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MODELLING OF ACOUSTIC TRANSMISSION THROUGH PERFORATED LAYER

V. Lukeš*, E. Rohan**

Summary: *The paper deals with transmission conditions imposed on the interface plane separating two halfspaces occupied by the acoustic medium. The conditions are obtained as the two-scale homogenization limit of the standard acoustic problem imposed in the layer with the perforated periodic structure embedded inside. The limit model involving some homogenized coefficients governs the interface discontinuity of the acoustic pressure associated with the two halfspaces and the magnitude of the fictitious transversal acoustic velocity. By numerical examples we illustrate this novel approach of modeling the acoustic impedance of perforated interfaces.*

1. Introduction

The purpose of the paper is to demonstrate the homogenization approach applied to computational modelling of the acoustic transmission through perforated planar structure. We consider the acoustic medium occupying domain Ω which is subdivided by perforated plane Γ_0 in two disjoint subdomains Ω^+ and Ω^-, so that $\Omega = \Omega^+ \cup \Omega^- \cup \Gamma_0$, see Fig. 4. In the differential form the problem for unknown acoustic pressures p^+, p^- reads as follows:

$$\begin{aligned} c^2\nabla^2 p^+ + \omega^2 p^+ = 0 \quad & \text{in } \Omega^+, \\ c^2\nabla^2 p^- + \omega^2 p^- = 0 \quad & \text{in } \Omega^-, \\ + \text{ boundary conditions} \quad & \text{on } \partial\Omega. \end{aligned} \tag{1}$$

In a case of no convection flow the usual transmission conditions are given by

$$\frac{\partial p^+}{\partial n^+} = -\mathrm{i}\frac{\omega\rho}{Z}(p^+ - p^-), \quad \frac{\partial p^-}{\partial n^-} = -\mathrm{i}\frac{\omega\rho}{Z}(p^- - p^+), \tag{2}$$

where n^+ and n^- are the outward unit normals to Ω^+ and Ω^-, respectively, ω is the frequency, ρ is the density and Z is the *transmission impedance*; this complex number is characterized by

* Ing. Vladimr Lukeš, Ph.D., doc. Dr. Ing. Eduard Rohan, New Technologies – Research Centre and Department of Mechanics, Faculty of Applied Sciences, University of West Bohemia in Plzeň, Univerzitní 22, 306 14 Plzeň; tel.: +420 377 632 365; e-mail: lukes@kme.zcu.cz

**doc. Dr. Ing. Eduard Rohan, New Technologies – Research Centre and Department of Mechanics, Faculty of Applied Sciences, University of West Bohemia in Plzeň, Univerzitní 22, 306 14 Plzeň; tel.: +420 377 632 320; e-mail: rohan@kme.zcu.cz

features of the actual perforation considered and is determined using experiments in the acoustic laboratories, see e.g. Kirby & Cummingset (1998).

The aim of our approach is to replace the transmission condition (2) by the two-scale homogenization limit of the standard acoustic problem and obtain some *homogenized coefficients* characterizing the perforated structure.

2. Problem formulation

By indices $^\varepsilon$ we denote dependence of variables on scale parameter $\varepsilon > 0$; similar convention is adhered in the explicit reference to the layer thickness $\delta > 0$. By the Greek indices we refer to the coordinate index 1 or 2, so that $(x_\alpha, x_3) \in I\!R^3$.

2.1. Geometry

Let $\Omega_\delta \subset I\!R^3$ be an open domain shaped as a layer bounded by $\partial\Omega_\delta$ which is split as follows

$$\partial\Omega_\delta = \Gamma_\delta^+ \cup \Gamma_\delta^- \cup \partial\Omega_\delta^\infty, \tag{3}$$

where $\delta > 0$ is the layer thickness, see Fig. 1. The acoustic medium occupies domain $\Omega_\delta \setminus S_\delta^\varepsilon$, where S_δ^ε is the solid obstacle which in a simple layout has a form of the periodically perforated sheet.

For homogenization technique, it is important to have a fixed domain, therefore the *dilatation* is considered, cf. Cioranescu & Saint Jean Paulin (1999); let Γ_0 be the plane spanned by coordinates $1, 2$ and containing the origin. Further let Γ_δ^+ and Γ_δ^- be equidistant to Γ_0 with the distance $\delta/2$ Therefore, $x_3 \in]-\delta/2, \delta/2[$ and we introduce the rescaling $x_3 = z\delta$.

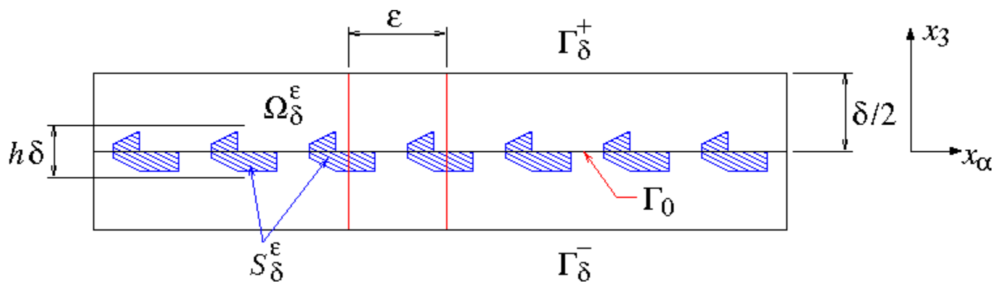

Figure 1: The layer Ω_δ of the acoustic medium with periodic "solid perforations" S_δ^ε.

2.2. Boundary value problem in the transmission layer

The problem of acoustics is defined in $\Omega_\delta^\varepsilon$. We assume a monochrome stationary incident wave with frequency ω and no convection velocity of the medium, so that

$$\begin{aligned} c^2\nabla^2 p^{\varepsilon\,\delta} + \omega^2 p^{\varepsilon\,\delta} = 0 \quad & \text{in } \Omega_\delta^\varepsilon, \\ c^2 \frac{\partial p^{\varepsilon\,\delta}}{\partial n^\delta} = -\mathrm{i}\omega g^{\varepsilon\,\delta\pm} \quad & \text{on } \Gamma_\delta^\pm, \\ \frac{\partial p^{\varepsilon\,\delta}}{\partial n^\delta} = 0 \quad & \text{on } \partial\Omega_\delta^\infty \cup \partial S_\delta^\varepsilon, \end{aligned} \tag{4}$$

where $c = \omega/k$ is the speed of sound propagation, $g^{\varepsilon\,\delta\pm}k^2$ is the interface normal acoustic velocity; by n^δ we denote the normal vector outward to Ω_δ.

3. Homogenization

For passing to the limit $\varepsilon \to 0$ we consider a proportional scaling between the period length and the thickness, so that $\delta = \varkappa\varepsilon$, for a fixed $\varkappa > 0$. Further, we need a convenient prepositions on the problem data involved in (4). Note that $g^{\varepsilon\,\delta\pm}$ is defined on Γ_0, which is equidistant to $\Gamma^\pm$; we assume

$$g^{\varepsilon\,\delta+} \rightharpoonup g^{0+}, \quad g^{\varepsilon\,\delta-} \rightharpoonup g^{0-}, \quad \frac{1}{\delta}\left(g^{\varepsilon\,\delta+} + g^{\varepsilon\,\delta-}\right) \rightharpoonup 0,$$

weakly in $L^2(\Gamma_0)$. It can be shown from the limit equation that

$$g^{0\pm} := g^{0+} = -g^{0-}. \tag{5}$$

3.1. Local microscopic problems

The homogenized coefficients are introduced using so called *corrector functions* $\pi^\beta, \xi^\pm \in H^1_{\#(1,2)}(Y)/I\!R$, $\beta = 1,2$ computed for the reference periodic cell $Y =]0,1[^2 \times]-1/2,+1/2[\subset I\!R^3$ which is perforated by the solid (rigid) obstacle T, so that the acoustic medium occupies domain $Y^* = Y \setminus T$. We refer to the upper and lower boundaries of Y by $I_y^+ = \{y \in \partial Y : z = 1/2\}$ and $I_y^- = \{y \in \partial Y : z = -1/2\}$.

The local microscopic problems can be formulated as: Find π^β and $\xi^\pm$ such that

$$\int_{Y^*}\left[\partial^y_\alpha \xi^\pm\,\partial^y_\alpha q + \frac{1}{\varkappa^2}\partial_z\xi^\pm\partial_z q\right] + \frac{|Y|}{c^2\varkappa}\left(\int_{I_y^+} q - \int_{I_y^-} q\right) = 0, \quad \forall q \in H^1_{\#(1,2)}(Y)/I\!R, \tag{6}$$

$$\int_{Y^*}\partial^y_\alpha(y^\beta + \pi^\beta)\,\partial^y_\alpha q + \frac{1}{\varkappa^2}\int_{Y^*}\partial_z\pi^\beta\partial_z q = 0, \quad \forall q \in H^1_{\#(1,2)}(Y)/I\!R,\ \beta = 1,2. \tag{7}$$

3.2. Macroscopic problem in transmission layer

Homogenized transmission behaviour is expressed in terms of *interface mean acoustic pressure* $p^0 \in H^1(\Gamma_0)$, and *fictitious acoustic velocity* $g^{0\pm}L^2(\Gamma_0)$ which satisfy the interface problem (to hold for all $q \in H^1(\Gamma_0)$ and $\psi \in L^2(\Gamma_0)$)

$$\begin{aligned}
&\int_{\Gamma_0} A_{\alpha\beta}\partial^x_\beta p^0\partial^x_\alpha q - \frac{|Y^*|}{|Y|}\omega^2\int_{\Gamma_0} p^0 q = -\mathrm{i}\omega\int_{\Gamma_0} B_\alpha\partial^x_\alpha q\, g^{0\pm},\\
&\int_{\Gamma_0}(p^+ - p^-)\psi - \int_{\Gamma_0} D_\beta\partial^x_\beta p^0\psi = \mathrm{i}\omega\int_{\Gamma_0} F^\pm g^{0\pm}\psi,
\end{aligned} \tag{8}$$

where the homogenized equations are expressed in terms of the corrector functions π^β and $\xi^\pm$:

$$A_{\alpha\beta} = \frac{c^2}{|Y|}\int_{Y^*} \partial^y_\gamma(y^\beta+\pi^\beta)\,\partial^y_\gamma(y^\alpha+\pi^\alpha) + \frac{c^2}{|Y|\varkappa^2}\int_{Y^*}\partial_z\pi^\beta\partial_z\pi^\alpha, \tag{9}$$

$$B_\alpha = \frac{c^2}{|Y|}\int_{Y^*}\partial^y_\alpha\xi^\pm, \tag{10}$$

$$D_\alpha = \frac{1}{|I_y|}\left(\int_{I_y^+}\pi^\alpha - \int_{I_y^-}\pi^\alpha\right) = \frac{\varkappa}{|I_y|}B_\alpha, \tag{11}$$

$$F^\pm = \frac{1}{|I_y|}\left(\int_{I_y^+}\xi^\pm - \int_{I_y^-}\xi^\pm\right). \tag{12}$$

Macroscopic acoustic behaviour in Ω is described by acoustic pressures p^+, p^- which satisfy equations (1) and by the transmission conditions on interface Γ_0 which are defined in terms of p^0 and $g^{0\pm}$ as

$$\begin{aligned} c^2\frac{\partial p^+}{\partial n^+} &= \mathrm{i}\omega g^{0\pm} \text{ on } \Gamma_0,\\ c^2\frac{\partial p^-}{\partial n^-} &= -\mathrm{i}\omega g^{0\pm} \text{ on } \Gamma_0. \end{aligned} \tag{13}$$

Terms p^0 and $g^{0\pm}$ satisfy interface problem (8). The transmission conditions (13) result from the homogenization limit of (4) and replace conditions (2).

4. Numerical examples

Examples introduced in this section were computed using our code based on Matlab system. We use Q1 finite element approximation for acoustic pressure in Ω and P1 line elements on Γ_0 to approximate p^0 and $g^{0\pm}$.

4.1. Homogenized coefficients for various perforations

For illustration, in Figs. 2 and 3 the local corrector functions $\xi^\pm$ (left) and π (right) are displayed for 2D and 3D examples of different shapes of the perforations.

4.2. Modelling acoustic waveguide – influence of perforation type

The following numerical example shows the global response presented by the acoustic pressure at the macroscopic scale and illustrate how this response is sensitive to the type of perforation. The geometry of the acoustic waveguide is depicted in Fig. 4 and the following boundary conditions are applied:

$$\begin{aligned} \mathrm{i}\omega\rho v + c\frac{\partial p}{\partial n} &= 0 \quad \text{on } \Gamma_{\text{in}},\\ \mathrm{i}\omega p + c\frac{\partial p}{\partial n} &= 0 \quad \text{on } \Gamma_{\text{out}}. \end{aligned} \tag{14}$$

The first condition prescribes the velocity of the incident wave on the input ($v = 1$ m/s) and the second one ensures the anechoic output. The acoustic medium has the density $\rho = 1.55$ kg/m^3 and the speed of sound propagation is $c = 343$ m/s. In Fig. 5 we show the modulus of acoustic pressure in the waveguide for perforation types #1, #2 and #3.

5. Conclusion

We presented the transmission conditions, see Rohan & Lukeš (2007), involving homogenized parameters (9)-(12) which reflect specific geometry of the periodic perforation. In 2D and 3D numerical examples we demonstrated the sensitivity of the acoustic transmission coefficients on the shape of perforation.

The presented model is motivated by simulation of muffler type structures, see e.g. Bonnet-Bendhia et al. (2004, 2005).

Acknowledgments

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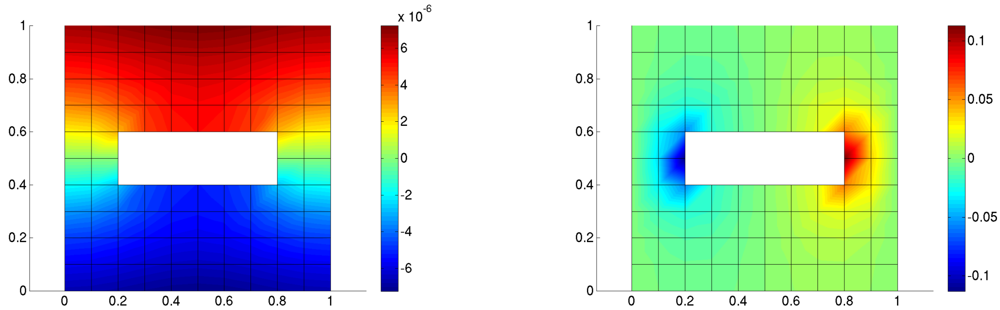
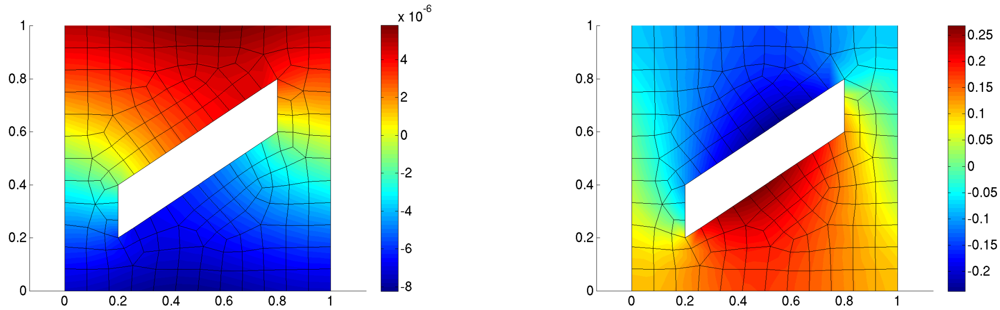


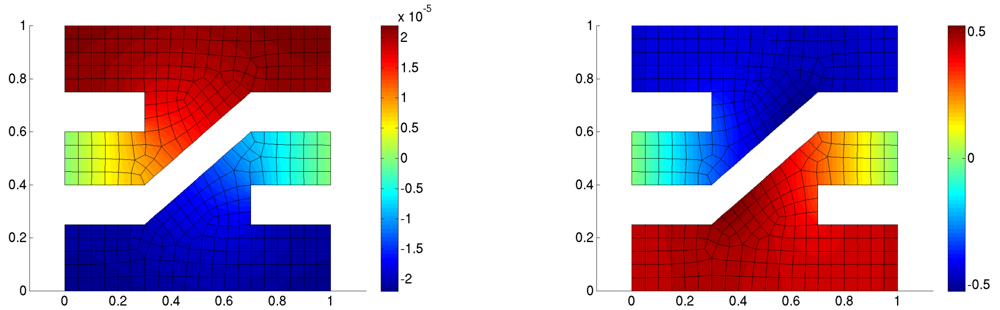
Mic. #1

$A = 1.1546 \cdot 10^5\ (\mathrm{m/s})^2$, $B = 0\ \mathrm{m}$, $F = 1.3913 \cdot 10^{-5}\ \mathrm{s}^2$



Mic. #2

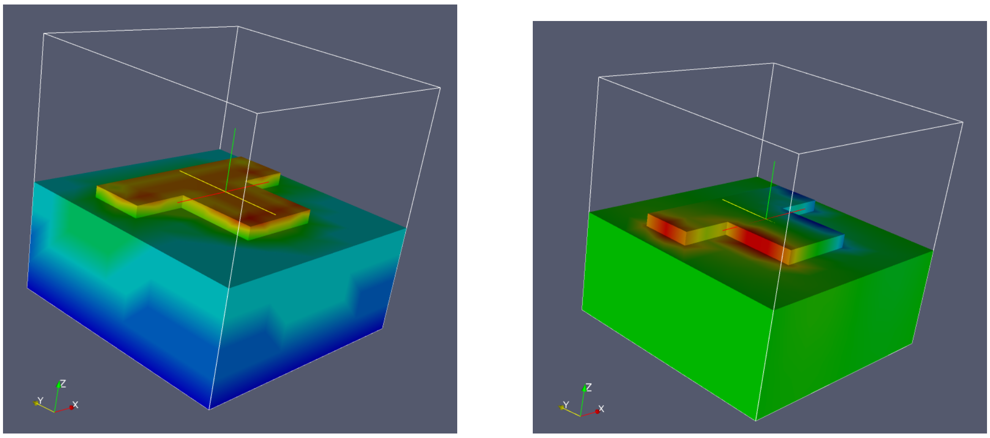
$A = 1.7035 \cdot 10^5\ (\mathrm{m/s})^2$, $B = -0.2509\ \mathrm{m}$, $F = 1.3237 \cdot 10^{-5}\ \mathrm{s}^2$



Mic. #3

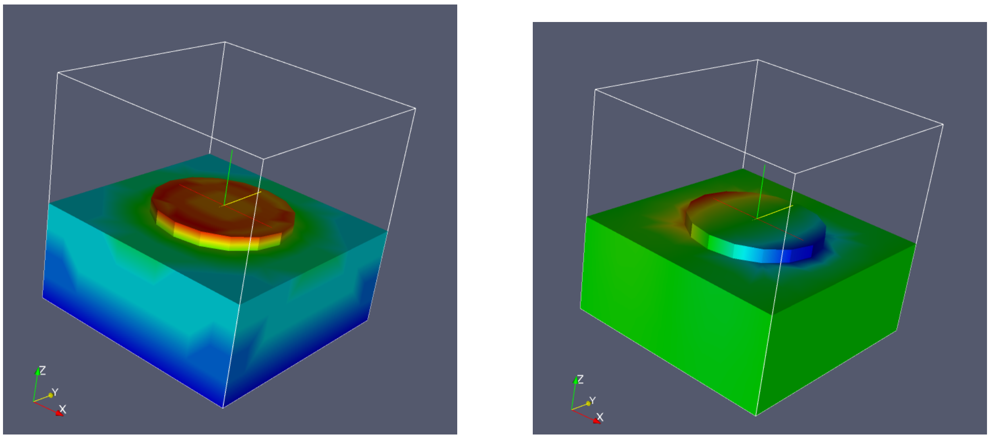
$A = 2.1855 \cdot 10^5\ (\mathrm{m/s})^2$, $B = -0.8974\ \mathrm{m}$, $F = 4.2653 \cdot 10^{-5}\ \mathrm{s}^2$

Figure 2: Distribution of $\xi^{\pm}$ (left), π (right) in Y^* and homogenized coefficients for three shapes of 2D perforations.



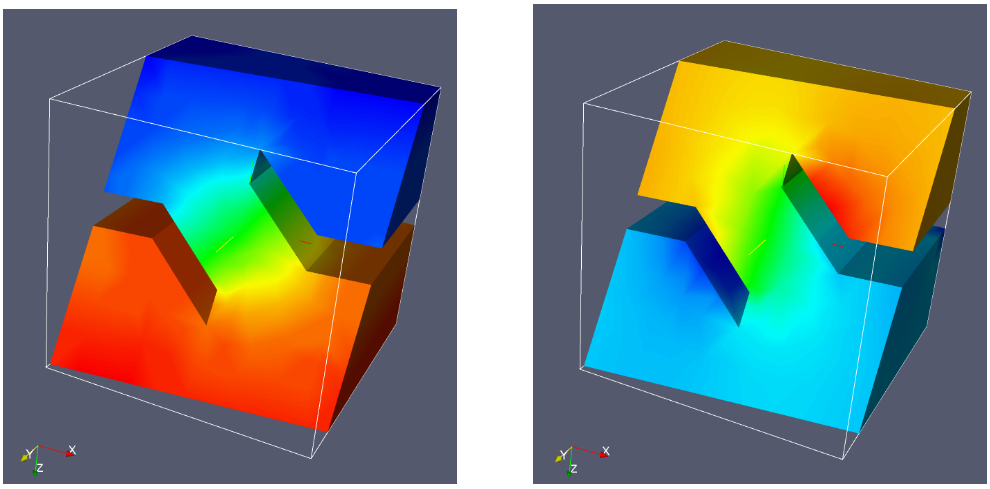
Mic. #4

$$A = \begin{bmatrix} 1.19 & 0.0 \\ 0.0 & 1.2 \end{bmatrix} \cdot 10^5 \ (\mathrm{m/s})^2, B = \begin{bmatrix} 0.0 & 0.0 \end{bmatrix} \ \mathrm{m}, F = 1.32 \cdot 10^{-5} \ \mathrm{s}^2$$



Mic. #5

$$A = \begin{bmatrix} 1.19 & 0.0 \\ 0.0 & 1.19 \end{bmatrix} \cdot 10^5 \ (\mathrm{m/s})^2, B = \begin{bmatrix} 0.0 & 0.0 \end{bmatrix} \ \mathrm{m}, F = 1.42 \cdot 10^{-5} \ \mathrm{s}^2$$



Mic. #6

$$A = \begin{bmatrix} 1.44 & -0.01 \\ -0.01 & 1.80 \end{bmatrix} \cdot 10^5 \ (\mathrm{m/s})^2, B = \begin{bmatrix} 0.303 - 0.011 \end{bmatrix} \ \mathrm{m}, F = 2.56 \cdot 10^{-5} \ \mathrm{s}^2$$

Figure 3: Distribution of $\xi^{\pm}$ (left), π_α (right) in Y^* and homogenized coefficients for three shapes of 3D perforations.

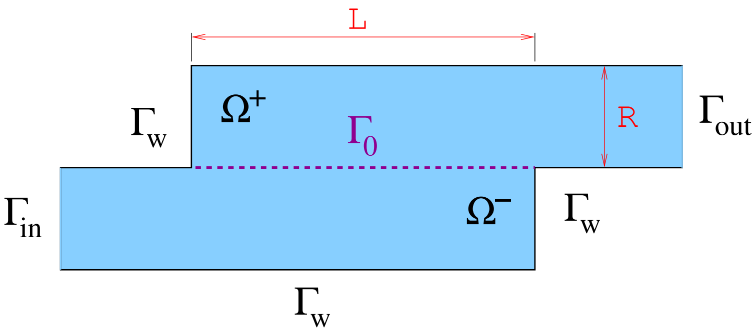
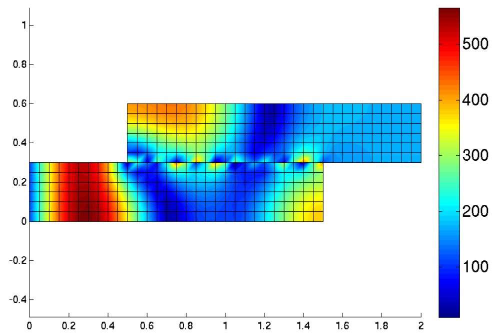
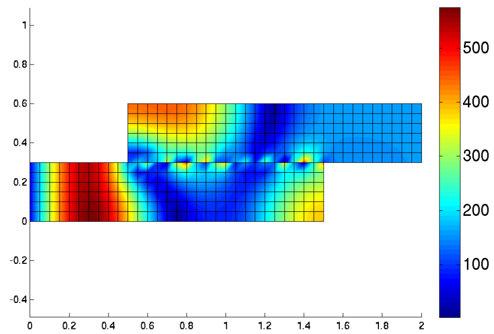


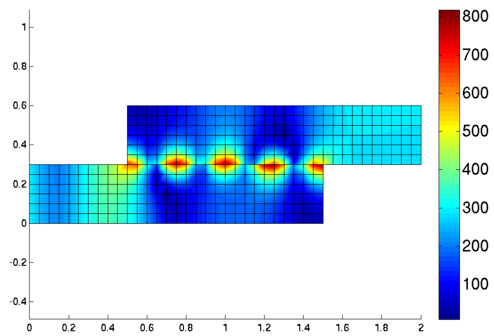
Figure 4: Macroscopic domain Ω; $L = 1\,\mathrm{m}$, $R = 0.3\,\mathrm{m}$.



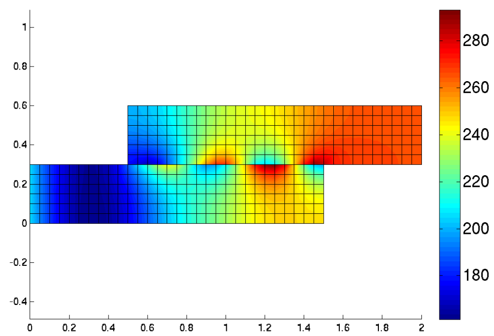
Mic. #1; $\omega = 5 \cdot c$



Mic. #2; $\omega = 5 \cdot c$



Mic. #3; $\omega = 5 \cdot c$



Mic. #3; $\omega = 1 \cdot c$

Figure 5: Modulus of the acoustic pressure in Ω .