ vs. $\Delta a_e$ graph. Curves for a cracked beam and a beam with the notch of the width $b_n$ = 4 mm ($K_{Ic}$ = 2 MPa·m$^{1/2}$) are displayed

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