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SIMPLIFIED MODELS OF PNEUMATIC LONG LINE

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Summary: *Pneumatic long line is a research object with complex parameters, described by a partial differential equation. Complexity of problems connected with an analysis of this type of objects leads to seeking different approximate ways of analyses. A partial differential equation can also be approximated by: system of difference equations, system of ordinary differential equations, and ordinary differential equation. This last procedure must be used of carefully, because we knowingly accept inaccuracy of the obtained model. Inaccuracy evaluation criterion of the simplified model is experimental comparison of responses obtained during the object testing with its model for a given case.*

1. Introduction

During developing the digital-analogue converter the converter dynamic properties were tested. In the first phase of tests attention was focused on dynamics of fluid elements which were produced in Poland on a semi-industrial scale three input elements of NOR type. Although the logical system was made up of forty elements, a detailed analysis revealed that about 98% of the air present in the built prototype of the device did not occur in working elements (NOR), but it appeared in connecting conduits and valve chambers of digital-analogue converter.

2. Approximation of partial differential equation by set of difference equations

Let u be a function of two or more variables which meets partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \tag{1}$$

Expanding (1) into Taylor series, for example for variable x in the vicinity of x_i we obtain

$$u\left(x_i + \Delta x\right) = u\left(x_i\right) + \frac{\Delta x}{1!} u'\left(x_i\right) + \frac{\left(\Delta x\right)^2}{2!} u''\left(x_i\right) + \frac{\left(\Delta x\right)^3}{3!} u^{(3)}\left(x_i\right) + \dots \tag{2}$$

$$u\left(x_i - \Delta x\right) = u\left(x_i\right) - \frac{\Delta x}{1!} u'\left(x_i\right) + \frac{\left(\Delta x\right)^2}{2!} u''\left(x_i\right) - \frac{\left(\Delta x\right)^3}{3!} u^{(3)}\left(x_i\right) + \dots \tag{3}$$

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Summing sides of equations (2) and (3) and after neglecting fourth and higher power of Δx we obtain

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{x_i} = \frac{2}{(\Delta x)^2}\left[u(x_i+\Delta x) - 2u(x_i) + u(x_i - \Delta x)\right] \tag{4}$$

It is the approximate value of second derivative of function u in point x_i. Subtracting sides of equations (2) and (3) and after neglecting third and higher power of Δx we obtain first derivative in the form

$$\left(\frac{\partial u}{\partial x}\right)_{x_i} = \frac{1}{\Delta x}\left[u(x_i+\Delta x) - u(x_i - \Delta x)\right] \tag{5}$$

Independent variable x takes discrete values $x_i = i\Delta x \left(i = 1,2,\ldots,N\right)$. For N points we obtain N equations. As we can see from (2) and (3) the approximate value of first or second derivative of function u in point x_i is determined by function values in three points x_{i-1}, x_i, x_{i+1}. This fact determines the shape of matrix (8); it is three diagonal.

From equations (2) and (3) we can obtain first approximation of function by taking into consideration only two initial components of the right side; so

$$\left(\frac{\partial u}{\partial x}\right)_{x_i} = \frac{1}{\Delta x}\left[u(x_i+\Delta x) - u(x_i)\right] \tag{6}$$

$$\left(\frac{\partial u}{\partial x}\right)_{x_i} = \frac{1}{\Delta x}\left[u(x_i) - u(x_i - \Delta x)\right] \tag{7}$$

Obviously, the obtained approximations are one order worse. After substitution of the above mentioned approximation, i.e. for second derivative (4) and for first derivative (5), (6) or (7) into equation (1) we obtain set of algebraic equations, which can be presented in the form

$$\begin{bmatrix} a_{11} & a_{12} & 0 & \cdots & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & \cdots & 0 & 0 & 0 \\ 0 & a_{32} & a_{32} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_{N-2,N-2} & a_{N-2,N-1} & 0 \\ 0 & 0 & 0 & \cdots & a_{N-1,N-2} & a_{N-1,N-1} & a_{N-1,N} \\ 0 & 0 & 0 & \cdots & 0 & a_{N,N-1} & a_{N,N} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_{N-2} \\ u_{N-1} \\ u_N \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{N-2} \\ d_{N-1} \\ d_N \end{bmatrix} \tag{8}$$

or in shorter form dependence

$$\mathbf{Au} = \mathbf{d} \tag{9}$$

Vector $\mathbf{d}$ is known in time j, whereas $\mathbf{u}$ is vector which is looked for in time $j+1$. Matrix $\mathbf{A}$ is a given matrix with constant entries, For $j = 0$, $\mathbf{d}$ denotes initial conditions.

After approximation of the left side by equation (4) and the right one by equation (6) we obtain a series of difference equations presented by formula:

$$u_{i,j+1} = a\left(u_{i+1,j} + u_{j-1,j}\right) + \left(1-2a\right)u_{i,j} \tag{10}$$

for $i = 1, 2, \ldots, N$

where $a = \dfrac{\Delta t}{\left(\Delta x\right)^2}$

Equation (10) is unstable for $a > 0.5$. Using approximations (4) and (7) we obtain

$$u_{i,j+1} = -a\left(u_{i+1,j+1} + u_{i-1,j+1}\right) + \left(1+2a\right)u_{i,j+1} \tag{11}$$

The approximation is unconditionally stable.

In order to obtain a better approximation – with second order error for both variables – we can use non-overt method developed by Crank and Nicolson according to which $\partial u / \partial t$ is expended in point $\left(i, j+1/2\right)$, and because the function value in this point is not known we take the average arithmetic value of function u, expanded according to (4) in points $\left(i, j\right)$ and $\left(i, j+1\right)$, thus

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j+1/2} = \frac{1}{2}\left[\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\left(\Delta x\right)^2} + \frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{\left(\Delta x\right)^2}\right] + 0\left(\Delta x\right)^2 \tag{12}$$

So, finally

$$-au_{i-1,j+1} + \left(2+2a\right)u_{i,j+1} - au_{i+1,j+1} = au_{i-1,j} + \left(2-2a\right)u_{i,j} - au_{i+1,j} \tag{13}$$

There is a possibility to create very many difference schemes which can be found e.g. in references - Collatz L., (1960), Demidowicz B.P., Maron L.A., Szuwałowa E.Z., (1965).

The procedure used for solving a set of equations regardless of used approximation, involves the following steps:

— for the first time period $j = 1$, $\mathbf{Au^1} = \mathbf{d}$;

— for the second time period $j = 2$, $\mathbf{Au}^2 = \mathbf{u^1}$

The value in time period $j+1$ is calculated on the basis known values in time j. This statement is also right when represents space dimension, not time.

Respecting (10), we can write e.g. parabolic dependence $\dfrac{\partial^2 P}{\partial x^2} = b\dfrac{w}{D}\dfrac{\partial P}{\partial t}$, $\quad P = p^2$ in the form

$$P_{i,j+1} = P_{i,j} + \frac{a}{\gamma}\left(P_{i-1,j} - 2P_{i,j} + P_{i+1,j}\right) \tag{14}$$

where: $\gamma = b\dfrac{w}{D}$, $P_{i,j}$ – pressure squared in i-th point, in j-th time period.

In turn

$$b = \frac{\hat{\Theta}\hat{Z}\hat{\rho}}{\Theta Z\hat{p}}\frac{\lambda}{2},$$

Difference scheme will be stable, when

$$\Delta t \le \frac{\hat{\Theta}\hat{Z}\hat{\rho}\lambda w\left(\Delta x\right)^2}{4\Theta Z\hat{p}D}, \quad \text{where } \Delta x = \frac{l}{N} \tag{15}$$

Velocity w – for flow Q_v (it is assumed that in time period $t = 0$ in the conduit there is constant flow) is calculated form the formula

$$w = \frac{4Q_v}{\pi D^2} \tag{16}$$

Respecting additionally dependence

$$Q_v = \frac{\hat{\Theta}\hat{Z}\hat{p}\hat{Q}_v 4}{\Theta Z p \pi D^2} \tag{17}$$

in which instead of p, in the conduit the pressure average value was substituted with the given constant flow.

In order to determine average pressure equation

$$p\left(x,0\right) = \sqrt{p^2\left(0,0\right) - \left[p^2\left(0,0\right) - p^2\left(l,0\right)\right]\frac{x}{l}}$$

must be integrated in the interval $\left[0,l\right]$, then

$$\overline{p} = \int_0^l p_x dx = \int_0^l \sqrt{p_{x=0}^2 - \left(p_{x=0}^2 - p_{x=l}^2\right)\frac{x}{l}}\,\mathrm{d}x$$

Finally

$$\overline{p} = \frac{2}{3}\left(p_{x=0} + \frac{p_{x=1}^2}{p_{x=0} + p_{x=1}}\right)$$

Accepting that in time $\left[t = 0\right]$

$$p\left(x,t\right) = f\left(x\right) \text{ for } x \in \left[0,l\right]$$

as well as $p(0,t) = \text{const}$, $Q_v\left(l,t\right) = Q_v\left(t\right)$, with assumption, that between points x_{N-1} and x_N flow is steady, from equation $p_1^2 - p_2^2 = \dfrac{16\lambda\Theta Z\hat{p}\hat{\rho}}{\pi^2 D^5\hat{\Theta}\hat{Z}}\hat{Q}_v^2 l$ we obtain:

$$P_{N,j+1} = P_{N-1,j+1} - \frac{\lambda A}{D^5}\hat{Q}_v^2\Delta x$$

where $A = \dfrac{16Z\hat{p}\hat{\rho}}{\pi^2\hat{\Theta}\hat{Z}}$.

3. Approximation of partial differential equation by a set of ordinary differential equations

Pneumatic transmission long line is presented in Fig. 1 in the form of a series connected pneumatic resistors R_i and capacitances C_i respectively. Laminar flow with flow rate q_i through all resistors is assumed. For these assumptions, the following dependencies are correct:

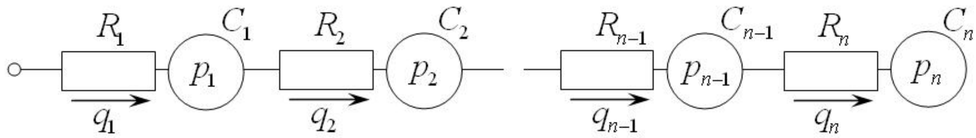

Fig. 1. Transmission line composed from n elements RC

$$\begin{aligned} C_1 \frac{\mathrm{d}p_1}{\mathrm{d}t} &= q_1(t) - q_2(t) \\ C_2 \frac{\mathrm{d}p_2}{\mathrm{d}t} &= q_2(t) - q_3(t) \\ &\vdots \\ C_{n-1} \frac{\mathrm{d}p_{n-1}}{\mathrm{d}t} &= q_{n-1}(t) - q_n(t) \\ C_n \frac{\mathrm{d}p_n}{\mathrm{d}t} &= q_n(t) \end{aligned} \tag{18}$$

as well as

$$\begin{aligned} q_1(t) &= \frac{1}{R_1}\left[p_p(t) - p_1(t) \right] \\ q_2(t) &= \frac{1}{R_2}\left[p_1(t) - p_2(t) \right] \\ &\vdots \\ q_{n-1}(t) &= \frac{1}{R_{n-1}}\left[p_{n-2}(t) - p_{n-1}(t) \right] \\ q_n(t) &= \frac{1}{R_n}\left[p_{n-1}(t) - p_n(t) \right] \end{aligned} \tag{19}$$

Set of equations (18) and (19) we can write in matrix form

$$\mathbf{C}\dot{\mathbf{p}} = \mathbf{A}\mathbf{p} + \mathbf{b}\mathbf{p}_{\mathrm{p}} \tag{20}$$

Where the particular matrixes are in the

$$\mathbf{C}\dot{\mathbf{p}} = \begin{bmatrix} C_1 \dfrac{\mathrm{d}p_1}{\mathrm{d}t} \\ C_2 \dfrac{\mathrm{d}p_2}{\mathrm{d}t} \\ \vdots \\ C_{n-1} \dfrac{\mathrm{d}p_{n-1}}{\mathrm{d}t} \\ C_n \dfrac{\mathrm{d}p_n}{\mathrm{d}t} \end{bmatrix} \quad \mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_{n-1} \\ p_n \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} \dfrac{1}{R_1} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} -\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right) & \dfrac{1}{R_2} & 0 & \dots & 0 & 0 & 0 \\ \dfrac{1}{R_2} & -\left(\dfrac{1}{R_2}+\dfrac{1}{R_3}\right) & \dfrac{1}{R_3} & \dots & 0 & 0 & 0 \\ 0 & \dfrac{1}{R_3} & -\left(\dfrac{1}{R_3}+\dfrac{1}{R_4}\right) & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\left(\dfrac{1}{R_{n-2}}+\dfrac{1}{R_{n-1}}\right) & \dfrac{1}{R_{n-1}} & 0 \\ 0 & 0 & 0 & \dots & \dfrac{1}{R_{n-1}} & -\left(\dfrac{1}{R_{n-1}}+\dfrac{1}{R_n}\right) & \dfrac{1}{R_n} \\ 0 & 0 & 0 & \dots & 0 & \dfrac{1}{R_n} & -\dfrac{1}{R_n} \end{bmatrix}$$

Matrix $\mathbf{A}$ is again three diagonal, which shows that signal in the j-th element is affected by signals in the previous and the next element. Other signals have no influence. It is consistent with expectations based on purely physical consideration.

4. Approximation of partial differential equation by ordinary differential equation

Previously it was proved, that a given object with distributed parameters may be described, with chosen accuracy, by a set of differential equations or a set of difference equations.

In some conditions we can completely neglect the fact, that the object parameters are of distributed parameters character. We knowingly accept inaccuracy of the obtained in this way model. One may use experimental results for checking if the system dynamics may be omitted or if one may derive an accurate model and carry out an accurate analysis of dynamic properties.

One may obtain an answer to the question, if a simplified model provides us with sufficient information. In references, there are many methods of approximate description of transmission line. The precision evaluation criterion, in most cases of simplified mathematical models with lumped parameters, is comparison of the object and the model responses in a concrete case.

Simplified models, assumptions for which have been obtained, and experimental results are presented below .

Model I

This model has been obtained by using following simplifications for hyperbolic trigonometric functions:

$$-\text{ for } |\gamma l| < 1 \quad \begin{cases} \text{ch}\gamma l \approx\sim 1 + \frac{1}{2}(\gamma l) \\ \text{sh}\lambda l \approx \lambda l \end{cases}$$

$$-\text{ for } |\gamma l| > 1 \quad \text{ch}\gamma l \approx \text{sh}\lambda l \approx \frac{1}{2}e^{\gamma l}$$

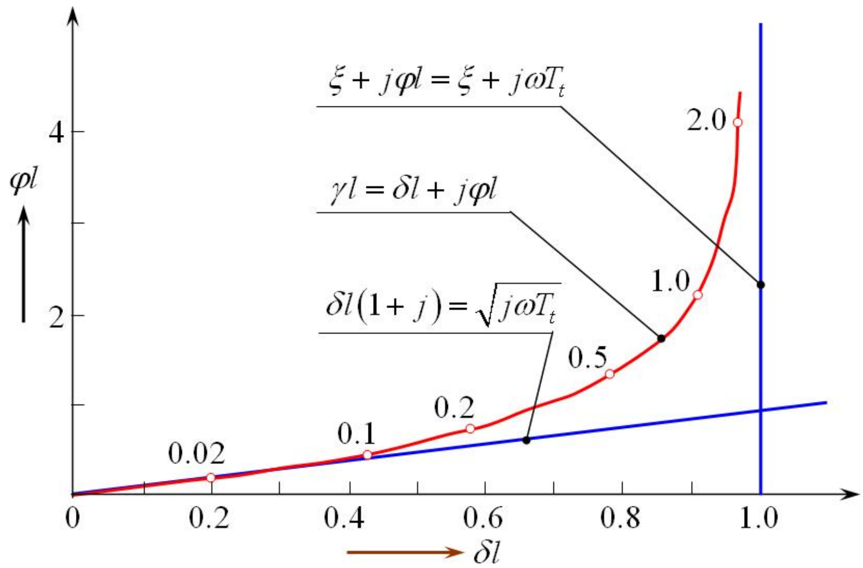

Fig. 2. Approximation of dependence $\gamma l = \delta l + j\varphi l$

Considering the course of propagation coefficient changes γ (Fig. 2) according to dependence

$$\gamma l = \delta l + j\varphi l$$

we can accept that

$$-\text{ for } |\gamma l| < 1 \quad \gamma l \approx \sqrt{sT_1}$$

$$-\text{ for } |\gamma l| > 1 \quad \gamma l \approx \xi + sT_t$$

Respecting the above simplifications, equation

$$Q_m(x,s)=\frac{p(0,s)}{Z_f}\frac{\mathrm{sh}\gamma l\left(1-\frac{x}{l}\right)+r\gamma l\,\mathrm{ch}\gamma l\left(1-\frac{x}{l}\right)}{\mathrm{ch}\gamma l+r\gamma l\,\mathrm{sh}\gamma l}$$ is given the form

$$\frac{p(x,s)}{p(0,s)}=\begin{cases}\dfrac{1+\left(1-\frac{x}{l}\right)\left(\frac{1}{2}+r\right)sT_1}{1+\left(\frac{1}{2}+r\right)sT_1}, & \text{for } |\gamma l|<1\\[2ex] e^{-(\xi+sT_t)\frac{x}{l}} & \text{for } |\gamma l|>1\end{cases}\tag{21}$$

Transient response for $x=l$ has the form

$$\frac{p(l,t)}{p(0,t)}=\begin{cases}1-e^{-\left(\frac{t-T_t}{T_1}\right)\left(\frac{1}{2}+r\right)}, & \text{for}\quad t>T_t\\ 0, & \text{for}\quad t\le T_t\end{cases}\tag{22}$$

Frequency characteristics (magnitude and phase) are expressed by dependencies:

$$A(\omega)=\begin{cases}\dfrac{1}{\sqrt{1+\left[\left(\frac{1}{2}+r\right)T_1\omega\right]^2}}, & \text{for}\quad \omega T_2>1\\[2ex] e^{-\zeta} & \text{for}\quad \omega T_2\le 1\end{cases}\tag{23}$$

$$\varphi(\omega)=\begin{cases}-arctg\left(\frac{1}{2}+r\right)\omega T_1, & \text{for}\quad \omega T_2>1\\ -\omega T_t & \text{for}\quad \omega T_2\le 1\end{cases}\tag{24}$$

Model II

For the assumption that volume on the end V_k is large, and frequency of input sinusoidal signals are small, we can assume that:

$$r=\frac{C_k}{lC'}\gg 1 \text{ as well as } l\ll\frac{2\pi c}{\omega}$$

Hence, we can write

$$\frac{\partial p}{\partial x}\approx\frac{\Delta p}{\Delta x}=\frac{p_{x=l}-p_{x=0}}{l}$$

Dependencies $\frac{\partial p}{\partial x}\approx\frac{\Delta p}{\Delta x}=\frac{p_{x=l}-p_{x=0}}{l}$ obtain the form

$$\begin{aligned}&\frac{p_{x=l}-p_{x=0}}{l}+R'Q_m+L'\frac{\partial Q_m}{\partial t}=0\\ &\frac{Q_{m(x=l)}-Q_{m(x=0)}}{l}+C'\frac{\mathrm{d}p}{\mathrm{d}t}=0\end{aligned}\tag{25}$$

After rearranging and taking into consideration that

$$\frac{C_k}{C'l} = r \text{ as well as } Q_m\left(l,t\right) = C_k \frac{\mathrm{d}p_{x=l}}{\mathrm{d}t}$$

we obtain

$$\begin{aligned} & p_{x=l} + RQ_{m(x=0)} + L\frac{\mathrm{d}Q_{m(x=0)}}{\mathrm{d}t} = p_{x=0} \\ & \left(1+r\right)C\frac{\mathrm{d}p_{x=l}}{\mathrm{d}t} = Q_{m(x=0)} \end{aligned} \tag{26}$$

Simplified differential equation of long line after respecting given dependences has the form

$$\left(1+r\right)T_1T_2\frac{\mathrm{d}^2 p_{x=l}}{\mathrm{d}t^2} + \left(1+r\right)T_1\frac{\mathrm{d}p_{x=l}}{\mathrm{d}t} + p_{x=l} = p_{x=0} \tag{27}$$

for which corresponding transfer function is as follows

$$\frac{p\left(l,s\right)}{p\left(0,s\right)} = \frac{1}{\left(1+r\right)T_1T_2s^2 + \left(1+r\right)T_1s + 1} \tag{28}$$

Respecting limited acoustic velocity, long line transfer function may be expressed by formula

$$\frac{p\left(l,s\right)}{p\left(0,s\right)} = \frac{e^{-sT_t}}{\left(1+r\right)T_1T_2s^2 + \left(1+r\right)T_1s + 1} \tag{29}$$

Transient response on input step function of long line described by equation (29) has the form

$$\frac{p\left(l,t\right)}{p\left(0,t\right)} = 1 - \left(\frac{T_a}{T_a - T_b}e^{-\frac{t-T_t}{T_a}} - \frac{T_b}{T_a - T_b}e^{-\frac{t-T_t}{T_b}}\right) \tag{30}$$

where $t \geq\sim T_t$

$$T_{a,b} = \frac{2T_2}{1 \mp \sqrt{1 - \frac{1}{\left(1+r\right)\zeta^2}}} \text{ for assumption, that } (1+\mathrm{r})\zeta^2 \geq 1$$

As we can see, dependences (30) and

$\frac{p\left(l,t\right)}{p\left(0,t\right)} = 1 - \sum_k \frac{2}{\beta_k\left(1 + r + r^2\beta_k^2\sin\beta_k\right)}\left(\frac{T_{1k}}{T_{1k} - T_{2k}}e^{-\frac{t-T_t}{T_{1k}}} - \frac{T_{2k}}{T_{1k} - T_{2k}}e^{-\frac{t-T_t}{T_{2k}}}\right)$ are not much different. After substituting $s = j\omega$ to (29), we obtain magnitude and phase frequency characteristics, respectively.

$$A\left(\omega\right) = \frac{1}{\sqrt{\left[1 - \left(1+r\right)T_1T_2\omega^2\right]^2 + \left[\left(1+r\right)T_1\omega^2\right]^2}} \tag{31}$$

(32)

Next were carried out analyses of simplified models I and II and full model described by dependencies $\frac{p(l,t)}{p(0,t)} = 1 - \sum_k \frac{2}{\beta_k \left(1 + r + r^2 \beta_k^2 \sin \beta_k\right)} \left(\frac{T_{1k}}{T_{1k} - T_{2k}} e^{-\frac{t-T_t}{T_{1k}}} - \frac{T_{2k}}{T_{1k} - T_{2k}} e^{-\frac{t-T_t}{T_{2k}}} \right)$ from reference Kamiński L. M. (1996) in relation to the real system. For pneumatic conduit with dimensions $D = 4{,}1$ mm, $l = 60$ m with volume on the end $V_k = 31$ (Fig. 3 – 7) were calculated line parameters

$$T_1 = 1{,}26\,\text{s} \qquad T_t = 0{,}207\,\text{s} \qquad r = 3{,}79$$
$$T_2 = 0{,}034\,\text{s} \qquad \zeta = 3{,}05$$

On the basis of the following equations

$$\frac{p(l,t)}{p(0,t)} = 1 - \sum_k \frac{2}{\beta_k \left(1 + r + r^2 \beta_k^2 \sin \beta_k\right)} \left(\frac{T_{1k}}{T_{1k} - T_{2k}} e^{-\frac{t-T_t}{T_{1k}}} - \frac{T_{2k}}{T_{1k} - T_{2k}} e^{-\frac{t-T_t}{T_{2k}}} \right),$$

as well as (22) and (30), which were simplified to forms

$$\frac{p(l,t)}{p(0,t)} = \begin{cases} 1 - 1{,}025 e^{-\frac{t-T_t}{5{,}1}} & \text{for} \quad t \geq 0{,}5s \\ 1 - e^{-\frac{t-T_t}{5{,}3}} & \text{for} \quad t > T_t \\ 1 - e^{-\frac{t-T_t}{5{,}7}} & \text{for} \quad t > T_t \end{cases}$$

pressure changes after step input ($\Delta p = 1000$ Pa have been determined, for average pressure in line $\overline{p} = 100$ kPa). Theoretical analyses results are compared with measurement results in Fig. 3. Then this procedure was repeated for the case when .

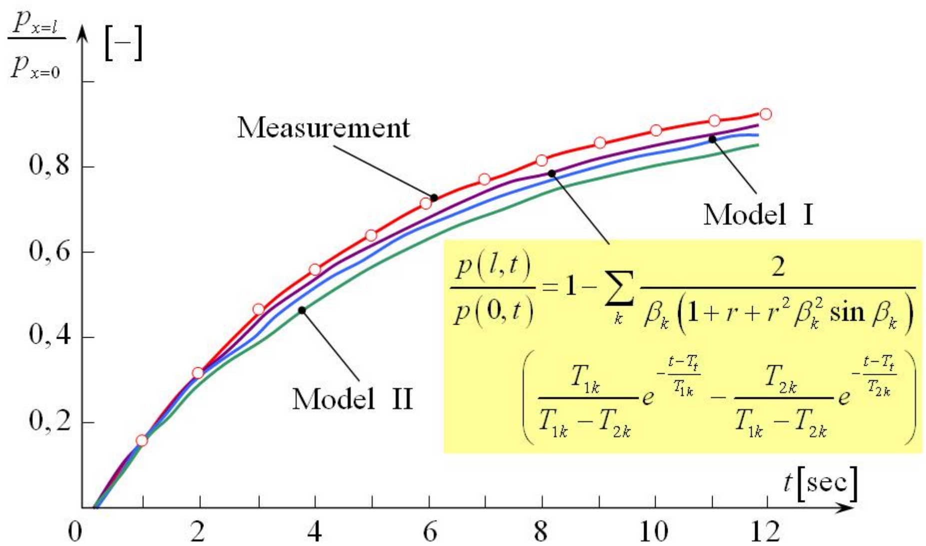

Fig. 3. Line pressure courses in case when $V_k \neq 0$

Results are presented in Fig. 4. Equation (30) was derived for assumption r □ 1. It is confirmed in Fig. 4, when we can see big discrepancy between experimental and calculated transient outputs after using equation (30) for case $V_k = 0$. From comparison of Figs 3 and 4 results a conclusion that from presented two simplified models of transmission line, the model described by equation (32) is favorable. The difference between the real object and its model is insignificant both for blank off conduit and for line with big volume on the end.

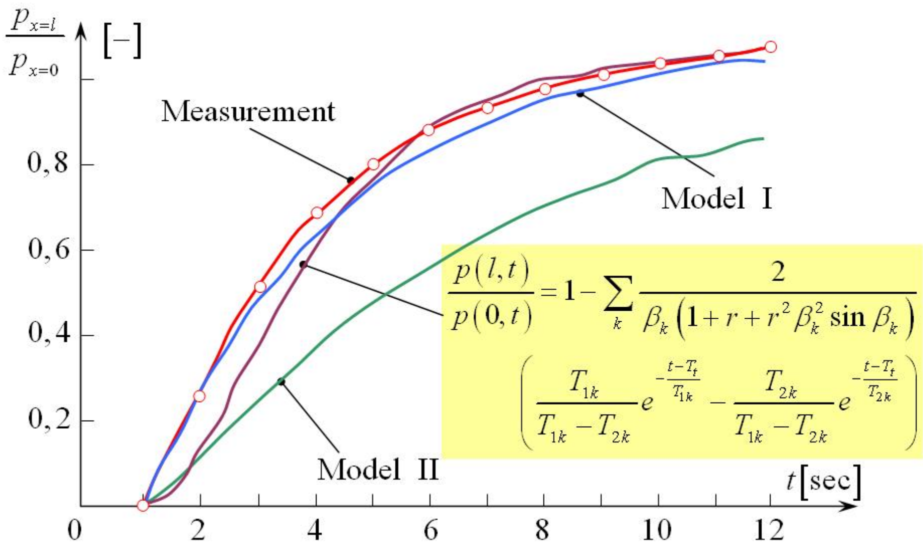

Fig. 4. Line pressure courses in case when $V_k = 0$

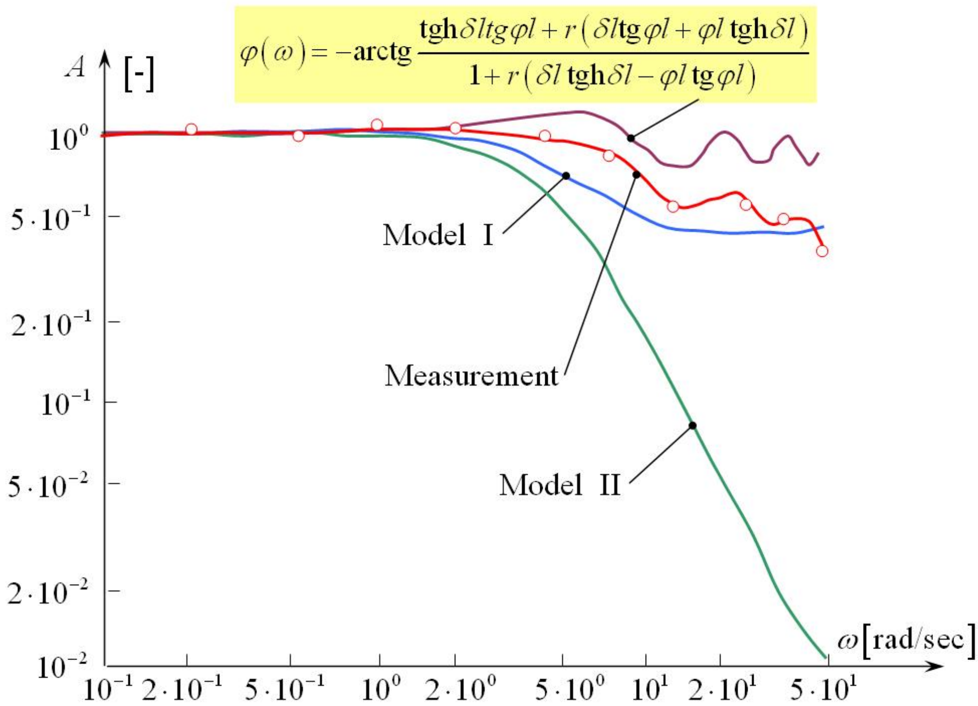

Fig. 5. Magnitude frequency characteristics of pressure changes in the line

Magnitude and phase frequency characteristics, both experimentally determined and calculated by using equations

$$A(\omega)=\frac{1}{\mathrm{ch}\delta l\left|\cos\varphi l\right|\sqrt{\left[1+r\left(\delta l\,\mathrm{tgh}\delta l-\varphi l\,\mathrm{tg}\varphi l\right)\right]^2+\left[\mathrm{tgh}\delta l\,\mathrm{tg}\varphi l+r\left(\delta l\mathrm{tg}\varphi l+\varphi l\,\mathrm{tgh}\delta l\right)\right]^2}},$$

$$\varphi(\omega)=-\mathrm{arctg}\frac{\mathrm{tgh}\delta l tg\varphi l+r\left(\delta l\mathrm{tg}\varphi l+\varphi l\,\mathrm{tgh}\delta l\right)}{1+r\left(\delta l\,\mathrm{tgh}\delta l-\varphi l\,\mathrm{tg}\varphi l\right)}$$

(23), and (24) as well as (32) and (33) are presented in Figs 5 and 6.

The object of comparison was conduit of length $l=60\,\mathrm{m}$, diameter $D=6$ mm, volume on the end $V_k=0$ and with pressure $\overline{p}=160$ kPa. Because model II was derived for assumption of small input frequency, we can see that for growing frequency, discrepancy of results between those calculated from equations (31), (32), and the accurate model, and experimental results grows.

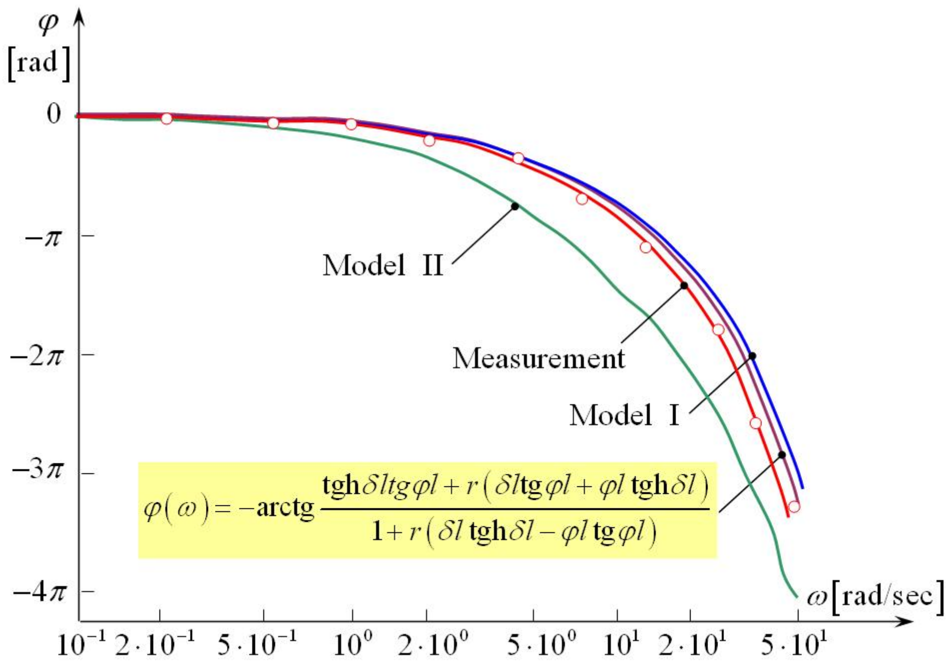

Fig. 6. Phase frequency characteristics of pressure changes in the line

The above figures confirm usefulness of simplified model I (equations (23) and (24)) although there is no possibility of calculation of magnitude caused by reflection in conduits.

5. Conclusions

Generally it can be concluded that consistence between experimental and computational data improves along with the rise of damping coefficient

Some results confirm usefulness of simplified model I although there is no possibility of calculation of magnitude caused by reflection in conduits.

Because model II was derived for assumption of small input frequency, we can see that for growing frequency, discrepancy of results between those calculated from model I equations, and the accurate model, and experimental results grows.

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