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MATHEMATICAL MODELS OF PNEUMATIC DAMPING SYSTEMS

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Summary: *The mathematical models of selected cascade systems presented in this article were obtained by means of analysis and experiment (to identify the relevant parameters). These models facilitate the investigation of such systems in the demodulation of pneumatic square-wave signals with modulated pulse width coefficient ($\gamma = 0 \div 1$) and various amplitudes ($p_0 = 20 \div 100\,\text{kPa}$) and frequencies ($f = 1 \div 100\,\text{Hz}$). Analytical investigations indicated the possibility of shaping the quality of the filter by the choice of geometric dimensions of the filter (d_{ij}, V_k) and the method of applying the input signal. The quality of filtration is indicated in this case by the limiting angular frequency (ω_{gr}), the settling time of the signal (T_u «7) and the linearity of the static characteristic ($p_{o\backslash k} = f\left(p_0, \gamma\right)$).*

1. Introduction

In many pneumatic measuring devices - Kaminski L.M. (1976), Werszko M. (1974), the information-carrying signal is a series of square pulses of variable amplitude p_0, frequency l, or pulse width coefficient $\gamma = \tau / T$ (Fig. 1). In all these cases, pneumatic low-pass filters are used. The role such filters is played by cascade systems built from resistors and chambers of fixed or variable volume.

The typical response of a dual-chamber pneumatic cascade to a series of square pulses

$$p\left(t\right) = \begin{cases} p_0 & \text{for } nT < t < nT + \tau \\ 0 & \text{for } nT + \tau < t < T\left(n+1\right) \end{cases} \left(n = 0,1,2,\ldots \right. \tag{1}$$

of parameters $p_0 = 100\,\text{kPa}$, $\gamma = 0.25$ and $\gamma = 0.75$ and $f = 2\,\text{Hz}$ is shown in Fig. 2.

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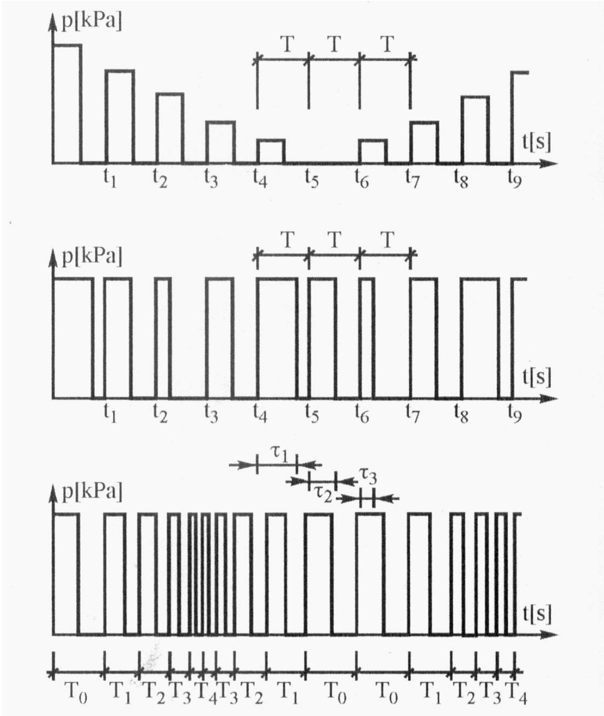

Fig. l. Periodic square-wave signal: a) amplitude modulation, b) pulse-width modulation, c) frequency modulation

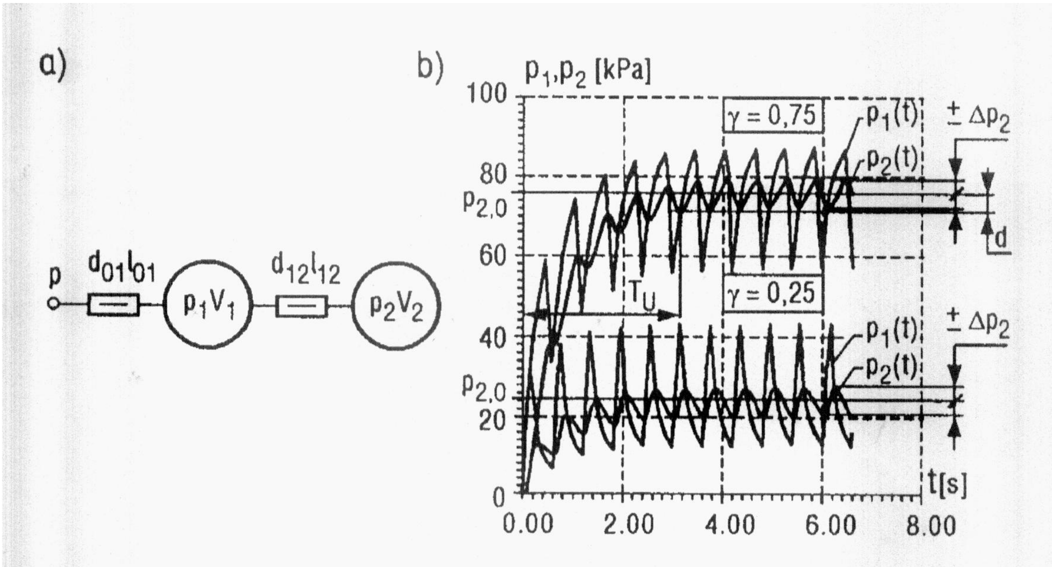

Fig.2. Response of dual chamber cascade to a series of square impulses, where d_{01}, d_{12}, l_{12} [mm], V_1 V_2 $\left[\mathrm{cm}^2\right]$ – cascade parameters

The properties of the output signal $p_1(t)$ or $p_2(t)$ are represented by:

- the static characteristic of the filter

$$p_1 = f_1\left(\gamma, p_0, f, d_{ij}, V_n\right)$$
$$p_2 = f_2\left(\gamma, p_0, f, d_{ij}, V_n\right) \tag{2}$$

− the dynamic error

T_u – the settling time of the output signal for the desired value of d (d – the permitted amplitude of the output signal) e. g.

$$\begin{aligned} &\text{for } t = T_u, \quad \left| p_{20} - p_2(t) \right| = d \\ &\text{for } t \neq T_u, \quad \left| p_{20} - p_2(t) \right| < d \end{aligned} \tag{3}$$

$f_{gr} = \dfrac{1}{T_{gr}}$ – the limiting frequency of the output signal (the frequency above which the amplitude of the output signal Δp_2 is less than the permitted value of d) e.g.

$$\text{for } f > f_{gr} \quad \Delta p_2 < d \tag{4}$$

As a measure of filtration quality, many researchers use the quality indicator defined as $QI = T_u \cdot \omega_{gr} = T_u \cdot 2\pi f_{gr} = 2\pi \dfrac{T_u}{T_{gr}}$.

2. Mathematical model of the experimental systems

Analysis of the functioning of pneumatic filters at moderate pressure (20 ÷ 100 kPa) leads to a mathematical description of the thermodynamic processes of a variable mass of air, based on mass-energy balance equations Gerc E.W. (1973). If we assume that the cascade is built from a series of chambers joined together by resistors, we can obtain a generalised mathematical description, which clearly depicts the modelled system and enables analysis and modification of the system. Changes in air pressure and temperature in the k(h chamber of fixed volume V_k are defined by the following equations:

$$\begin{aligned} \frac{dp_k}{dt} &= \frac{\kappa \cdot R}{V_k}\left[\sum_{i=1}^{n} \Theta_i \dot{m}_{ik} - \Theta_k \sum_{i=1}^{n} \dot{m}_{kj}\right] \\ \frac{dp_k}{dt} &= \frac{\kappa \cdot R}{V_k}\left[\sum_{i=1}^{n}\left(\kappa - \frac{\Theta_k}{\Theta_i}\right)\Theta_i \dot{m}_{ik} - (\kappa - 1)\Theta_k \sum_{i=1}^{n} \dot{m}_{kj}\right] \end{aligned} \tag{3}$$

where: i, j are the indices of the parameters of the air before and after the chamber, $\Theta_i, \Theta_k\,[\mathrm{K}]$ – the air temperature before the resistor and in the chamber, $R = 287\,[\mathrm{N \cdot m/kg \cdot K}]$ – the gas constant of air, κ –politropy coefficient, $\dot{m}_{ik}, \dot{m}_{kj}\,[\mathrm{kg/s}]$ – mass flow of air through the resistor before and after the k-*th* chamber.

The mass flow characteristic of the pneumatic resistors are defined by the equation:

$$\mathrm{M}_{ij} = \alpha_{ij} \cdot f_{ij} \sqrt{\frac{2}{R\Theta}} \sqrt{\frac{\kappa}{\kappa - 1}\left(\frac{2}{\kappa + 1}\right)^{\frac{2}{k+1}}} \sqrt{\frac{1}{b_i}}\, p_i \sqrt{1 - \frac{\left(\frac{a_j}{a_i} - B_{ij}\right)^2}{\left(1 - B_{ij}\right)^2}}. \tag{4}$$

In building a mathematical model for a chosen cascade structure, it is important to take into account the correct direction of air flow from the i*th* to the k*th*, or from the k*th* to the i*th* chamber, from the j*th* to the k*th* or from the k*th* to the j*th* chamber. In this work, single and dual chamber pneumatic cascade systems were investigated (parametric identification and optimization). The following notation is used in equations:

$$a_1 = \frac{p_1}{p_0},\ a_2 = \frac{p_2}{p_0},\ b_1 = \frac{\Theta_1}{\Theta_0},\ b_2 = \frac{\Theta_2}{\Theta_0},$$

$$\alpha_{01} = \alpha_{10}, \alpha_{12} = \alpha_{21}, \alpha_{01K} = \alpha_{10K}, \alpha_{02K} = \alpha_{20K},$$

$$f_{01} = f_{10}, f_{12} = f_{21}, f_{01K} = f_{10K}, f_{02K} = f_{20K}.$$

For a single-chamber pneumatic cascade (Fig. 3) the equations take the form:

$$\begin{aligned} \frac{\mathrm{d}a_1}{\mathrm{d}t} &= \frac{\kappa R \Theta_0}{V_1 p_0}\left(\sum_i D^i_{p_1} - \sum_j O^j_{p_1}\right) \\ \frac{\mathrm{d}b_1}{\mathrm{d}t} &= \frac{R\Theta_0 bl1}{V_1 p_0 a_1}\left(\sum_i D^i_{\Theta_1} - \sum_j O^j_{\Theta_1}\right) \end{aligned} \tag{5}$$

where:

$D^i_{p_1}$, $D^i_{\Theta_1}$ – intakes to the first chamber (in accordance with equation (3)),

$O^j_{p_1}$, $O^j_{\Theta_1}$ – outflows from the first chamber (in accordance with equation (3)).

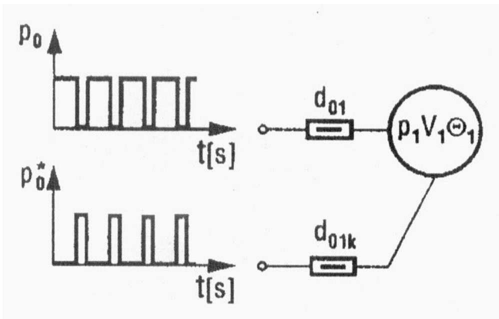

Fig.3. Single chamber pneumatic cascade

For a dual-chamber pneumatic cascade (Fig 4) the equations take the form

$$\begin{aligned} \frac{\mathrm{d}a_1}{\mathrm{d}t} &= \frac{\kappa R \Theta_0}{V_1 p_0}\left(\sum_i D^i_{p_1} - \sum_j O^j_{p_1}\right) \\ \frac{\mathrm{d}a_2}{\mathrm{d}t} &= \frac{\kappa R \Theta_0}{V_2 p_0}\left(\sum_i D^i_{p_2} - \sum_j O^j_{p_2}\right) \\ \frac{\mathrm{d}b_1}{\mathrm{d}t} &= \frac{R\Theta_0 b_1}{V_1 p_0 a_1}\left(\sum_i D^i_{\Theta_1} - \sum_j O^j_{\Theta_1}\right) \\ \frac{\mathrm{d}b_2}{\mathrm{d}t} &= \frac{R\Theta_0 b_2}{V_2 p_0 a_2}\left(\sum_i D^i_{\Theta_2} - \sum_j O^j_{\Theta_2}\right) \end{aligned} \tag{6}$$

where:

$D^i_{p_1}$, $D^i_{p_2}$, $D^i_{\Theta_1}$, $D^i_{\Theta_2}$ – intakes to the first and second chambers (according to (3)),

$O^j_{p_1}$, $O^j_{p_2}$, $O^j_{\Theta_1}$, $O^j_{\Theta_2}$ – outflows from the first and second chambers (according to (3)).

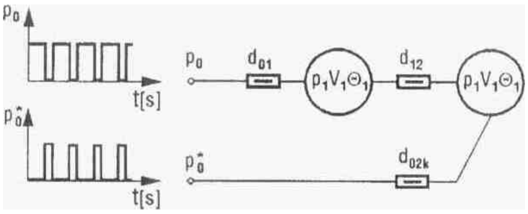

Fig.4. Dual chamber pneumatic cascade

The values (α_{ij}, B_{ij}) of the flow characteristics of the pneumatic resistors were obtained by identification [3]. The vector of the identifying parameters has the form:

- for a single chamber cascade

$$\bar{P} = \left[\alpha_{01}, \alpha_{01K}, B_{01}, B_{01K}\right]$$

- for a dual chamber cascade

$$\bar{P} = \left[\alpha_{01}, \alpha_{12}, \alpha_{02K}, B_{01}, B_{12}, B_{02K}\right]$$

The quality indicator of identification e , which is a measure of the agreement of the properties of the system as obtained experimentally and analytically is obtained using the least squares method in the form:

$$\varepsilon = \int_0^T h(t)\,\mathrm{d}t = \int_0^T e^T(t)\cdot\mathbf{1}\cdot e(t)\,\mathrm{d}t \tag{7}$$

where:

$h(t)$ – identification error function,

$e(t)$ – difference between the output signal of the system (experiment) and the output signal of the mathematical model (calculation) dependent on the vector $\bar{P}$ of identified parameters,

1 – unit matrix.

3. Dynamic Error of the Filter

Although a measure of the dynamic error of the filter is the minimum of the quality indicator (QI), the interpretation of the numerical value of this indicator, or the comparison of the indicators of different pneumatic systems is problematic. Therefore in this work, the factors (ω_{gr}, T_n) of the indicator are presented in the form of two interdependent graphs: $T = f(SP)$, $\omega_{gr} = f(SP)$ where: SP are system parameters. The results of the investigation of the single chamber system (Fig. 3, $d_{01K} = 0$) are shown in Fig. 5, and the dual chamber system (Fig. 4, $d_{02K} = 0$) in Fig. 6. From the graphs, we can state that to obtain $\omega_{gr} = 5$ rad/s, for example, it is necessary to use a dual-chamber cascade with parameters e.g. $V_1 = V_2 = 100\,\text{cm}^3$, $d_{01} = 0.3\,\text{mm}$, $d_{12} = 0.42\,\text{mm}$, which gives a settling time of $T_u = 7.4\,\text{s}$.

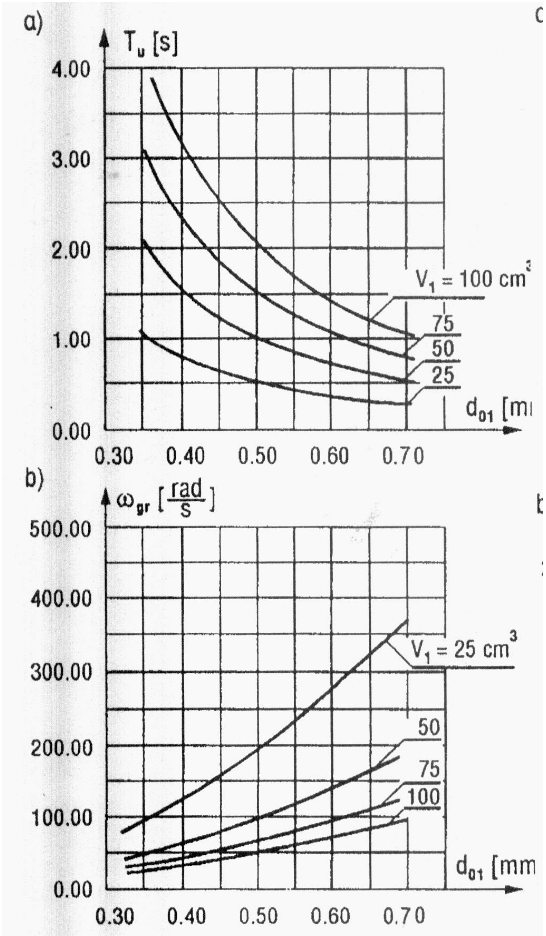

Fig.5. Dependency of dynamic error parameters on the constructional parameters of a single chamber cascade: a) $T_u = f(d_{12}, V_1)$;
b) $\omega_{gr} = f(d_{12}, V_1)$

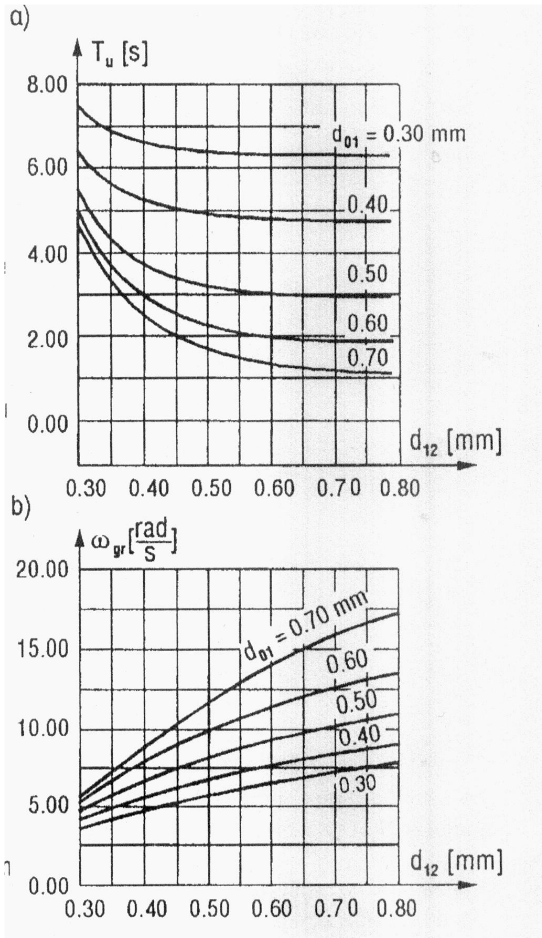

Fig.6. Dependency of dynamic error parameters on the constructional parameters of a dual chamber cascade: $V_1 = V_2 = 100\,\text{cm}^3$:
a) $T_u = f(d_{12}, d_{01})$;
b) $\omega_{gr} = f(d_{12}, d_{01})$

4. Static Characteristic of the Filter

In impulse devices working in the pressure range $20 \div 100\,\text{kPa}$, pneumatic resistors give a turbulent flow. Hence the static characteristic of the filter is nonlinear in both the pulse-width coefficient γ and the amplitude p_0 of the input signal. The results of research by the author and others ViktorowV.V. (1982) indicate that it is necessary to optimize the parameters of the filter in order to maximize the linearity of the static characteristic. The method of least square differences is used. With this criterion, the minimum of the sum of the square differences between the pressure values obtained from the mathematical simulation and the values defined by the straight line ($ay + b$), where:

$$a = \frac{\sum_{i=1}^{n} \gamma_i p_i - \frac{1}{n}\sum_{i=1}^{n} \gamma_i \sum_{i=1}^{n} p_i}{\sum_{i=1}^{n} \gamma_i \gamma_i - \frac{1}{n}\sum_{i=1}^{n} \gamma_i \sum_{i=1}^{n} \gamma_i} \tag{8}$$

The deciding variables in the function are the diameters of the resistors $\left(d_{ij}\right)$ and the amplitude of the input signal p_0^*.

Fig. 7 shows the static characteristic of a single-chamber pneumatic filter (Fig. 3) in which the optimized parameters are:

- line 1 – $d_{01} = 0.60\,\text{mm}$, $d_{01K} = 0.43\,\text{mm}$, $p_0 = 100\,\text{kPa}$, $p_0^* = 93.76\,\text{kPa}$,
- line 2 – $d_{01} = 0.60\,\text{mm}$, $d_{01K} = 0.48\,\text{mm}$, $p_0 = p_0^* = 100\,\text{kPa}$.

In both cases $V_1 = 100\,\text{cm}^3$.

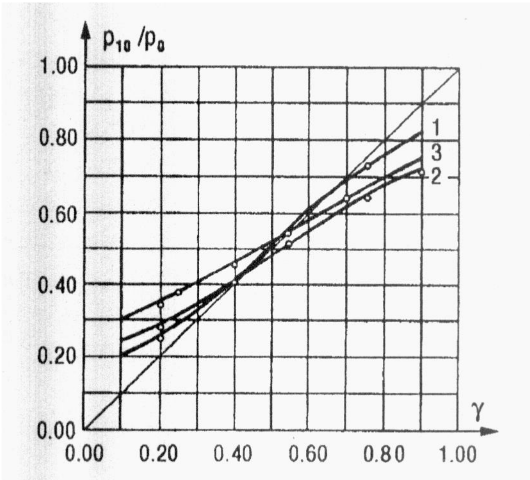

Fig.7. Static characteristic of single chamber filter after optimization

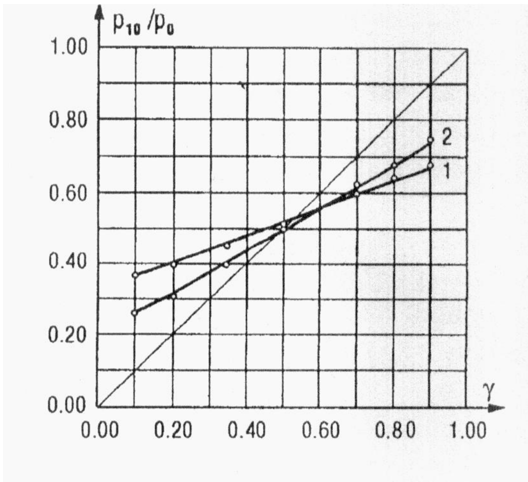

Fig. 8. Static characteristic of dual chamber filter after optimization

Fig. 8 shows the static characteristic of a dual chamber pneumatic filter (Fig. 4) in which the optimized parameters are:

- line 1 – $d_{01} = 0.70\,\text{mm}$, $d_{12} = 0.78\,\text{mm}$, $d_{02K} = 0.29\,\text{mm}$, $p_0 = p_0^* = 100\,\text{kPa}$,
- line 2 – $d_{01} = 0.70\,\text{mm}$, $d_{12} = 0.78\,\text{mm}$, $d_{02K} = 0.43\,\text{mm}$, $p_0 = 100\,\text{kPa}$, $p_0^* = 92.10\,\text{kPa}$,
- line 3 – $d_{01} = 0.70\,\text{mm}$, $d_{12} = 0.69\,\text{mm}$, $d_{02K} = 0.41\,\text{mm}$, $p_0 = p_0^* = 100\,\text{kPa}$.

In all cases $V_1 = V_2 = 100\,\text{cm}^3$.

It is worth noting that, depending on the permitted range of the parameters (limits on the upper and lower values) different values of the parameters are obtained, and also different shaped characteristics.

5. Conclusion

The results presented in this work of research on the use of single and dual chamber cascades in the filtration of periodic pneumatic signals demonstrate the possibility of shaping their properties (quality, static characteristic) by:

- appropriate choice of the geometric dimensions of the resistors,
- selection of the method of application of the input signal to the chambers,
- appropriate choice of amplitude of the correction signal ($p_0^* / p_0 \approx\sim 0.93$).

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