

## **MULTIBODY SIMULATIONS OF TROLLEYBUS VERTICAL DYNAMICS AND INFLUENCE OF TIRE INFLATION**

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**Summary:** *Vertical dynamic properties of the ŠKODA 21 Tr low-floor trolleybus were investigated on an artificial test track when driving with a real vehicle and when simulating driving with a multibody model along a virtual test track. Driving along the artificial test track was aimed at determining vertical dynamic properties of the real trolleybus and on the basis of them at verifying computer trolleybus models. Time histories and extreme values of the air springs relative deflections are the monitored quantities. The influences of the determined tire radial characteristics at varied tire inflation are evaluated.*

### **1. Introduction**

Generally, dynamic properties play a decisive role in the overall quality of every road vehicle. However, dynamics of road vehicles represents a wide spectrum of problems and engineering applications (e.g. Genta, 2003).

Optimum dynamic properties of the vehicle intended for the public transport can usually be achieved in dependence on its structural design by the proper choice of axles suspension elements (in some cases in combination with the proper choice of seats suspension elements). It must be the compromise of the requirements for the vehicle behaviour during driving manoeuvres, for the riding comfort and for the body and the chassis parts lifetime when driving along an uneven road surface, and for the passenger safety (e.g. Vlk, 2000).

Driving along the uneven road surface can reveal a lot about the vehicle vertical dynamic properties and about the suitability of the applied axles suspension elements. Especially time histories of relative deflections of springs, relative velocities in the shock absorbers, stress acting in the axles radius rods or radius arms and acceleration in various points in the vehicle interior are the monitored quantities (Gillespie & Karamihas, 2000). On the basis of relative deflections of springs, relative velocities in shock absorbers and stress acting in radius rods or radius arms it is possible to determine the time histories and the extreme values of the forces acting in those suspension elements of axles, which can be utilized in connection with the suitable numerical methods for the stress analysis of structures, for the prediction of the fatigue life of the body and the chassis parts of the tested vehicle. The frequency domain responses of the acceleration in the vehicle interior can be used for the assessment of a riding comfort (e.g. Vlk, 2000).

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Fig. 1: The ŠKODA 21 Tr low-floor trolleybus.

In order to evaluate the vertical dynamic properties of the vehicle when driving along the uneven road surface it is necessary to know the surface characteristics, i.e. statistical properties of unevennesses of the surface or direct its geometry (e.g. Vlček, 2000). The geometry of the uneven surface profile of the run through the section is known in test polygons (the best known polygon focused on testing the public transport vehicles is Altoona in Pennsylvania, USA). Test tracks, which are created by distributing artificial vertical unevennesses (obstacles) on the smooth road surface, also are often used (e.g. Polach et al., 2001).

Vertical dynamic properties of the ŠKODA 21 Tr low-floor trolleybus (see Fig. 1; its design concept is described in Polach & Hajžman, 2005b), which was produced by ŠKODA OSTROV s.r.o. (the Czech company with the tradition of trolleybuses and buses production), were investigated on the artificially created test track when driving with the real vehicle and when simulating driving with the computer models along the virtual test track. Driving along the artificial test track was aimed at determining the vertical dynamic properties of the real trolleybus and on the basis of them at verifying computer models. The verified computer models will be further utilized for the simulations of driving along the virtual uneven road surfaces, which will be generated on the basis of the statistical evaluation of the measured quantities in the course of driving along the real city road with the real trolleybus (Hejman & Lukeš, 2005).

This paper continues the work presented in Polach & Hajžman (2005a), Lukeš et al. (2005), Hajžman et al. (2005), Polach & Hajžman (2005b), Polach & Hajžman (2006) and Polach & Hajžman (2007). Those papers deal with the influences of the shock absorbers characteristics, of the shock absorbers bushings characteristics, of the air spring characteristics, of the tire radial stiffness characteristics and of the multibody models complexity on the computed extreme values of the time histories of the air springs relative deflections determined by the simulations with the selected multibody models of the empty trolleybus.

## 2. Experimental measurements with the real trolleybus

The experimental measurements on the empty ŠKODA 21 Tr low-floor trolleybus were carried out in the depot of Hradec Králové Public City Transit Co. Inc. (Dopravní podnik města Hradce Králové, a.s., Czech Republic) in October 2004.

The test track consisted of three standardized artificial obstacles (in compliance with the Czech Standard ČSN 30 0560 Obstacle II:  $h = 60$  mm,  $R = 551$  mm,  $d = 500$  mm – see Fig. 2) spaced out on the smooth road surface 20 meters one after another. The first obstacle was run over only with right wheels, the second one with both and the third one only with left wheels (see Fig. 3).

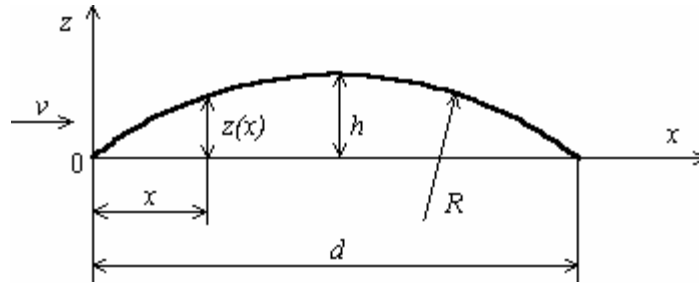


Fig. 2: Standardized artificial obstacle.

Vertical co-ordinates of the standardized artificial obstacle  $z(x)$  are given by the formula

$$z(x) = \sqrt{R^2 - \left(x - \frac{d}{2}\right)^2} - (R - h) \quad , \quad (1)$$

where  $R$  is the obstacle radius,  $h$  is the obstacle height,  $d$  is the obstacle length and  $x$  is the obstacle co-ordinate in the vehicle driving direction.

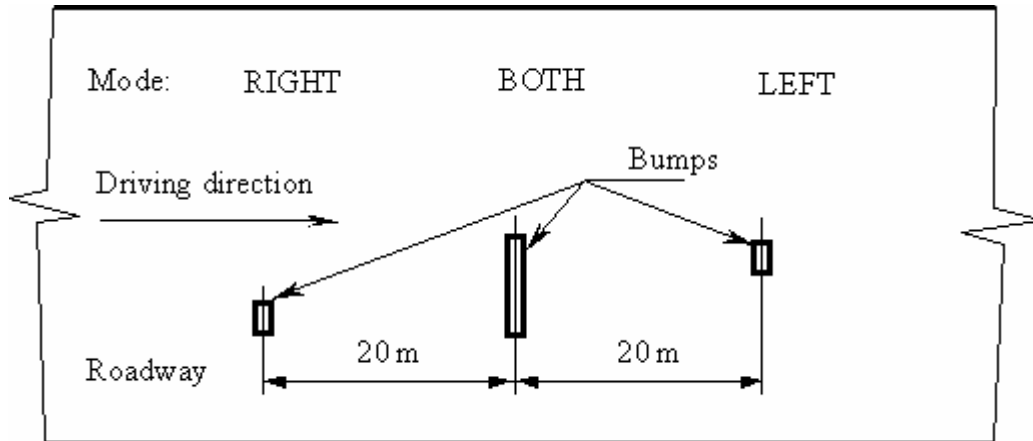


Fig. 3: A track scheme.

In the course of the test driving the already mentioned time histories of the relative displacements between the axles and the chassis frame were recorded (altogether four displacement transducers, which were placed in the lateral direction approximately on the level of the air springs: on the left front half-axle, on the right front half-axle, on the rear axle to the left and on the rear axle to the right were used). Further time histories of stress on twelve places of the trolleybus structure and time histories of the vertical acceleration on

seven places of the trolleybus structure were recorded during the test drives. The records of the time histories of the measured quantities were made during three test drives. Trolleybus speed moved within the range from 43 km/h to 47 km/h at that drives.

### 3. Trolleybus multibody model

In order to simulate drives along the virtual test track, which corresponded with the artificially created test track in the depot of Hradec Králové Public City Transit Co. Inc., the most complex multibody model (Polach, 2003) created in the **alaska 2.3** simulation tool (Maißer et al., 1998) is used to investigate influences of the tire radial characteristics at varied tire inflation.

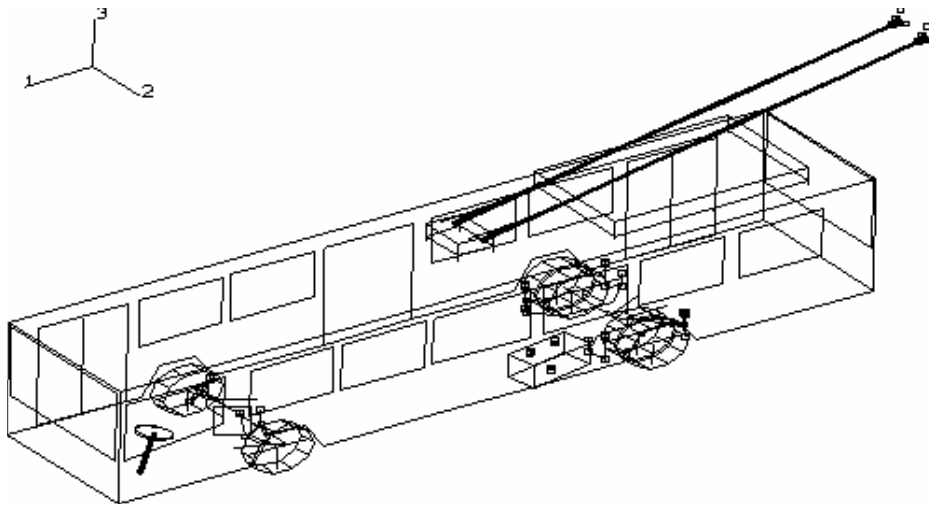


Fig. 4: Visualization of the ŠKODA 21 Tr low-floor trolleybus multibody model in the **alaska 2.3** simulation tool.

#### 3.1. Structure of multibody model

The multibody model of the ŠKODA 21 Tr low-floor trolleybus is formed by 35 rigid bodies and two superelements (2x4 bodies) mutually coupled with kinematic joints. The rigid bodies correspond generally to the vehicles individual structural parts. The superelements correspond to the flexible parts of the chassis frame. The bodies are mutually coupled with 52 kinematic joints. The number of degrees of freedom in kinematic joints is 136. Rigid bodies are defined by inertia properties (mass, centre of mass co-ordinates and moments of inertia). Air springs and hydraulic shock absorbers in axles suspension and bushings in the places of mounting some trolleybus structural parts are modelled by connecting the corresponding bodies by nonlinear spring-damper elements. When simulating driving along the uneven road surface the contact point model of tires is used in the multibody model; radial stiffness and radial damping properties of tires are modelled by nonlinear spring-damper elements considering the possibility of bounce of the tire from the road surface (Kovanda et al., 1997).

#### 3.2. Characteristics of spring-damper structural elements

Dynamic properties of road vehicles are most influenced by the suspension springs, shock absorbers, bushings and tires (e.g. Vlk, 2000). In order that vehicle virtual computer model should reliably approximate kinematic and dynamic properties of the real vehicle, knowledge of the characteristics of those decisive spring-damper structural elements is the important

presumption (besides the proper approach to the model creating and knowledge of all the substantial vehicle parameters).

The characteristics of the air springs (force in dependence on deflection) of the ŠKODA 21 Tr trolleybus were determined on the basis of the test reports of ŠKODA OSTROV s.r.o. and of the Hydrodynamic Laboratory of the Technical University of Liberec (Polach, 2003).

In the multibody model of the ŠKODA 21 Tr trolleybus the damping force dependence on the relative velocity of compression and rebound of the shock absorbers is used as the shock absorbers characteristics. The characteristics were measured by BRANO Co. Inc. (the Czech shock absorbers producer) on the Schenck testing device. On the basis of the results of the simulations in Polach & Hajžman (2005a) it was concluded, that the shock absorbers characteristics measured on the laboratory testing device under specified conditions (i.e. under harmonic exciting and piston stroke 100 mm) do not fully correspond in rebound field with the loading conditions of the shock absorbers in the real vehicle in the course of running over the large road unevenness. That is why virtual “less steep” force-velocity characteristics of the shock absorbers (especially of the shock absorbers of the rear axle) in the rebound field were subsequently determined (Polach & Hajžman, 2005b). Determination of the “less steep” characteristics of the shock absorbers influenced the improvement of accordance of the simulations results with the results of experimental measurements only partially. In the multibody model, by the using of which the results mentioned in this paper were obtained, the characteristics of the shock absorbers measured on the mentioned Schenck testing device were applied again.

In the shock absorbers structure rubber bushings are used in the places of mounting to the chassis frame and to the axles of the trolleybus (Polach & Hajžman, 2005a). In the multibody model the bushings are modelled by means of spring elements, the nonlinear force-deformation characteristics of which were determined under the laboratory conditions (they were taken over from Kopenec, 2002) and which are coupled in series to the damping elements representing the hydraulic shock absorbers themselves.

#### **4. Tire characteristics**

Various tire models need different input data depending on their complexity. The review of the tire models used in the field of vehicle multibody dynamics can be found in the monograph Pacejka (2002). The supposed application of the multibody model is the important factor that determines the type of the necessary characteristics. The most important tire characteristics needed for solving the vehicle vertical dynamics tasks are their radial properties (Vlk, 2000). The used tire model for the vertical dynamics is the already mentioned contact point model based on the tire substitution by a single parallel spring and a damper.

The way to determine the tire radial characteristics was the experimental measurement performed in the Dynamic Testing Laboratory ŠKODA VÝZKUM s.r.o. in December 2005 (Hajžman & Polach, 2006; Polach & Hajžman, 2007). The standard ŠKODA 21 Tr trolleybus tires MICHELIN 275/70 R 22,5 XZU at varied tire inflation were measured.

The scheme of the measurement set is in Fig. 5. The experimental set was composed of the tire (P) mounted to the rigid frame and of the movable weight. The weight was placed over the tire in a given height and then released. The vertical position of the weight over the tire was measured by the displacement transducer on the axis ( $o$ ). The force between the tire and the weight was measured using the force transducer (S).

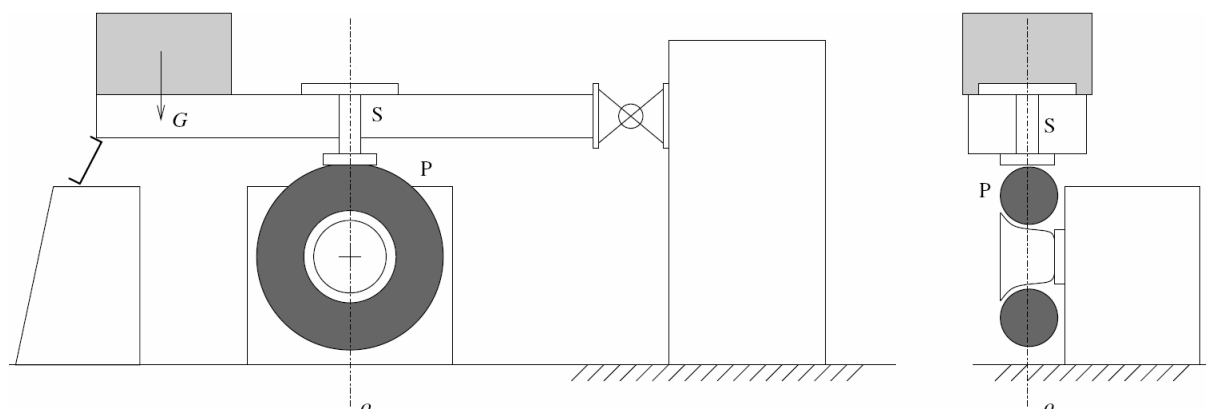


Fig. 5. Scheme of tire radial properties measurement.

The effects of the tire inflation pressure, the mass loading and the shape of uneven surfaces on the tire radial characteristics were investigated at the measurement. Measurements were performed for various parameters combinations. Two values of the tire inflation were considered: specified inflation 900 kPa (100 %) and underinflation (approx. 90 % of the specified one). Underinflation lower than 90 % of the specified inflation cannot occur during the standard vehicle operation (according to the information of the Head of the Maintenance Department of Hradec Králové Public City Transit Co. Inc.). In order to determine the influence of the unevenness shape on the tire behaviour three various contact surfaces falling on the tire – smooth surface, cylindrical section corresponding to the standardized obstacle and a triangular unevenness were used (see Fig. 6). The last variable parameter at measurement was the mass of the weight acting on the tire (2 or 3 tons). Due to the paper extent only the values of the tire radial characteristics determined for the smooth contact surface are used (see Tab. 1).



Fig. 6. Illustrative photograph of the measurement and various shapes of the contact surfaces.

For the purpose of the tire radial damping identification the weight and the tire were simply substituted by the single degree of freedom system

$$m \cdot \ddot{x} + b \cdot \dot{x} + k \cdot x = 0 , \quad (2)$$

rewritten as

$$\ddot{x} + 2 \cdot D \cdot \Omega \cdot \dot{x} + \Omega^2 \cdot x = 0 , \quad (3)$$

with the parallel spring (of the stiffness  $k$ ) and the damper (of the damping coefficient  $b$ ) characterizing the tire radial properties and the vibrating weight (of the mass  $m$ ) over the tire.

The damping coefficient  $b$  was evaluated on the basis of the logarithmic decrement  $\delta$  (e.g. Zeman & Hlaváč, 1999), which can be used for the damping ratio  $D$  calculation

$$D = \frac{\delta}{\sqrt{4 \cdot \pi^2 + \delta^2}} . \quad (4)$$

Each measurement was processed by means of the in-house software programmed in the MATLAB system (MathWorks, 2004), which allows to read the extreme values of a signal and their sampling times and to calculate the damping ratio  $D$  and the corresponding frequency  $f$ . In each case, the measured time history of the force acting between the contact surface and the tire (see Fig. 7 for illustration) was used to determine the moment, from which no bounce of weight occurs, and the time history of the displacement between the contact surface and the tire (see Fig. 7 for illustration) was used to evaluate the frequency  $f$  and the damping ratio  $D$ . Then the radial damping coefficient was

$$b = 2 \cdot D \cdot \Omega \cdot m = 4 \cdot \pi \cdot D \cdot f \cdot m . \quad (5)$$

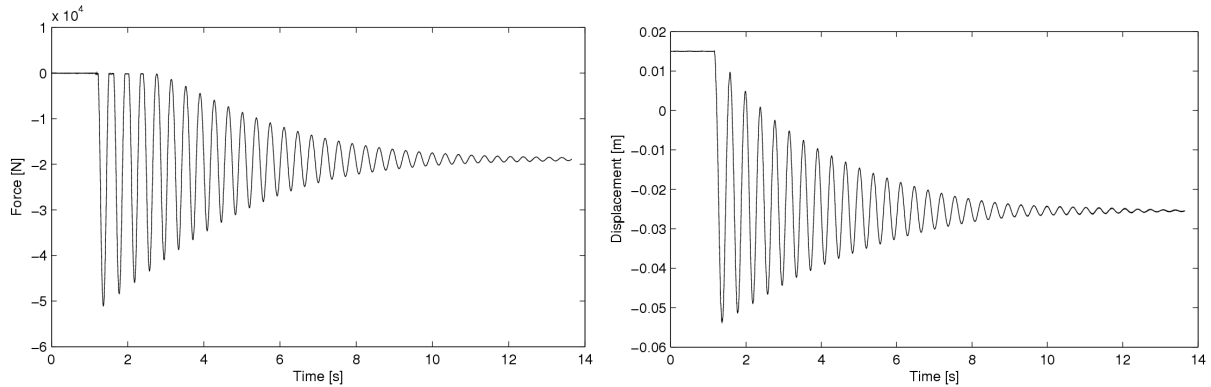


Fig. 7: Illustrative plots of the time histories of the force and the displacement between the contact surface and the tire.

The tire radial stiffness characteristics were supposed to be nonlinear. The measured force  $F(x)$  can be plotted with respect to the measured deformation  $x$  and these points can be fitted by the chosen curve. Approximation of the tire radial stiffness characteristic was achieved by the quadratic fitting function

$$F(x) = \begin{cases} c_1 \cdot x^2 + c_2 \cdot x & \text{for } x < 0 \\ 0 & \text{for } x \geq 0 . \end{cases} \quad (6)$$

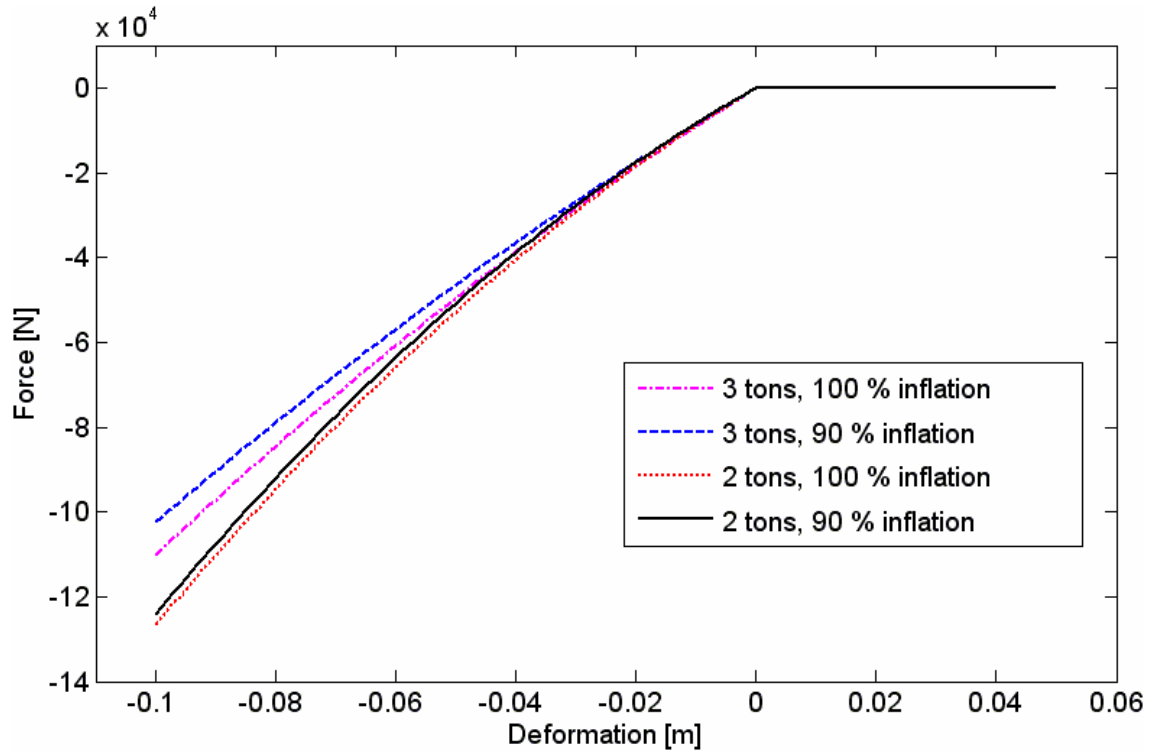


Fig. 8: The tire radial stiffness characteristics.

The identified tire radial stiffness characteristics are shown in Fig. 8 and the coefficients of the quadratic fitting functions are given in Tab. 1.

The tire radial damping characteristics were considered to be linear. The tire radial damping coefficient obtained from the processed signals for the MICHELIN 275/70 R 22,5 XZU tire are given in Tab. 1.

Tab. 1: Coefficients of the tire radial stiffness characteristics and the tire radial damping coefficients.

Mass of the weight	Tire inflation	Coefficient $c_1$ of fitting function (6)	Coefficient $c_2$ of fitting function (6)	Damping coefficient $b$ [N·s/m]
3 tons	100 %	-2 209 108	879 526	1 767.00
	90 %	-1 881 241	835 600	1 636.73
2 tons	100 %	-4 227 027	843 265	1 473.22
	90 %	-4 504 970	789 277	1 407.08

At simulations the characteristics of all tires in the trolleybus multibody model always are identical, i.e. the case with e.g. only one underinflated tire is not considered.

The identified tire radial characteristics have to be used carefully because the presented measurement was proposed for the smooth contact surface interacting with the tire. This surface corresponds rather to the smoother surfaces such as the test track consisted of artificial obstacles.



## 5. Results of the experimental measurement and the simulations

Results of the first documented test drive at trolleybus speed 44.13 km/h are given in this paper.

Time histories of the relative displacements between the axles and the chassis frame (on the right side) measured in the course of driving with the real empty trolleybus are given in Fig. 9. After the run over the last obstacle of the test track step-by-step fading of the recorded relative displacements to zero does not occur. It is caused by the subsequent trolleybus braking, which is necessary due to the deviation of the traction line from a straight direction approx. 50 meters after the end of the last obstacle.

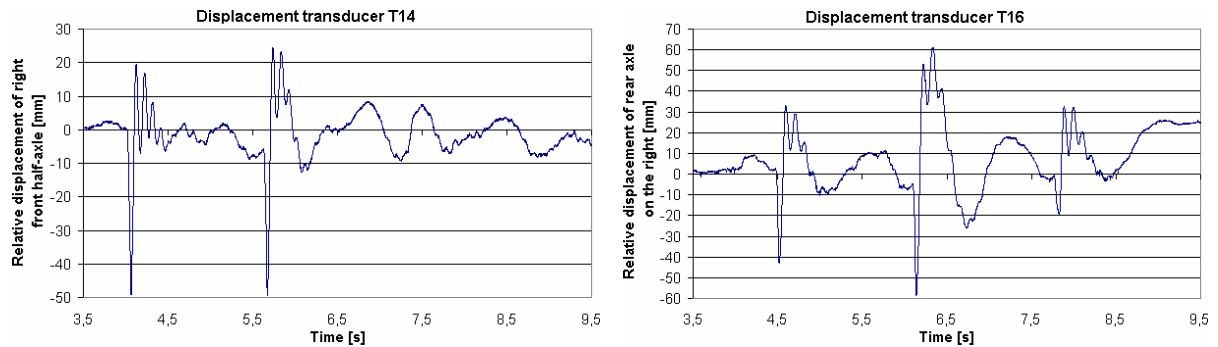


Fig. 9: Experimentally measured time histories of the relative displacement between the trolleybus front right half-axle and the chassis frame and the rear axle and the chassis frame on the right side.

When simulating movement with the multibody models, nonlinear equations of motion, which are solved by means of numerical time integration, are generated. Results of the simulations were obtained using the Shampine-Gordon integration algorithm (Maißer et al., 1998). Figs 10 and 11 show the time histories of the air springs relative deflections when simulating the test drive with the tire radial characteristics identified at 100 % inflation and at 90 % inflation, both with the mass of the weight 3 tons. Character of time histories of the air springs relative deflections when simulating with the tire radial characteristics measured at 2 tons mass of the weight is the same. The extreme values of the air springs relative deflections read from the time histories are in Tab. 2.

Tab. 2: Extreme values of the relative displacements and deflections.

EXPERIMENT	Value	Relative displacement between the axles and the chassis frame [mm]			
		On right front side	On left front side	On right rear side	On left rear side
1st obstacle	min.	-49	-16	-43	-18
	max.	19	-2	32	27
2nd obstacle	min.	-49	-47	-59	-61
	max.	24	18	61	57
3rd obstacle	min.	-9	-48	-19	-46
	max.	8	14	32	29

SIMULA- TIONS	Obstacle	Value	Relative deflection of air springs [mm]			
			Right front	Left front	Right rear	Left rear
Radial tire characteristics at 100 % inflation and the mass of the weight 3 tons	1st	min.	-56	-6	-66	-21
		max.	23	5	20	13
	2nd	min.	-54	-53	-68	-66
		max.	27	27	32	30
	3rd	min.	-6	-53	-21	-63
		max.	5	25	13	20
Radial tire characteristics at 90 % inflation and the mass of the weight 3 tons	1st	min.	-54	-5	-63	-20
		max.	22	5	19	13
	2nd	min.	-51	-51	-65	-62
		max.	26	26	31	29
	3rd	min.	-6	-50	-21	-60
		max.	5	24	13	20
Radial tire characteristics at 100 % inflation and the mass of the weight 2 tons	1st	min.	-61	-6	-72	-22
		max.	24	6	22	15
	2nd	min.	-59	-58	-75	-73
		max.	29	29	36	34
	3rd	min.	-7	-58	-22	-69
		max.	5	26	15	22
Radial tire characteristics at 90 % inflation and the mass of the weight 2 tons	1st	min.	-61	-6	-71	-22
		max.	24	6	21	15
	2nd	min.	-58	-57	-74	-72
		max.	29	29	36	34
	3rd	min.	-7	-58	-22	-69
		max.	5	26	15	22

It is evident from the time histories of the monitored relative displacements and deflections shown in Figs 9 to 11, from the extreme values of the time histories given in Tab. 2 and from Polach & Hajžman (2007) that considering the measured tire radial characteristics at different tire inflation in the computational models did not improve (as assumed) the accordance of the experimental measurements and the computer simulation results. The tire radial characteristics at different tire inflation influenced especially the extreme values of the time histories of the air springs relative deflections (except considering the tire radial characteristics with the 2 tons mass of the weight acting on the tire and with the smooth contact surface when the influence of tire inflating shows only minimally – see Tab. 2); the character of the time histories of the air springs relative deflections is less influenced. In order to achieve the better accordance of the experimental measurements and

the computer simulations results it would be necessary to obtain higher extreme values of the air springs relative deflections in the rebound field keeping the same or changing minimally the extreme values of the air springs relative deflections in the compression field (Polach & Hajžman, 2005a; Polach & Hajžman, 2005b; Polach & Hajžman, 2006 and Polach & Hajžman, 2007). But the determined tire radial characteristics at varied tire inflation influence the extreme values of the air springs relative deflections in the rebound field and in the compression field approximately in the same way. When improving the accordance of the experimental measurements results and the computer simulations results in the rebound field of the air springs relative deflections the accordance in the compression field of the air springs relative deflections decreased and vice versa.

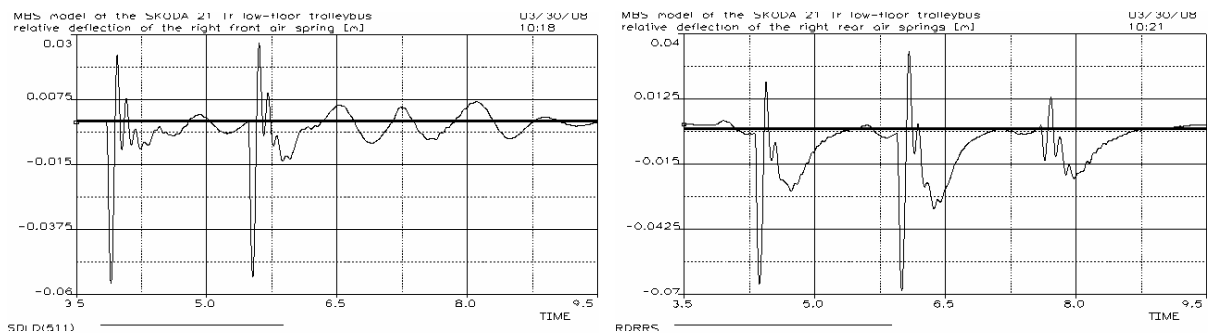


Fig. 10: Time histories of the front right and the rear right air springs relative deflection when simulating the test drive with the tire radial characteristic model at 100 % inflation and the mass of the weight 3 tons.

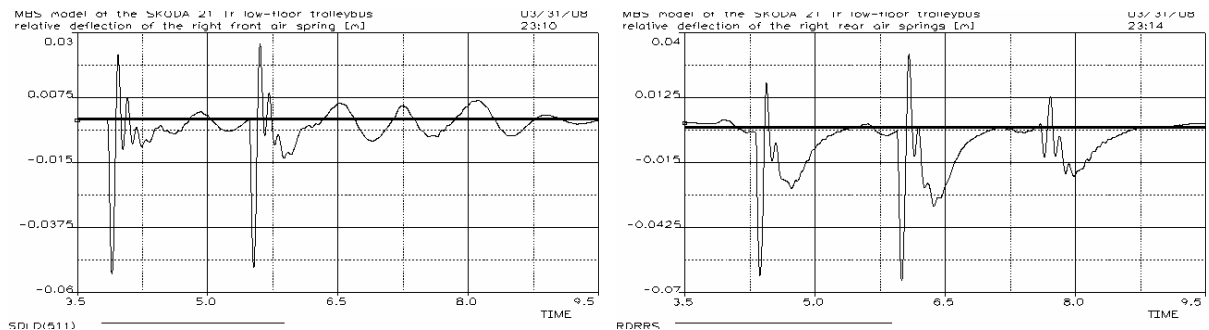


Fig. 11: Time histories of the front right and the rear right air springs relative deflection when simulating the test drive with the tire radial characteristic model at 90 % inflation and the mass of the weight 3 tons.

## 6. Sensitivity analysis of the multibody model

The sensitivity analysis of the multibody model for the change of the tire inflation was performed.

The sensitivity analysis of the selected parameters characterizing the system behaviour for the change in various system parameters is applied especially in the field of optimization, identification and correction of the mathematical models of the investigated systems. By means of that it is possible to determine which parameters influence the change of the selected quantities most significantly and subsequently to select the parameters as the optimizing ones and to try to define them more precisely or to correct them.

The results of the computer simulations and the experimental measurements with the ŠKODA 21 Tr low-floor trolleybus, compared on the basis of the evaluation of the accordance of extreme values of the time histories of the air springs relative deflections with the measured extreme values of the relative displacements during the run along the test track, are not identical especially in the course of the spring elements rebound (i.e. in the field of the positive values – see Tab. 2). It is obvious that this fact is influenced by the course of the characteristics of the spring-damper structural elements. That is why the sensitivity analysis of the extreme values of the time histories of the air springs relative deflections to the change in the most problematic force-velocity characteristics of hydraulic shock absorbers, the force-deflection characteristic of the air springs and force-deformation characteristics of the shock absorbers bushings were performed in Polach & Hajžman (2006). It was found out from the test calculations (Polach & Hajžman, 2006) that the sensitivity of the extreme values of the time histories of the air springs relative deflections to the change of the tire radial characteristics shows up in a different way than with the characteristics of the spring-damper elements investigated in Polach & Hajžman (2006). During the sensitivity analysis the bounce of the tire from the road surface shows up very considerably, which really occurs in the course of the vehicle relative speeding along the relatively demanding test track. The influence of the change in the tire radial characteristics is not fully deterministic in the course of the simulation of drive along this test track and comparison of the results of the sensitivity analysis of its influence on the monitored quantities with the results of the influence of the characteristics of the spring damper elements investigated in Polach & Hajžman (2006) would be biased. That is why the sensitivity of the extreme values of the time histories of the air springs relative deflections to the tire radial characteristics change at varied tire inflation is not compared with the influence of the spring-damper structural elements investigated in Polach & Hajžman (2006).

The influence of the changes in the parameters of the tire radial characteristics on the extreme values of the relative deflections of all the air springs when running over each obstacle of the artificial test track was monitored.

### 6.1. The sensitivity analysis of the dynamic response of the trolleybus multibody models

As in most cases of the complicated multibody systems it is not possible to derive analytical relations to express the dynamic response of the given multibody model to the general excitation. It is not possible to derive analytical formulas to calculate the sensitivity of the dynamic response to the change in the system parameters either. In order to express the partial derivative of the certain monitored quantity  $y = y(\mathbf{p})$  regarding the vector of the  $S$  selected parameters of the system  $\mathbf{p} = [p_1, p_2, \dots, p_S]^T$  it is necessary to use relations for the numerical calculations of sensitivity, so called difference formulas (Gill et al., 1981).

Change  $\Delta y$  of the monitored quantity  $y$  can be expressed with a small change  $\Delta \mathbf{p}$  of the initial parameters vector  $\mathbf{p}_0$ , when the specific conditions of the continuity of derivations of the monitored quantity  $y$  are fulfilled, using the Taylor formula (approx. by two terms)

$$\Delta y = y(\mathbf{p}_0 + \Delta \mathbf{p}) - y(\mathbf{p}_0) = \sum_{j=1}^S \frac{\partial y(\mathbf{p}_0)}{\partial p_j} \cdot \Delta p_j \quad . \quad (7)$$

After the modification of relation (7) it is obtained

$$\frac{\Delta y}{y(\mathbf{p}_0)} = \sum_{j=1}^S \frac{\partial y(\mathbf{p}_0)}{\partial p_j} \cdot \frac{p_{j0}}{y(\mathbf{p}_0)} \cdot \frac{\Delta p_j}{p_{j0}} \quad (8)$$

From relation (8) it is possible to get relative sensitivity  $\Delta \bar{y}_j$  of quantity  $y$  to the change in parameter  $p_j$

$$\Delta \bar{y}_j = \frac{\partial y(\mathbf{p}_0)}{\partial p_j} \cdot \frac{p_{j0}}{y(\mathbf{p}_0)} \quad (9)$$

Partial derivative in relation (9) is approximated using the finite difference

$$\frac{\partial y(\mathbf{p}_0)}{\partial p_j} = \frac{y(\mathbf{p}_0 + \Delta \mathbf{p}_j) - y(\mathbf{p}_0)}{\Delta p_j} \quad (10)$$

where vector  $\Delta \mathbf{p}_j = [0, \dots, 0, \Delta p_j, 0, \dots, 0]^T$ .

Then differential relation for the calculation of relative sensitivity  $\Delta \bar{y}_j$  of quantity  $y$  to the change in parameter  $p_j$ , using relations (9) and (10), can be written in the final form

$$\Delta \bar{y}_j = \frac{y(\mathbf{p}_0 + \Delta \mathbf{p}_j) - y(\mathbf{p}_0)}{\Delta p_j} \cdot \frac{p_{j0}}{y(\mathbf{p}_0)} \quad (11)$$

Thus in case of the sensitivity analysis of the ŠKODA 21 Tr trolleybus multibody model when driving along the artificial test track the relative deflections of the air springs of axles were successively the monitored quantities  $y$  and the relative changes in the tire radial characteristics at different tire inflation were the vectors of parameters  $\mathbf{p}$ .

## 6.2. Sensitivity analysis results

The tire radial characteristics at varied tire inflation were the parameters of the sensitivity analysis, during which the influence of parameter changes of those characteristics on the extreme values of the relative deflections of the air springs was monitored. The results of the sensitivity analysis in the course of simulating with the multibody model of the ŠKODA 21 Tr low-floor trolleybus showed that the tire radial force-deformation characteristics have a greater influence on the results of driving along the virtual test track than the tire radial damping characteristics (i.e., damping coefficient). Sensitivity of the monitored extreme values of the relative deflections of the air springs to the tire inflation change is very low.

The relative sensitivities of the relative deflections of the air springs to the change in the tire radial characteristics at varied tire inflation in the course of the simulations of driving along the virtual test track are given in Figs 12 and 13 (2t or 3t = the mass of the weight acting on the tire; 90Inf or 100Inf = percentage of the specified tire inflation; Stf = sensitivity analysis of the tire radial force-deformation characteristics; Dmp = sensitivity analysis of the tire radial damping coefficients; 1st to 3rd Obst = obstacle sequence; Comp = compression of air springs; Reb = rebound of air springs).

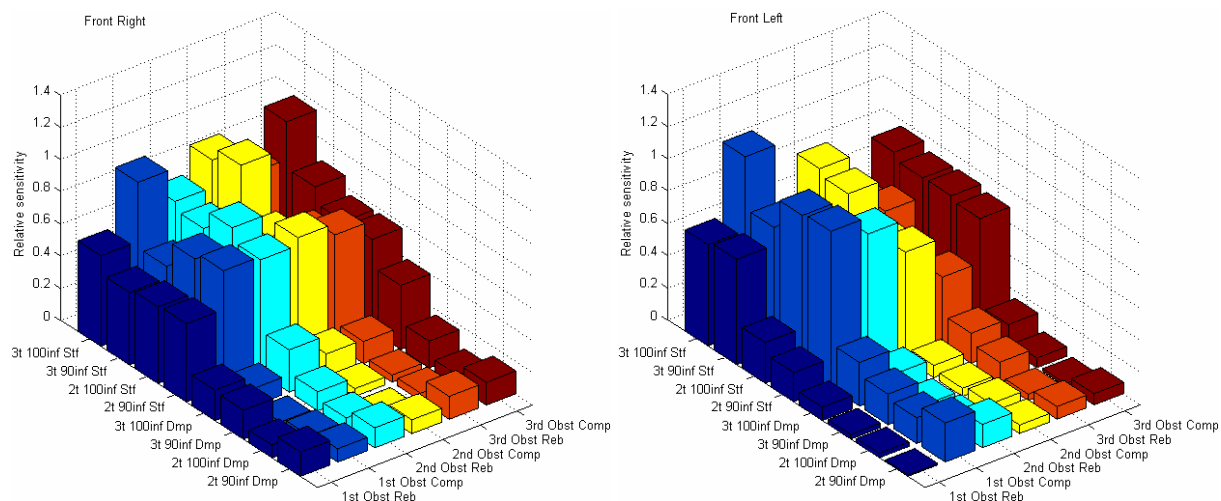


Fig. 12: Relative sensitivity of the relative deflections of the front air spring (right and left) to the change of the tire radial characteristics.

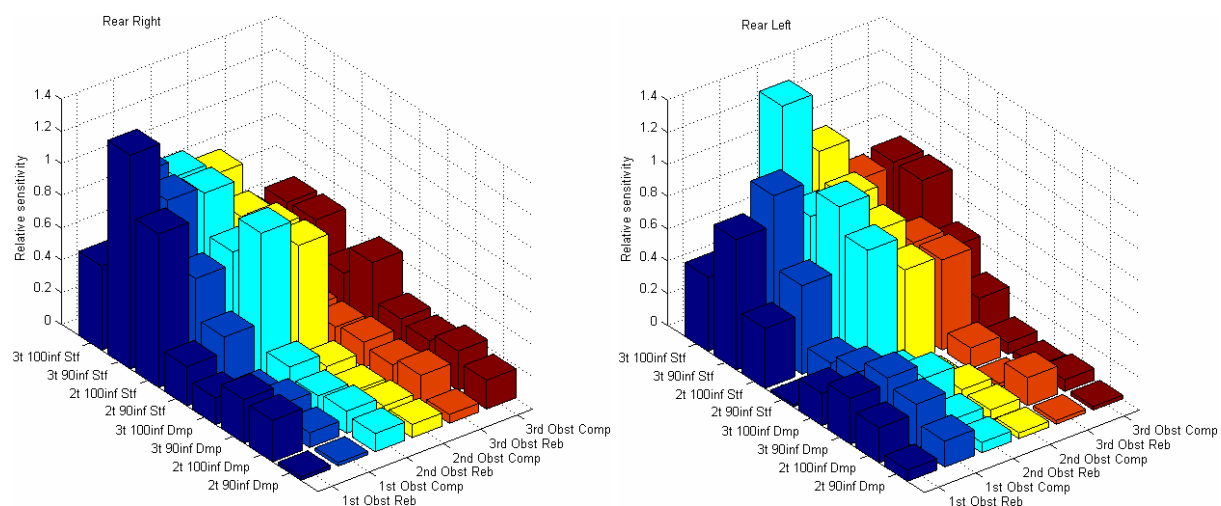


Fig. 13: Relative sensitivity of the relative deflections of the rear air springs (right and left) to the change of the tire radial characteristics.

## 7. Conclusions

The vertical dynamic properties of the ŠKODA 21 Tr low-floor trolleybus were investigated on the artificially created test track when driving with the real vehicle and when simulating driving along the virtual test track with the most complex multibody model (Polach, 2003) created in the **alaska 2.3** simulation tool (Maiber et al., 1998). The results of the simulations and the experimental measurement were compared on the basis of the evaluation of the accordance of the extreme values of time histories of the air springs relative deflections with the measured extreme values of the relative displacements between the axles and the chassis frame during the run over each obstacle of the test track. Even influences of the determined tire radial characteristics at varied tire inflation did not improve the accordance of the results of the simulations with the results of the experimental measurement. The dynamic responses of the ŠKODA 21 Tr low-floor trolleybus to the vertical kinematic wheel excitation when considering different tire inflation in reasonable limits (i.e. between 90 % of specified tire inflation and specified tire inflation) do not differ significantly.

The cause of the deviations of the simulations results from the experimental measurement results could also be in the ignorance of all the conditions of test drives with the real trolleybus needed for the more precise performing of the simulations (a real static height of the air springs was not measured and the vehicle was not weighted – the structural design data were used in the multibody model; in addition the characteristics were measured on new spring-damper elements, not on those used in the structure of the vehicle, on which the experimental measurement was performed) – e.g. Polach & Hajžman (2005a).

The sensitivity analysis of the multibody model to the change of the tire radial characteristics at varied tire inflation was performed. The tire force-deformation characteristics have a greater influence on the results of simulations of driving along the virtual test track than the tire radial damping characteristics (i.e., damping coefficient).

Change of the monitored extreme values of the relative deflections of the air springs to the change of the tire inflation is very low. In the field of the investigation of the tire radial characteristics influence on the results of the simulations of driving with the trolleybus multibody model along the virtual test track attention will also be paid to the shape of the contact surface falling on the tire during the measurement of its radial characteristics – see Fig. 6 and Hajžman & Polach (2006).

But generally it is necessary to confirm (Polach & Hajžman, 2006; Polach & Hajžman, 2007) that the simulations results are substantially influenced by the force-velocity characteristics of the hydraulic shock absorbers. The measuring of the characteristics is planned in the Hydrodynamic Laboratory of the Technical University of Liberec for the 3rd quarter of 2008. The determination of new characteristics of the hydraulic shock absorbers and their implementation into the existing trolleybus multibody models are expected to lead to the substantial improvement in accordance of the results of the simulations with the results of the experimental measurements.

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