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ABOUT A PRECEDENCE OF THE TANDEM OF OPPOSITE RUNNING GYROSCOPES

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Summary: *It was demonstrate in the previous author's paper, that the gyroscopic stabilization on vibroisolation system of ambulance couch is possible and useful. But this conclusion was made by assumption, that the kinematic excitation does not include any vertical rotation. In the case, when this assumption is not satisfied, it is possible substitute every gyroscopes which are connected by antiparallelogram and the vertical rotation will be practically compensated.*

1. Introduction

The vibroisolation system of the ambulance couch has been analyzed in the paper (Šklíba et al. 2006) this system is determined for a terrain ambulance car. Besides we suppose that the loading area of ambulance car has three degrees of freedom: We have limited on the vertical translation and two rotations about horizontal axes of carriage. Later this system was completed by the gyroscopic stabilizer of the Cardan suspension (see fig.1a). The preliminary analysis was provided in (Šklíba 2007).

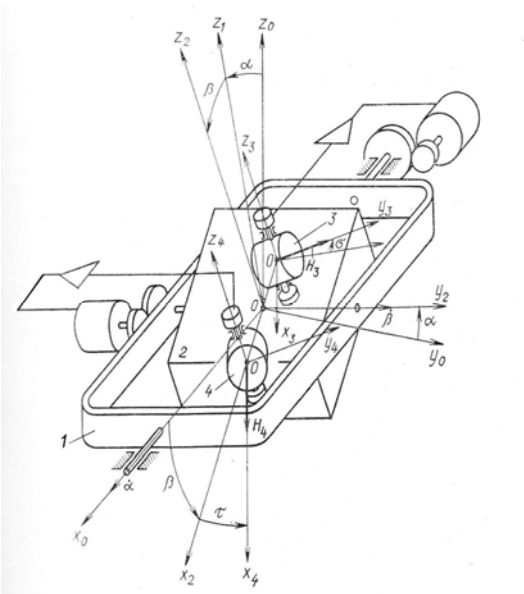

Fig. 1a

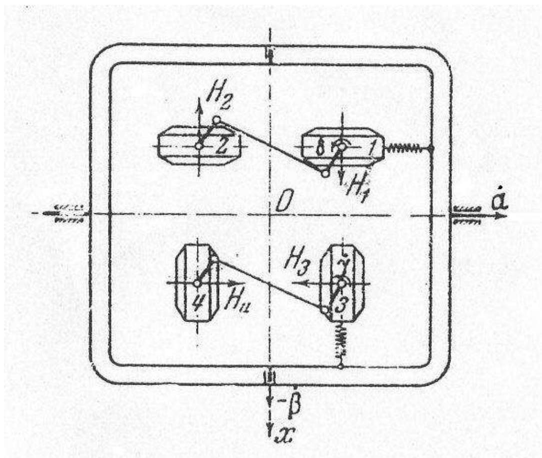

Fig. 1b

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There is a question: If the two gyroscopic platform will be in correct functionality in the case, when the above mentioned assumption will not be satisfied – concretely by rotation of the carriage about the vertical axis? The first solution, (that is very difficult) is a three gyroscopic platform, which is stabilized in the all three axes. The second solution lays in possibility of the application of a tandem of the opposite rotating gyroscopes, in which the precession frames are connected by an antiparallelogram (see fig.1b).

2. Application of tandems with horizontal axes of gyroscopes

The principle of the tandem has been patented in 1923 from M. Schuler, almost in the same time, as a principle of the spherical gyroscopic pendulum with the period 84.4min. J. N. Rojtenberg gave an application for naval stabilized platform in 1942 (Rojtenberg 1945). A four gyroscope platform with the vertical axes of precession tandems was analyzed by the general motion of the ship – including its vertical rotation. It was proved big decreasing of the velocity and ballistic deviations.

3. Application of tandems with vertical axes of gyroscopes

I. We introduce coordinate systems *Oxyz* (connected with carriage) and $Ox_iy_iz_i$, connected with Cardan frame (*i=0*), with first (*i=1*) and with second (*i=2*) precession frames (see fig.2). We denote J_{xi}, H_i, Ω_{xi} inertia moments, Ω_{xi}, Ω_{yi}, Ω_{zi} components of the vector of absolute angle velocity, *Hi* impulsmoment of gyroscope. After the substitution into the Euler dynamic equations we obtain

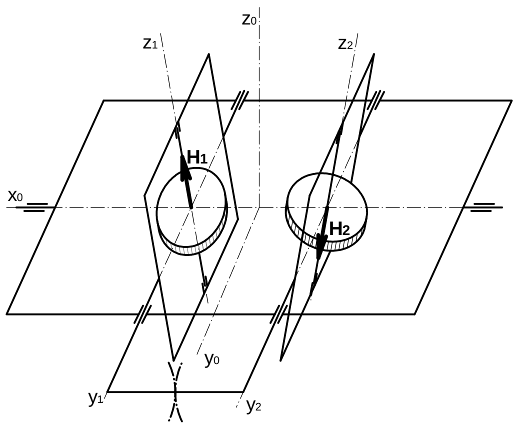

Fig. 2

$$J_{x0}\dot{\Omega}_x + \left(J_{z0} - J_{y0}\right)\Omega_y\Omega_z - J_{y1}\dot{\varepsilon}\Omega_z + J_{y2}\dot{\varepsilon}\Omega_z + H_1(\Omega_y + \dot{\varepsilon})\cos(\varepsilon) - H_2(\Omega_y - \dot{\varepsilon})\cos(-\varepsilon) = M_{x0} \tag{1}$$

$$J_{y1}\left(\dot{\Omega}_y + \dot{\varepsilon}\right) + \left(J_{x1} - J_{z1}\right)\left(\frac{\Omega_x^2 - \Omega_z^2}{2}\sin(2\varepsilon) + \Omega_x\Omega_z\cos(2\varepsilon))\right) - H_1\Omega_x\cos(\varepsilon) + H_1\Omega_z\sin(\varepsilon) = M_{y1} \tag{2}$$

$$J_{y2}\left(\dot{\Omega}_y - \dot{\varepsilon}\right) + \left(J_{x2} - J_{z2}\right)\left(\frac{\Omega_x^2 - \Omega_z^2}{2}\sin(-2\varepsilon) + \Omega_x\Omega_z\cos(-2\varepsilon))\right) + H_2\Omega_x\cos(-\varepsilon) - H_2\Omega_z\sin(-\varepsilon) = M_{y2} \tag{3}$$

According to the assumption H_1=H_2=H and after the subtraction equation (2) and (3) we receive

$$2J_y\ddot{\varepsilon}+(J_{x1}-J_{z1})(\Omega_x^2-\Omega_z^2)\sin(2\varepsilon)-2H\Omega_x\cos(\varepsilon)=M_{y1}-M_{y2} \tag{4}$$

$$Jx\dot{\Omega}_x+(J_z-J_y)\Omega_y\Omega_z+2H\dot{\varepsilon}\cos(\varepsilon)=M_{x0} \tag{5}$$

The compensation of vertical component of angle velocity ω_{z0} by an application of the tandem of opposite gyroscopes was proved by one axis gyroscopic platform.

II. It is possible such compensation to make also by two axis platform:

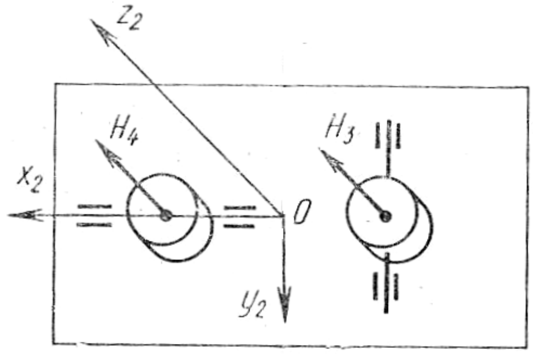

Fig. 3

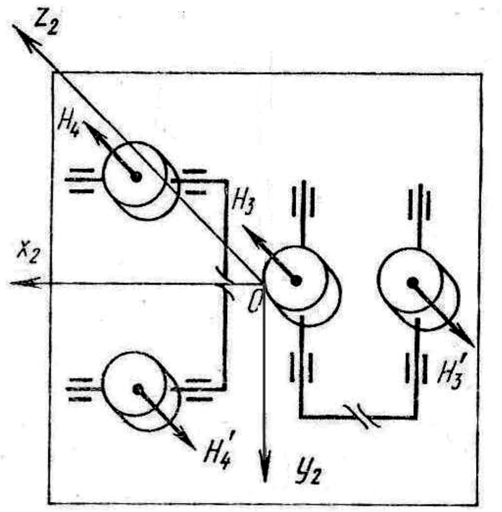

Fig. 4

If we use the theorem on moment of movement step by step for first, second Cardan frame on for two precession frames, it will be:

$$J_{x1}\dot{\omega}_{x1}+\left(J_{z1}-J_{y1}\right)\cdot\omega_{z1}\omega_{y1}+\left[J_{x2}\dot{\omega}_{x2}+\left(J_{z2}-J_{y2}\right)\omega_{y2}\omega_{z2}\right]\cdot\cos(\beta)+$$
$$+\left[J_{z2}\dot{\omega}_{z2}+\left(J_{y2}-J_{x2}\right)\omega_{x2}\omega_{y2}\right]\cdot\sin(\beta)=M_{x1}+R_{x1} \tag{6}$$

$$J_{y2}\dot{\omega}_{y2}+\left(J_{x2}-J_{z2}\right)\omega_{x2}\omega_{z2}=M_{y2}+R_{y2} \tag{7}$$

$$J_{y3}\left(\ddot{\sigma}+\dot{\omega}_{y2}\right)+H_3\left(\omega_{x2}\cos(\sigma)-\omega_{z2}\sin(\sigma)\right)+\left(J_{x3}-J_{z3}\right)\cdot\left[\frac{\varpi_{x2}^2-\omega_{z2}^2}{2}\sin(2\sigma)+\omega_{z2}\omega_{x2}\cos(2\sigma)\right]=M_{y3} \tag{8}$$

$$J_{x4}\left(\bar{\tau}+\dot{\omega}_{x2}\right)-H_4\left(\omega_{z2}\cos(\tau)+\omega_{z2}\sin(\tau)\right)-\left(J_{y4}-J_{z4}\right)\cdot\left[\frac{\omega_{y2}^2-\omega_{z2}^2}{2}\sin(2\tau)+\omega_{z2}\omega_{y2}\cos(2\tau)\right]=M_{x4} \tag{9}$$

where for absolute angle velocity components we have

$$\omega_{z1}=\omega_{z0}\cos(\alpha)-\omega_{y0}\sin(\alpha)\qquad\omega_{x1}=\omega_{x0}+\dot{\alpha}\qquad\omega_{y1}=\omega_{y0}\cos(\alpha)-\omega_{z0}\sin(\alpha) \tag{10}$$

$$\omega_{z2} = \omega_{x1}\sin(\beta) + \omega_{z1}\cos(\beta) \qquad \omega_{x2} = \omega_{x1}\cos(\beta) - \omega_{z1}\sin(\beta) \qquad \omega_{y2} = \omega_{y1} + \dot{\beta} \tag{11}$$

For gyroscopic reaction of the first and second precession frame we obtain

$$\begin{aligned} R_{3x2} &= \left[\left(J_{z3}\cos^2(\sigma) + J_{x3}\sin^2(\sigma) - J_{y3}\right)\cdot \omega_{z2} + H_3\cos(\sigma)\right]\cdot\left(-\omega_{y2} - \dot{\sigma}\right) - \\ &- \left(J_{x3}\cos^2(\sigma) + J_{z3}\sin^2(\sigma)\right)\cdot \omega_{z2}\dot{\sigma} + \left(J_{z3} - J_{x3}\right)\frac{\dot{\omega}_{z2}}{2}\sin(2\sigma) \end{aligned} \tag{12}$$

$$\begin{aligned} R_{3z2} &= \left[\left(J_{z3} - J_{x3}\right)\frac{\omega_{z2}}{2}\sin(2\sigma) + H_3\sin(\sigma)\right]\cdot\left(-\omega_{y2} + \dot{\sigma}\right) + \left(J_{z3} - J_{x3}\right)\dot{\sigma}\sin(2\sigma) - \\ &- \left(J_{x3}\cos^2(\sigma) + J_{x3}\sin^2(\sigma)\right)\dot{\omega}_{z2} \end{aligned} \tag{13}$$

$$\begin{aligned} R_{4yz} &= \left[\left(J_{z4}\cos^2(\tau) + J_{y4}\sin^2(\tau) - J_{x4}\right)\cdot \omega_{z2} + H_4\cos(\tau)\right]\cdot\left(\omega_{x2} + \tau\right) - \\ &\left(J_{y4}\cos^2(\tau) + J_{z4}\sin^2(\tau)\right)\cdot \omega_{z2}\dot{\tau} - \left(J_{z4} - J_{y4}\right)\frac{\dot{\omega}_{z2}}{2}\sin(2\tau) \end{aligned} \tag{14}$$

$$\begin{aligned} R_{4z2} &= \left[\left(J_{z4} - J_{y4}\right)\frac{\omega_{z2}}{2}\sin(2\tau) + H_4\sin(\tau)\right]\cdot\left(\omega_{x2} + \dot{\tau}\right) - \left(J_{z4}\cos^2(\tau) + J_{y4}\sin^2(\tau)\right)\cdot \dot{\omega}_{z2} + \\ &+ \left(J_{z4} - J_{y4}\right)\frac{\omega_{z2}}{2}\dot{\tau}\sin(2\tau) \end{aligned} \tag{15}$$

$$R_{x1} = R_{3x2} + (R_{3z2} + R_{4z2})\sin(\beta) \tag{16}$$

$$R_{y2} = R_{4y2}$$

Now we substitute the schema of fig (3) by schema of fig (4) – the precession frames by tandems.

The second gyroscope in every tandem has impulsmoment $-H_3$, resp. $-H_4$ and precession angle deflection $-\sigma$ resp. $-\tau$.

We attach the equation for second gyroscope to equation (8) *($H_3=H_4=H$)*

$$\begin{aligned} &J_{y3}\left(-\ddot{\sigma} + \dot{\omega}_{y2}\right) - H\left(\omega_{x2}\cos(\sigma) + \omega_{z2}\sin(\sigma)\right) + \left(J_{x3} - J_{z3}\right)\cdot \\ &\cdot\left[-\frac{\omega_{x2}^2 - \omega_{z2}^2}{2}\sin(2\sigma) + \omega_{z2}\omega_{x2}\cos(2\sigma)\right] = M'_{y3} \end{aligned} \tag{17}$$

and both two equation we subtract

$$\begin{aligned} &2J_{y3}\ddot{\sigma} + 2H\left[\left(\omega_{x0} + \dot{\alpha}\right)\cos(\beta) - \left(\varpi_{z0}\cos(\alpha) - \omega_{y0}\sin(\alpha)\right)\sin(\beta)\right] + \left(J_{x3} - J_{z3}\right)\cdot \\ &\cdot\left(\omega_{z2}\omega_{x2}\cos(2\sigma)\right) = M_{y3} - M'_{y3} \end{aligned} \tag{18}$$

In the equations (6-9) of gyroscopic systems there are called gyroscopic members (proportional H) the dominant influence. The same consideration we can make for the second frame and consequent tandem. Also in the equations (12)-(15) we can substitute the precession frames by tandems. If we consider only gyroscopic members in these equations, we get

$$
\begin{aligned}
&H\cos(\sigma)(-\omega_{y2}+\dot{\sigma})+(-H\cos(\sigma))\cdot(-\omega_{y2}-\dot{\sigma})=2H\dot{\sigma}\cos(\sigma)\\
&H\sin(\sigma)(-\omega_{y2}+\dot{\sigma})+H\sin(\sigma)(-\omega_{y2}-\dot{\sigma})=-2H\omega_{y2}\sin(\sigma)\\
&H\cos(\tau)(\omega_{x2}+\dot{\tau})+(-H\cos(\tau))\cdot(\omega_{z2}-\dot{\tau})=2H\dot{\tau}\cos(\tau)\\
&H\sin(\tau)(\omega_{x2}+\dot{\tau})+H\sin(\tau)(\omega_{x2}-\dot{\tau})=2H\omega_{x2}\sin(\tau)
\end{aligned}
\tag{19}
$$

And according to (10), (11) and (16), the deflections produced by drag angular velocity ω_{z0} are quantities of second orders (proportional $\sin(\alpha), \sin(\sigma), \sin(\tau)$).

4. Conclusion

The application of the tandems of an opposite turning gyroscopes with the vertical axes in the one axis and two axes stabilized platform contributes the important resultant: a practical compensation of the vertical rotation of the platform base.

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