

## **BIAXIAL STRENGTH TEST OF FLAT GLASS**

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**Summary:** *One of the important physical characteristics of glass is its strength. The biaxial flexural strength test is used for strength evaluation usually. The biaxial test offers several advantages over uniaxial test because it critically explores surface flaws – as it does not matter in which orientation the crack lies. However, it does minimize the volume or surface area investigated and also the edge effect. In the paper modification of strength testing process is suggested based on both ball-on-three-balls and ball-on-ring-of-balls principle. This work demonstrates that the biaxial flexure test can be used for the testing of glass.*

### **1. Introduction**

Conventional strength tests in the sphere of mechanical engineering use for description usually the failure behaviour of defined specimen under simple stress conditions (mostly under uniaxial stresses). However, mechanical loading applied to a real product often induces more general state of stress. To be able to analyse the strength and the deformation behaviour under multiaxial stress conditions special experimental methods were developed. For brittle materials strength testing the different modifications of biaxial test are used frequently. The wide variety of biaxial test assemblies is possible. The attention has been paid to the ball-on-three-balls and the ball-on-ring-of-balls tests in this work.

### **2. Materials and methods**

The biaxial test offers several advantages over three-point bending because it critically explores surface flaws - it does not matter which orientation the crack lies in. In uniaxial strength measurements such as the 3 or 4 point bending tests, the cracks, which are parallel to the tensile direction (parallel to the longitudinal direction) in the bar type specimen, do not reduce the fracture strength of the specimen. However, in a biaxial strength measurement, more reliable fracture strength can be measured because cracks existing in the disc type specimen lower the strength, independently of the directions of the crack arrangement. The maximum tensile stress is induced at the bottom center of the specimen in a biaxial strength test so that edge polishing of the specimen is not necessary, whereas it is very important for bar specimens in the uniaxial strength test.

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Tab. 1: Typical testing assemblies for biaxial strength test of discs

Axis-symmetric stress distributions	Non axis-symmetric stress distributions
Ring on ring	Punch on three balls
Ball on ring	Ball on three balls
Punch on ring	Three balls on three balls
Ball with flat on ring	Ring of balls on ring of balls
	Ball on ring of balls

## 2.1. Biaxial strength test modification

The wide variety of biaxial test assemblies is possible (Tab. 1); among the most frequently used belong:

- **the ring-on-ring test (RR)** the specimen in the shape of the disc is supported by a ring and it is loaded from the opposite side by another, smaller concentric one (Fig. 1a),
- **the ring-on-ring-of-balls test (RRB)** the supporting ring is in this case replaced by a ball bearing ring (Fig. 1b). This design configuration reduces friction between disc and supporting construction.

Both of mentioned modifications results in well-defined state tension inside inner ring only providing to security ideal loading conditions. Therefore high accurate parallel samples are required. If this is not the case, a three-point contact between ring and sample will reach. Only at high loads a continuous line contact will be developed, resulting in the correct stress distribution.

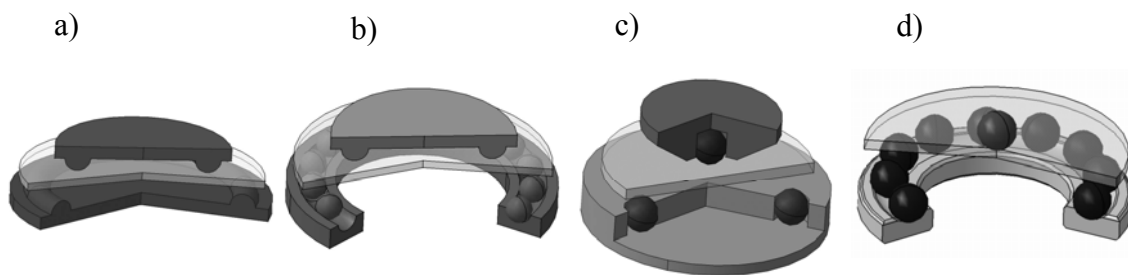


Fig. 1: Test modification: a) ring-on-ring test, b) ring-on-ring-of-balls test, c) ball-on-three-balls test, d) ball-on-ring-of-balls test

- **the ball-on-three-balls test (B3B)** the outer ring is replaced by three balls (Fig. 1c). This configuration results in statically exactly given conditions,
- **the ball-on-ring-of-balls test (BRB)** (Fig. 1d) the supporting ring is in this case replaced by a ball bearing ring and loading the central portion of the plate by the ball.

At these tests it is however relatively small effective surface tested. With increasing distance from the sample centre, the stresses quickly decline. The regions at greater distance from the centre thus spawn seldom sample breakdown.

In the paper modification of strength testing process is suggested based on both ball-on-three-balls and ball-on-ring-of-balls principle.

Measured equipment is placed into laboratory machine LLoyd LR50K plus that is used for putting and controlling of loading force. Construction details of measuring equipments for ball-on-three-balls and ball-on-ring-of-balls test are shown in Fig. 2.

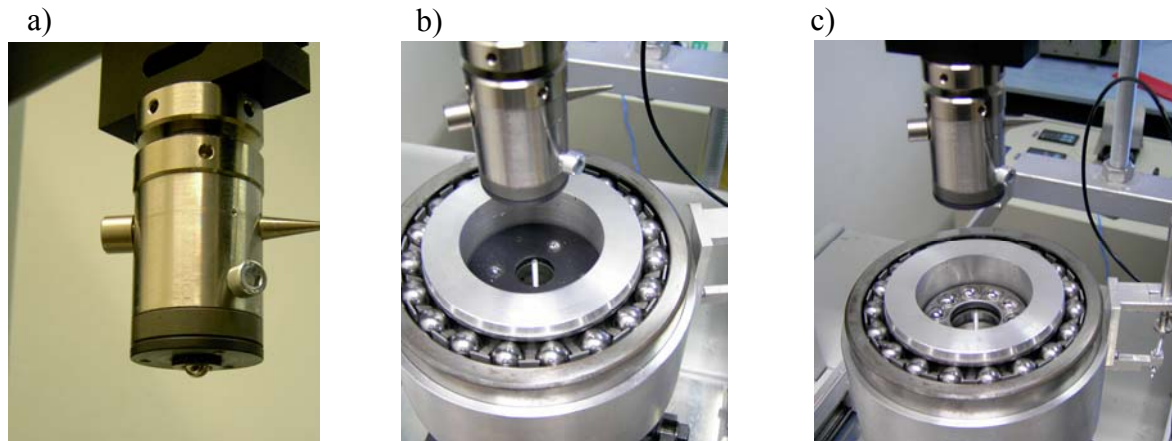


Fig. 2: Construction details of measuring equipment: a) upper jaw with loading ball, b) three balls support, c) ring of balls support

## 2.2. Preparation of the specimens

Strength tests with disc specimens were carried out with flat glass (used technology – traction system Fourcault). Samples of 50 mm diameter with a thickness of 2.5 mm were used. Twenty samples using the ball-on-three-balls method and further twenty by means of ball-on-ring-of-balls were tested. Balls of a ball bearing of 8 mm in diameter were used. The supporting balls were located on a circle with radius of 20 mm.

## 2.3. Fracture behaviour of glass samples

View of typical fracture forms by ball-on-three-balls test are shown in Fig. 3. In all fractured specimens, the fracture was initiated in the tensile surface plane underneath the loading ball. Failure with the threefold symmetry dominates, but in a few cases the disc breaks into four pieces.



Fig. 3: Fracture patterns of glass samples tested in the ball-on-three-balls test assembly

The ball-on-ring-of balls test of glass specimens showed that all specimens failed in the centre region. Discs breaks into eight or more pieces. It was found the number of fracture pieces increases with strength (Fig. 4).

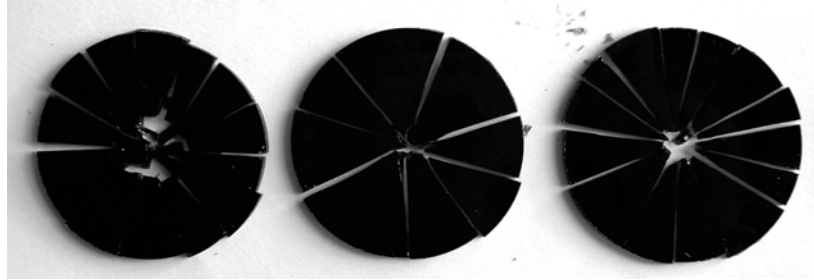


Fig. 4: Fracture patterns of glass samples tested in the ball-on-ring-of-balls test assembly

## 2.4. Calculation of biaxial strength

The biaxial flexural strength for ball-on-three-balls test and ball-on-ring-of-balls test is determined by the following relation:

$$\sigma_{\max} = \frac{3 \cdot F \cdot (1 + \nu)}{4 \cdot \pi \cdot t^2} \left[ 1 + 2 \ln \frac{R_a}{b} + \frac{(1 - \nu)}{(1 + \nu)} \cdot \left( 1 - \frac{b^2}{2 \cdot R_a^2} \right) \cdot \frac{R_a^2}{R^2} \right], \quad (1)$$

Where  $F$  is applied load;  $t$ , sample thickness;  $\nu$ , Poisson's ratio of the disc material;  $R$ , radius of the disc;  $R_a$ , support radius;  $b$ , contact radius.

## 3. Results

Two-parametric Weibull fracture probability distribution for evaluation results was applied. The Weibull parameters (characteristic strength  $\sigma_o$  and Weibull modulus  $m$ ) are compiled in Table 2.

Tab. 2: Weibull parameters

TEST	$\sigma_o$ [MPa]	$m$
Ball on three balls	220.3	4.9
Ball on ring of balls	217.6	5.1

### 3.1. Weibull statistics

A Weibull regression analysis computer program was used on the strength data (Fig. 5) to find the Weibull modulus  $m$  and the strength levels at a 5% probability of failure in order to assess materials reliability. The description of the Weibull distribution is given by the formula:

$$P_f = 1 - \exp\left[-(\sigma_f / \sigma_o)^m\right], \quad (2)$$

where  $P_f$  is probability of failure;  $\sigma_f$ , strength at a given  $P_f$ ;  $\sigma_o$ , characteristic strength;  $m$ , Weibull modulus.

By ranking statistics, the strength have been ordered from weakest to strongest with each assigned a probability of failure based on ranking

$$P_{(\sigma)} = \frac{n}{N+1}, \quad (3)$$

where  $N$  is the number of specimens ( $N = 20$ ). Equation (2) can be rearranged to a straight line by transforming the variables  $P_f$  and  $\sigma_f$  to:

$$1/(1 - P_f) = 1/\exp\left[-(\sigma_f / \sigma_o)^m\right]. \quad (4)$$

Taking integral logs:

$$\ln[1/(1 - P_f)] = (\sigma_f / \sigma_o)^m, \quad (5)$$

$$\ln \ln[1/(1 - P_f)] = m \ln \sigma_f - m \ln \sigma_o. \quad (6)$$

Therefore a plot of  $\ln \ln[1/(1 - P_f)]$  vs  $\ln \sigma_f$  will yield a slope with value  $m$  and an intercept  $m \ln \sigma_o$ .

The cumulative probability versus the fracture strength is shown in Fig. 6.

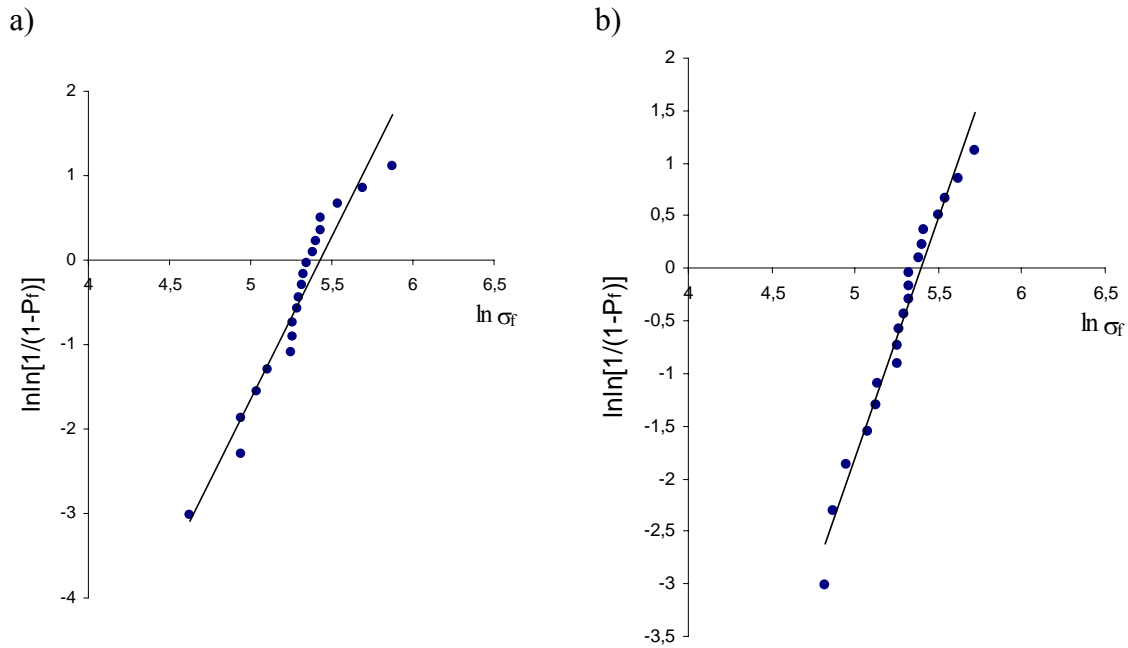


Fig. 5: Weibull plots: a) ball-on-three-balls test, b) ball-on-ring-of-balls test

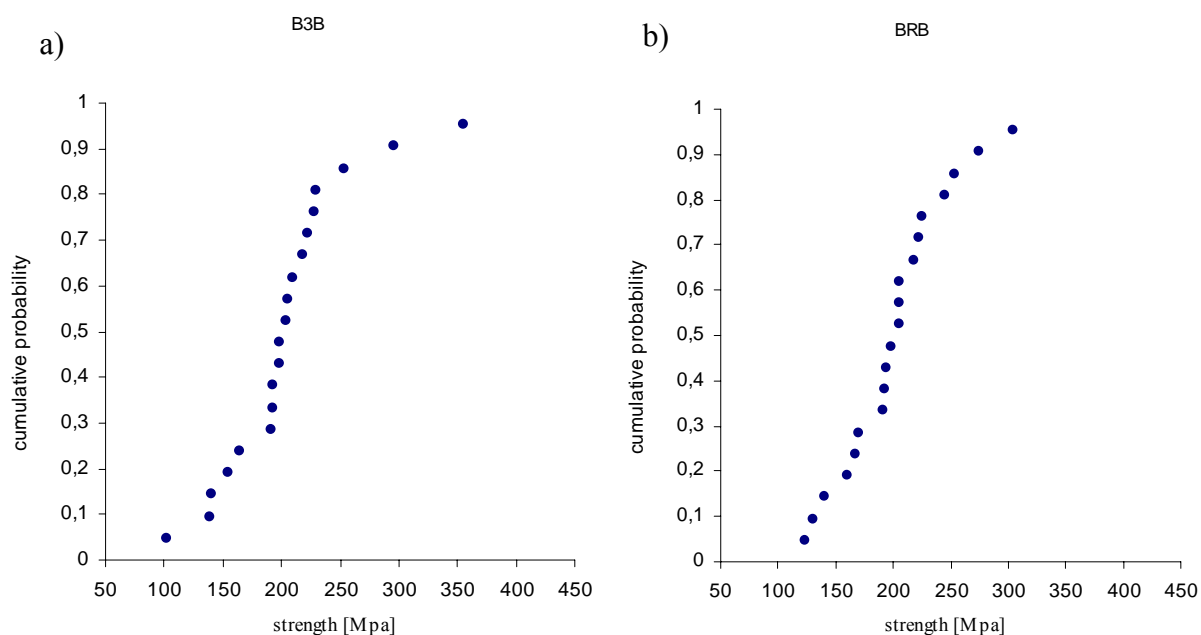


Fig. 6: Plots of fracture cumulative probability: a) ball-on-three-balls test,  
b) ball-on-ring-of-balls test

#### 4. Conclusions

The realized analyses show that the main advantages of biaxial strength test (ball-on-three-balls and ball-on-ring-of balls) are unambiguous distribution of stresses generated in the specimen analysed, simple testing equipment and specimen shape. Therefore the stress distribution in the sample can be fairly, quickly and relatively accurately calculated. Main disadvantage of tests significant decline of pressure with distance from sample centre is. So probability, that any defect causes sample fracture declines with distance from its centre markedly. The difference in strength between the ball-on-three-balls and the ball-on-ring-of balls tests was explained on the basis of a simple Weibull analysis and small number of specimens. This work demonstrates that the biaxial flexure tests (ball-on-three-balls and ball-on-ring-of balls) can be used for the testing of glass.

#### References

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