National Conference with International Participation

ENGINEERING MECHANICS 2008

Svratka, Czech Republic, May 12 – 15, 2008

COMPUTATIONAL AND EXPERIMENTAL ANALYSIS THE INTERACTION OF AN ELASTIC BODY WITH AN LIQUID ACCORDING THE LARGE DISPLACEMENT AND STRAINS

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Summary: *This contribution is focused on the analysis of dynamic behavior of elastic body moving in liquid. In many technical applications this motion is with large displacements. Some technical applications can be vibration of blades or rotors in centrifugal pumps or water turbines. But on other hand presented approach has more general application. In general case dynamic behavior are the modal behavior, steady state response and nonstationary response. Because of are assumed the large displacements, this analysis is nonlinear and modal behavior depends on a parameters. There is very difficult or impossible to do this analysis using commercial programme systems. It is caused by limited number of boundary conditions for contact between body and liquid in these systems. In this contribution is presented the mathematical model of a new type of boundary conditions which allowed the modal analysis and computing the steady state response. In principle this analysis is if the frequency domain. It is necessary provided some testing, because this approach is new. For this case, the curvilinear co-ordinates were chosen. The Bézier body was chosen for the description of geometrical configuration and also for approximation the solution. MATLAB programme code was chosen for software processing.*

Nomenclature

m_{ij}, b_{ij}, k_{ij} - elements of local matrices of mass, damping and stiffness, u_i - i - base function (see appendix), S_E - area of element with pressure lay - out, $\mathbf{p},\boldsymbol{\sigma}$ - vectors of pressure and viscous forces in the i direction, reached on surface unit, p - pressure, Π_{ij} - nonreversible stress tensor, n_i, n_j - one - unitary vector of external normal line element with regard to liquid, f - function dependent on $\mathbf{p}$ and $\boldsymbol{\sigma}$ as a consequence of FEM, η_1 - dynamic viscosity, c_i - velocity, x_i, z - coordinates, q_l - time function for l^{th} shape of vibration, v_{il} - i^{th} deformation parameter for l^{th} shape of vibration, $S, \Gamma_1, \Gamma_2, \Gamma_3$ - denotation of surfaces enclosing liquid volume, $\alpha_{il}, \boldsymbol{\alpha}, \boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}$ - velocity functions, $\beta_{1_{il}}, \beta_{2_{il}}, \boldsymbol{\beta}, \boldsymbol{\beta_1}, \boldsymbol{\beta_2}, \boldsymbol{\beta_3}$ - pressure

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functions, δ - Dirac function, t, τ - time, ρ - density, $\mathbf{\bar{A}}, \mathbf{\bar{B}}, \mathbf{\bar{C}}, \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{0}$ - matrices, $\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3$ - right side vectors, R_0, R_1, R_2, L_0, L_1 - geometrical dimensions, $\mathbf{r}$ - state vector, $\mathbf{r}(u, v, w) = \mathbf{r}$ - value of geometrical or unknown value by the Bezier body application, $\mathbf{r}_{ijk}$ - value of geometrical or unknown value in control points by the Bezier body application, a^k - contra variant velocity component, u^i, u, v, w - curvy – linear co – ordinates, $B_i^n.$ – Bernstein multinomial, ω_{ijk} - weight of values in control points by the rational Bézier body application.

Keywords:

Fluid structure interaction, finite element method, added mass, added damping, experimental analysis, and finite element method

1. Introduction

A solution a problem of fluid - elastic structure interaction belongs to most difficult in mechanics. From the point of view, they are three basic tasks. As the first it is the eigen value problem, as the second can be the solution of steady state response do to harmonic (periodic) excitation and at last the solution of unsteady state response (computational simulation).

It is evident in the last time a takeovers and mergers the computational packages, where were interested only in the individual and limited parts of mechanics. As a sample is the merging the ANSYS (solid mechanics) and FLUENT (hydromechanics). This process is inevitable and makes the development and creation a new mathematical and computational models and algorithms of solution.

It is necessary to have two different types of mesh by the solution the fluid – elastic structure interaction. One mesh is for a solid or structure and the second one for a fluid or surroundings. According the solving problem is almost already necessary to do some changing of mesh during the solution. It is evident, that this step leads to increasing the computational time consuming. In substance they are three basic types of changes of mesh

a. Layering

b. Smoothing

c. Remeshing

When are used the commercial programme package, especially ANSYS and FLUENT, these approaches are in detail presented in (FLUENT 6.2 1988). General overview of methods of computational modelling is presented by Axise (2007).

Problem of fluid structure interaction needs the different approach to the computational modelling. General has bad numerical stability and is very time consuming. That is why a lot of scientists deal with the idea how to achieve better numerical stability and shorter tome of calculation.

Daneshmand & Niroomandi (2006) presented a ne method to simulation fluid – structure interaction. It is based on the use of a meshless technique named as Natural Element Method or natural neighbor Galerkin method in which the natural neighbor interpolation is used for the construction of test and trial function. The eigen value problem arising from the computation of the free vibrations of a coupled fluid – structure system is solved. Displacement variables for both the solid and the fluid domains are used, but the fluid displacements are written as gradient of potential function. One classical example is considered: free vibration of a flexible cavity filled with liquid.

One of the possibilities how to achieve this, is presented by Stein et al. (2007). In computation of fluid-structure interactions, is used mesh update methods consisting of mesh-moving and remeshing – as - needed. When the geometries are complex and the structural displacements are large, it becomes even more important that the mesh moving techniques are designed with the objective to reduce the frequency of remeshing. To that end, is present here mesh moving techniques where the motion of the nodes is governed by the equations of elasticity, with selective treatment of mesh deformation based on element sizes as well as deformation modes in terms of shape and volume changes. It is also presented some results from application of these techniques to a set of two-dimensional test CASE.

Legay & Kölke (2006) presented new approach to the solution, where velocity and pressure are solved on base the weak formulation of the governing equations of viscous and incompressible fluid flow (Navier – Stokes equations) is discretized by finite space – time elements using discontinuous Glerkin scheme for time integration. To capture the occurring moving discontinuities from embedding a thin solid body into the flow field, a locally enriched space time finite element method is applied to ensure a fluid mesh independent from the current configuration of the structure. Based on the concept of the extended finite element method, the space – time approximation of the pressure is enriched to present strongly discontinuous solution at the position of the structure. The similar approach is presented by Kölke & Legay (2006). A numerical method for investigation challenging interaction phenomena of viscous fluid flow and flexible structures of negligible thickness like membranes and plates on a topologically fixed fluid discretization is presented. Since the formulation of fluid, structure and coupling conditions uniformly uses velocities as unknown and the integration of the governing equations is performed on the deformed space – time mesh, the realization of a strong coupling of the physical domains becomes very comfortable and results in a monolithic system.

Sigrist et al. (2004) presented an approach to him solution of the fluid structure interaction with a finite element discretization or with modal approach. The structure problem is modeled in the CFD code with various FORTRAN subroutines. Fluid is solved using finite volume discretization. For the achieving better numerical stability the special algorithm for the discretization in time and spatial domains is suggested.

Giannopapa & Papadakis (2004) presented the first stage of development of such a method, in which the solid equations are formulated so as to be solved for velocity and pressure i.e. for the same unknowns as the ones for the liquids equations.

In many cases the governing of the fluid are expressed in an Arbitrary – Lagrangian – Eulerian (ALE) frame reference that in a natural way treats the complex movement of the interface between the fluid and the structure without the need for surface tracking procedures. Also Lund et al. (2004) presented approach for analysis and semi – analytical design sensitivity analysis of time dependent fluid – structure interaction problem discretized by finite element methods. The aim of the method is to provide a general design tool than can be used for both analysis and synthesis of fluid - structure interaction where the dynamic interaction of a flexible structure and a viscous flow is in focus.

In immersed interface methods, solid in a fluid are presented by Sheng & Wang (2007), by singular forces in the Navier – Stokes equations, and flow jump conditions induced by the singular forces directly enter into numerical schemes. The article is focused on the implementation of an immersed interface method for simulation fluid – solid interaction in the 3D space. The method employs the method of control volumes for the spatial discretization and method of Runge Kutha the 4 order for the time integration. The FFT – based Poisson solver for the pressure Poisson equation is used. A fluid – solid interface is tracked by Lagrangian markers.

A Lagrangian model for the numerical simulation of fluid – structure interaction problems is proposed by Antoci et al. (2007). In the method both fluid and solid phases are described by smoothing particle hydrodynamics: fluid dynamics is studied in the inviscid approximation, while solid dynamics is simulated through an incremental hypoelastic relation. The interface condition between fluid and solid is enforced by a suitable term, obtained by an approximate smoothed particle hydrodynamics evaluation of a surface of fluid pressure. The method is validated by comparing numerical results with laboratory experiments where an elastic plate is deformed under the effect of a rapidly varying fluid flow.

The newly developed immersed object method is presented by Tai et al. (2007). Parallel computation of unsteady incompressible viscous flows around moving rigid bodies using an immersed object method with overlapping grids is solved. Approach to parallel calculation is presented by Tai et al. (2005). Newly is extended for 3D unsteady flow simulation with fluid – structure interaction, which is made possible by combining it with a parallel unstructured multigrid Navier – Stokes solver using a matrix – fee implicit dual time stepping and finite volume method. An object mesh is immersed into the flow domain to define the boundary of the object. The advantage of this is that bodies of almost arbitrary shapes can be added without grid restructuring, a procedure which is often time – consuming and computationally expensive.

How is evident from this research study, all tasks the fluid - elastic structure interaction are solved as coupled. To achieve better numerical stability and shorter time computing they are used special algorithms.

Another of possibilities for achievement this is application of the new type of boundary conditions for contact between continuum and liquid. Approach is based on the application the expansion the solution according to eigen shapes of continuum vibration. The authors are many years interested in the possibilities, how is possible to separate the continuum and liquid from each other. They proved that this is possible for the solid body. The summarizing results more then sixth year's research are presented by Pochylý & Malenovský (2004) and also Malenovský & Pochylý (2004). The possibility for separation is based on the approximation of solution for velocity and pressure functions in the form of convolutory integrals. In this contribution is presented application on an elastic continuum. For simplicity and possibility of software performing was chosen cantilever beam with circular cross section vibrating in water. The objective is to determine the expressions for local matrices of added mass and damping of liquid and suggest algorithm for solution the elastic structure – liquid interaction.

Similar model sample is presented by Levy & Wilkinson (1976). Vibration a shaft in water is solved as coupled problem. Authors are mainly focused on the determination of added mass. Only the potential flow is taken into account and the water is considered as ideal. The finite element method for the both structure and liquid is used.

Introductory study to the solution of this problem is presented by Pochylý & Malenovský (2008). In this contribution are derived the expressions for added mass and damping. General approach to the analysis of coupled vibrations a beam which is submersed in a liquid is presented by Pochylý & Malenovský (2008). The both ideal and real liquid is taken into account. The expressions for added mass and damping are derived from the point of view of application FEM.

2. Mathematical model

The way in order to draw up the mathematical and computational model will be demonstrated on the vibration of the bar in liquid. It is possible to generalize it to the case of vibration of any elastic continuum in liquid, eventually in any surrounding. The target is to determine the relations of local matrices of additional effects by the liquid. The motion equation for the finite element of the bar – the free undamped vibration in liquid has the form:

$$m_{ij} \ddot{r}_j + b_{ij} \dot{r}_j + k_{ij} r_j = -\int_{S_e} u_i \, f dS_e \tag{1}$$

and also according application the FEM is

$$\begin{aligned} (\mathbf{p})_i &= p n_i \\ (\boldsymbol{\sigma})_i &= \Pi_{ij} n_j \end{aligned} \tag{2}$$

The above mentioned nonreversible stress tensor for uncompressible liquid has the form:

$$\begin{aligned} \Pi_{ij} &= 2\eta_1 c_{ij} \\ c_{ij} &= \frac{1}{2}\left(\frac{\partial c_i}{\partial x_j} + \frac{\partial c_j}{\partial x_i}\right) \end{aligned} \tag{3}$$

The principle of the solution is based on the modal transformation of the bar. Let’s extend the state vector of the bar with the help of eigenvectors on this principle. Then, with this presumption, the following is valid:

$$r_i(z,t) = v_{il}(z) q_l(t) \tag{4}$$

Further, it is important to define the boundary conditions for liquid. On the boundaries of the area, filled with liquid, the following is valid:

$$\begin{aligned} &S : c_i = \dot{r}_i \\ &\Gamma_1 : p = 0 \\ &\Gamma_2 : c_i = 0 \\ &\Gamma_3 : c_i = 0 \end{aligned} \tag{5}$$

Establishing new defining relations for velocities and pressure in the form of convolutory integrals is a very important step for drawing up a mathematical model. With establishing velocity and pressure functions depending on the normal bar coordinates it is possible to separate continuum and liquid movements. The velocity of liquid and the pressure in an arbitrary place are defined by these relations:

$$
\begin{aligned}
p &= \int_0^t \beta_k (t-\tau) q_k^{\bullet}(\tau)\, d\tau \\
c_i &= \int_0^t \alpha_{il} (t-\tau) q_l^{\bullet}(\tau)\, d\tau
\end{aligned}
\tag{6}
$$

where α_{il}, β_k are new variables. The boundary condition for the liquid velocity on the bar surface using this presumption has the form:

$$
\int_0^t \alpha_{il}(t-\tau) q_l^{\bullet}(\tau) d\tau = v_{ik}(z) q_k^{\bullet}(t) \tag{7}
$$

Now we can analyze the separated liquid. It is necessary to mention in this context, that the shape of the continuum vibration doesn't only influence the velocity boundary conditions, but also by this the geometrical configuration is given in the given instant time. The modal features of the vibration continuum in the surroundings, in general, depend on the vibration amplitude for the given shape of vibration. Let's assume the liquid is real and incompressible. The initial Navier – Stokes and continuity equations for a linear task have this form:

$$
\begin{aligned}
&\rho \frac{\partial c_i}{\partial t} - \frac{\partial \Pi_{ij}}{\partial x_j} + \frac{\partial p}{\partial x_i} = 0 \\
&\rho \frac{\partial c_k}{\partial x_k} = 0
\end{aligned}
\tag{8}
$$

The following is valid in the finite dimension space using Bezier body features:

$$
\begin{aligned}
&\bar{\mathbf{A}}\boldsymbol{\alpha}^{\bullet} - \bar{\mathbf{B}}\boldsymbol{\alpha} + \bar{\mathbf{C}}\boldsymbol{\beta} = \mathbf{0} \\
&\bar{\mathbf{D}}\boldsymbol{\alpha} = \mathbf{0}
\end{aligned}
\tag{9}
$$

The solution is possible to suppose regarding the character of the differential equation in this form:

$$
\begin{aligned}
\boldsymbol{\alpha} &= \boldsymbol{\alpha}_1 \delta + \boldsymbol{\alpha}_2 (t) \\
\boldsymbol{\beta} &= \boldsymbol{\beta}_1 \delta^{\bullet} + \boldsymbol{\beta}_2 \delta + \boldsymbol{\beta}_3 (t)
\end{aligned}
\tag{10}
$$

We obtain after the substitution (10) into (9) and with respect to boundary conditions:

$$
\begin{aligned}
&\mathbf{A}\left(\boldsymbol{\alpha}_1 \delta^{\bullet} + \boldsymbol{\alpha}_2^{\bullet}\right) - \mathbf{B}\left(\boldsymbol{\alpha}_1 \delta + \boldsymbol{\alpha}_2\right) + \mathbf{C}\left(\boldsymbol{\beta}_1 \delta^{\bullet} + \boldsymbol{\beta}_2 \delta\right) = -\mathbf{f}_1 \delta^{\bullet} - \mathbf{f}_2 \delta \\
&\mathbf{D}\left(\boldsymbol{\alpha}_1 \delta + \boldsymbol{\alpha}_2\right) = -\mathbf{f}_3 \delta
\end{aligned}
\tag{11}
$$

We obtain the next equation comparing elements of the general Dirac function derivation in the motion equation and the general Dirac function derivation in the continuity relation:

$$\mathbf{A}\boldsymbol{\alpha}_1 + \mathbf{C}\boldsymbol{\beta}_1 = -\mathbf{f}_1$$
$$\mathbf{D}\boldsymbol{\alpha}_1 = -\mathbf{f}_3 \tag{12}$$

and the next:

$$-\mathbf{B}\boldsymbol{\alpha}_1 + \mathbf{C}\boldsymbol{\beta}_2 = -\mathbf{f}_2 \tag{13}$$

Hence the pressure function $\boldsymbol{\beta}_2$ is determined:

$$\boldsymbol{\beta}_2 = -\mathbf{C}^{+}\mathbf{f}_2 + \mathbf{C}^{+}\mathbf{B}\boldsymbol{\alpha}_1 \tag{14}$$

The velocity and pressure functions are calculated after substitution of the boundary conditions for the instant shape of vibration. Neglecting the influence of $\boldsymbol{\alpha}_2$ and $\boldsymbol{\beta}_3$ functions the following is valid for the velocities and pressures for the l^{th} shape of vibration:

$$c_i = \alpha_{1_{il}} q_l^{\bullet}$$
$$p = \beta_{1_l} q_1^{\bullet\bullet} + \beta_{2_l} q_1^{\bullet} \tag{15}$$

Now it is possible to proceed with composing local matrices of additional effects by the liquid. We obtain the next equation by the substitution of relations (15) for the velocities and pressure into equation (1) using eqs. (2) and (3):

$$m_{ij} r_j^{\bullet\bullet} + b_{ij} r_j^{\bullet} + k_{ij} r_j = -\int_{S_e} u_i \left[\beta_{1_l} q_l^{\bullet\bullet} + \beta_{2_l} q_l^{\bullet} + \eta_l \left(\frac{\partial \alpha_{1ml}}{\partial x_n} + \frac{\partial \alpha_{1nl}}{\partial x_m} \right) q_l^{\bullet} \right] dS_e \tag{16}$$

We obtain this formula with the presumption that the solution develops into the free vibration form i.e. the substitution of relation (4):

$$m_{ij} r_j^{\bullet\bullet} + b_{ij} r_j^{\bullet} + k_{ij} r_j = -\int_{S_e} u_i \beta_{1_l} v_{jl}^{-1} r_j^{\bullet\bullet} dS_e - \int_{S_e} u_i \left[\beta_{2_l} + \eta_l \left(\frac{\partial \alpha_{1ml}}{\partial x_n} + \frac{\partial \alpha_{1nl}}{\partial x_m} \right) \right] v_{jl}^{-1} r_j^{\bullet} dS_e \tag{17}$$

Hence it is obvious that local matrices of additional effects by the liquid for the l^{th} shape of vibration are determined by these equations:

$$m_{t_{ij}} = \int_{S_e} u_i \beta_{1_l} v_{jl}^{-1} dS_e$$
$$b_{t_{ij}} = \int_{S_e} u_i \left[\beta_{2_l} + \eta_l \left(\frac{\partial \alpha_{1ml}}{\partial x_n} + \frac{\partial \alpha_{1nl}}{\partial x_m} \right) \right] v_{jl}^{-1} dS_e \tag{18}$$

It is evident, that real liquid has influence on mass and damping.

3. Model sample

The model exercise is a cantilever bar in liquid. This model was chosen with regard to the possibility of comparing with an experiment. Scheme of this is on the Fig. 1. The geometrical properties are presented in table I. All the dimensions are in millimeters.

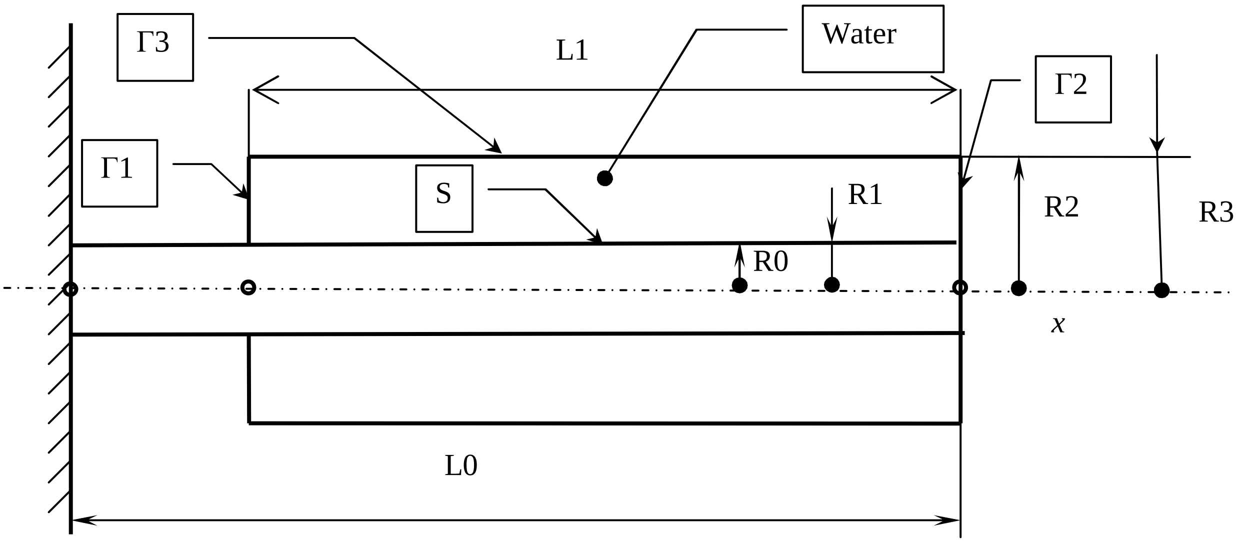

Fig. 1: Scheme of the model task

4. Experimental analysis

For verification the mathematical and calculating model was provided the experimental analysis. Experimental kit consists from the cantilever steel tube (beam) which is submersed into liquid, in this case into the water. Properties of the beam are presented in Tab. I.

Tab. I: Properties of the beam

Length L0 [mm]	1100
Inner radius R0 [mm]	16,85
Outer radius R1 [mm]	17,85

Vessel with the water is also a tube with different diameters. On the whole the 5 vessels was manufactured, but in this contribution are presented some results only from the 3 types. Properties of these vessels are evident from Tab. II. Material of vessels was chosen plexiglass. Through shine material was chosen for the possibility of control the water level to which is beam submersed. Outer vessel was stiffened during the experiment, as possible to achieved highest stiffness. This condition is necessary to have for the zero velocity of liquid on the outer surface. On the other hand, how was evident from the experiment, this requirement is very difficult to realize. The whole arrangement of the experimental kit was such as the free end of beam was close the bottom of outer vessel.

Tab. II: Properties of the vessels

Type of vessels	3	4	5
Inner radius R2 [mm]	35	50	105
Outer radius R3 [mm]	40	55	110

On Fig. 2 is a view on the vessels and on Fig. 3 is a general view on the assemblage of kit.

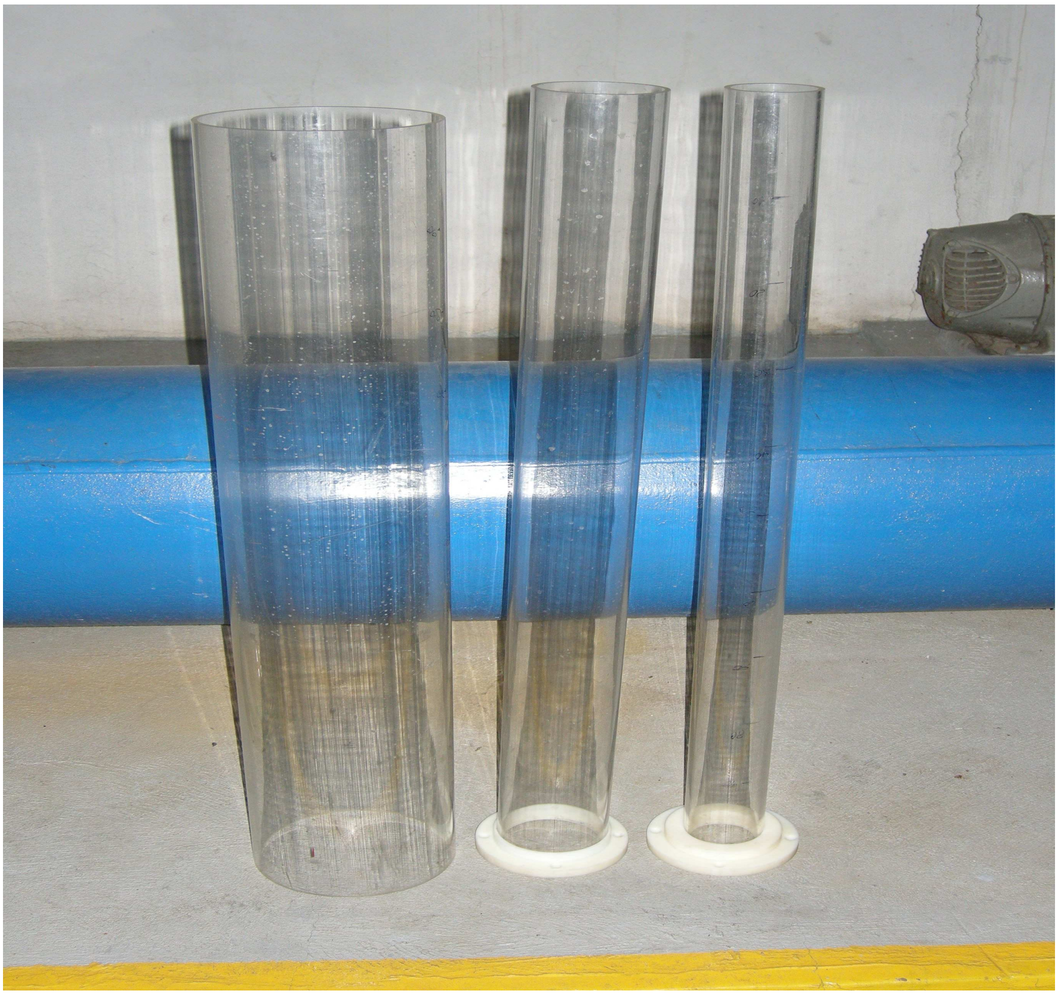

Fig. 2: View on the outer vessels

Fig. 3: General view on the assemblage of the kit

As an exciter was chosen the harmonic exciter type Brüel & Kjaer 4824, whereas the suitable point on the upper part of beam was chosen for excitation. It is evident from fig. 4 the connection between exciter and beam. How is also evident from this figure, the exciter is free – hanged, whereas the moved part of exciter is fixed with the excited beam. Direction of excitation was not changed during the experiment.

Fig. 4: Excitation the beam

The two acceleration sensors were chosen for recording the data of vibrating beam. The both are type Brüel & Kjaer 4374 and were clued to the inner surface of beam near the free end, before the closing the tube.

Evaluation of experiment

Only the eigen frequency of submersed beam in the water was evaluated during the first stage of experiment. The beam vibrates near centered position with relatively small amount of vibration. Only the results and comparison of this experiment and calculation (eigen frequency) are presented in this contribution. On the whole, the 11 measurements were provided with the height of water from 0 to 1000 mm with step 100 mm. For the first shape of vibration the frequency the bandwidth 10 - 30 Hz was chosen and 90 - 140 Hz for the second one and the both with linear changing the frequency of excitation (constant acceleration). The time of increasing the frequency was chosen 32 s for the both shapes of vibration. It means that the angular acceleration for the first shape is $\alpha = 0.625$ Hz/s and for the second shape is $\alpha = 1.562$ Hz/s. Only for illustration is on fig. 5 drawn response for the first shape of vibration in the same direction as was excitation. The height of water vas 1000 mm and tube Nr. 3 was chosen.

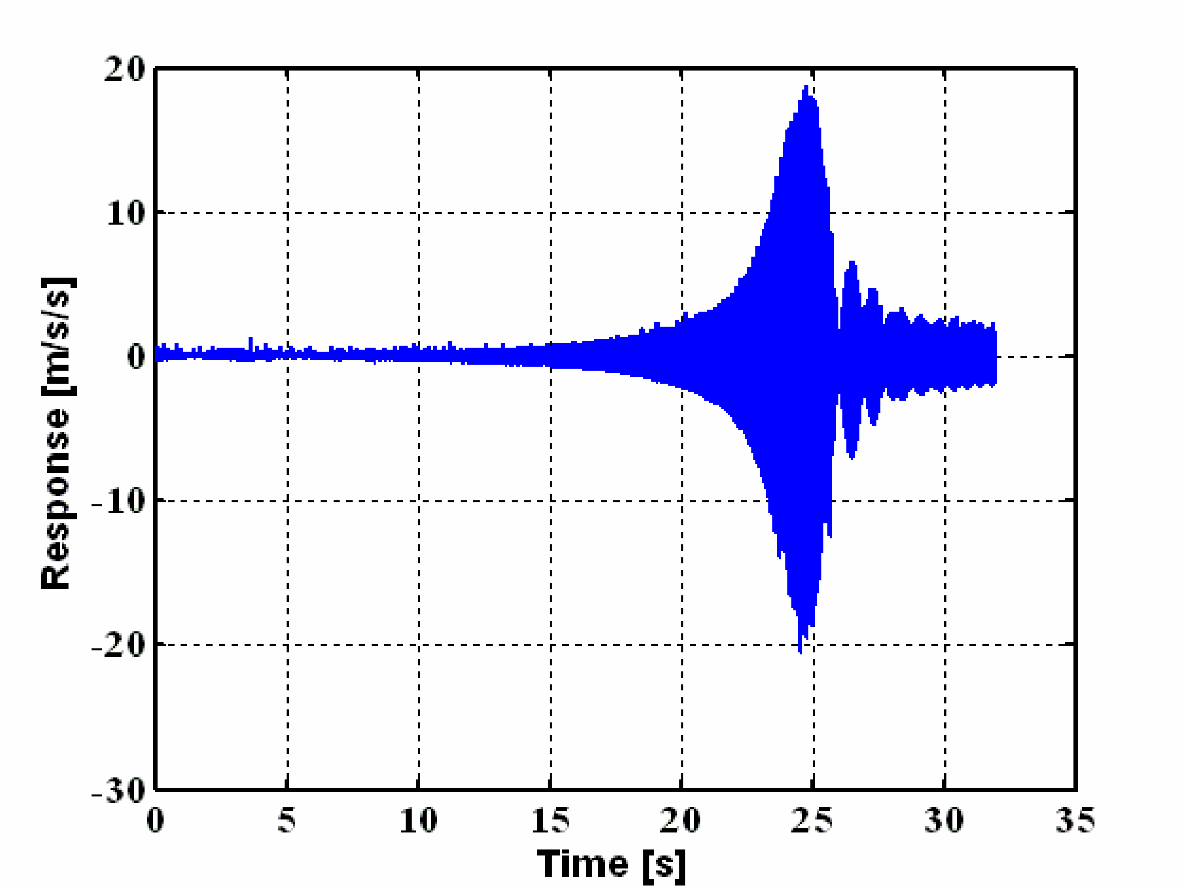

Fig. 5: Time dependant acceleration

From the time dependant acceleration was calculated the Fourier spectra. The systems Brüel & Kjaer HW – 3560D Pulse and Brüel & Kjaer SW – 7700 Pulse Labshop were used for the recording of data and numerical treatment of accelerations. Only for illustration is on fig. 6 drawn the Fourier spectra, corresponds to the time dependence which is drawn on fig. 5.

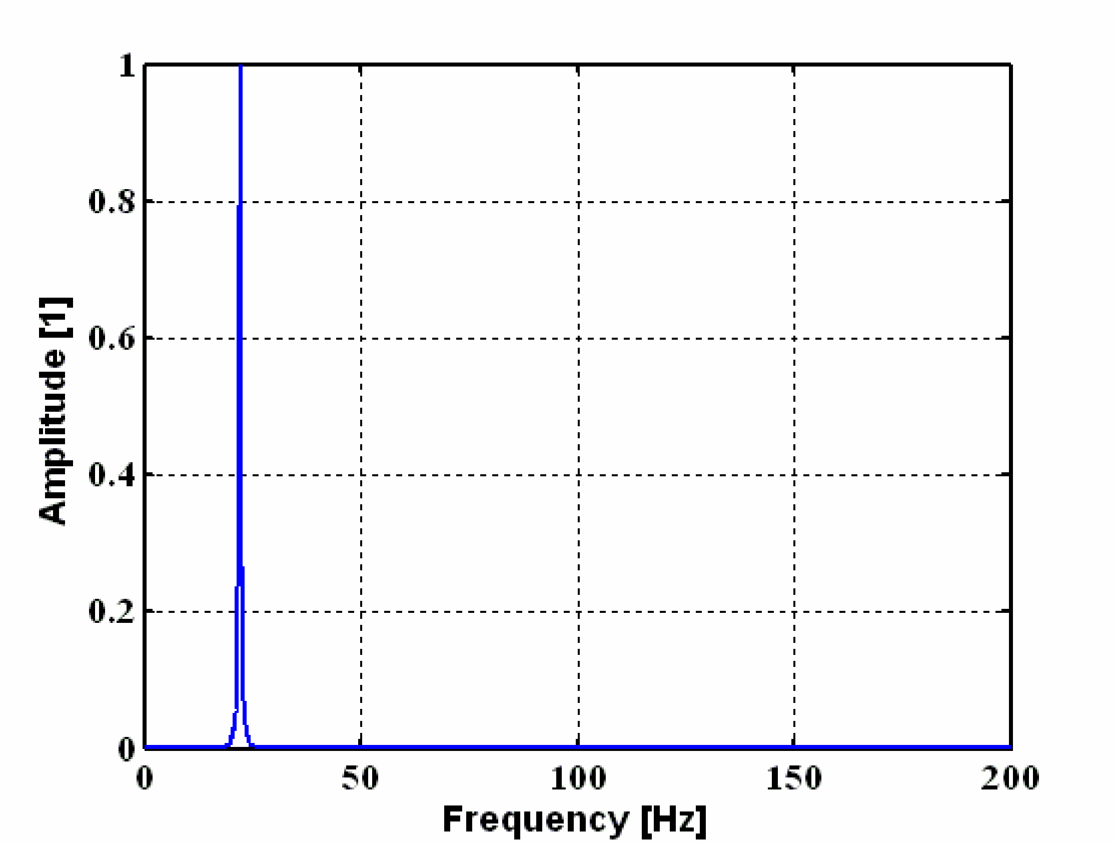

Fig. 6: The Fourier spectra

On behalf of identification the maximum of amplitude in the Fourier spectra and the frequency bandwidth near the resonance peak was determined the eigen frequency and modal damping for given shape of vibration. In this contribution is presented the comparison only the eigen frequencies. On figures 7 and 8 is presented the comparison for the first and the second shape of vibration. In the legend the abbreviation „exp“ means that the results concerned to the experiment and abbreviation „calc“ means the results concerned to the numerical solution.

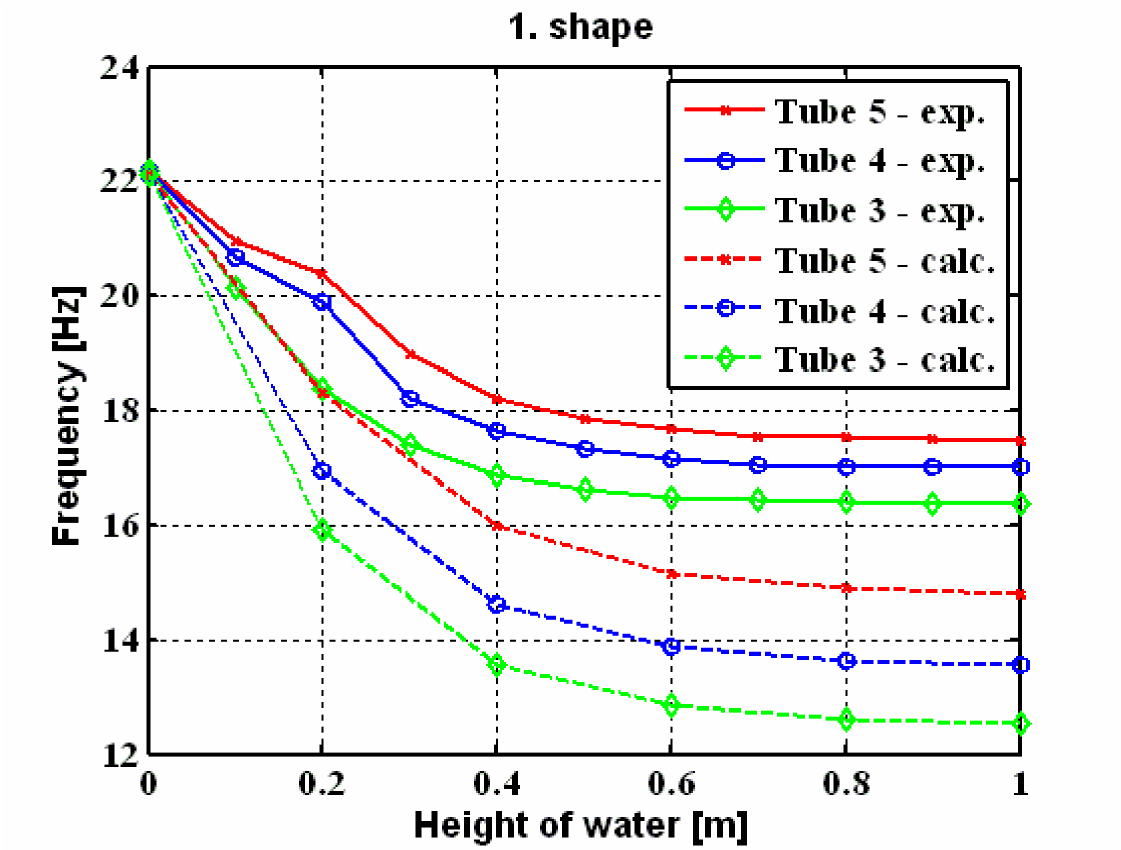

Fig. 7: Comparison for the first shape

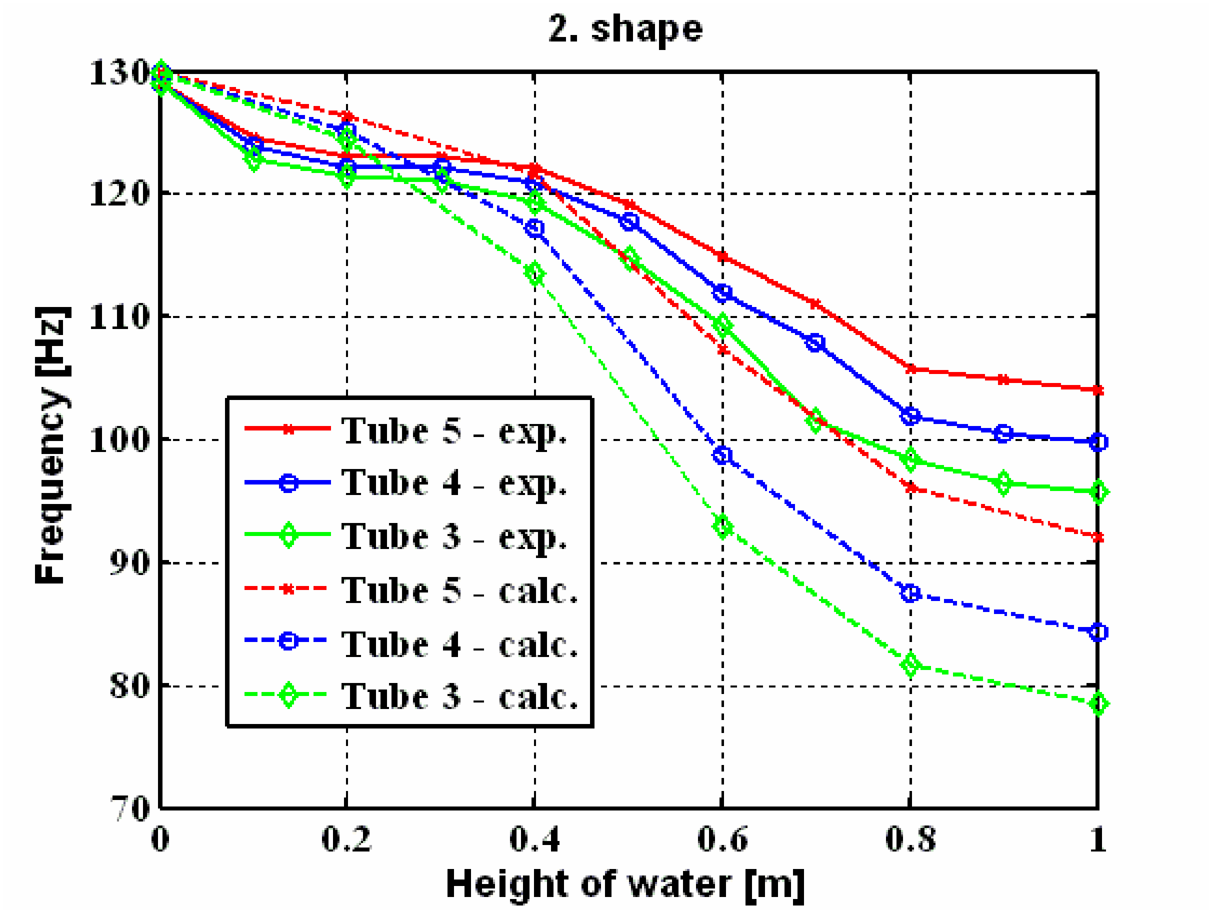

Fig. 8: Comparison for the second shape

5. Computational analysis

The results of the model task are only for matter – of - fact purposes and that is to present the possibilities of computational modeling. The whole analysis is carried out only for the first and the second shapes of vibration. The velocity and pressure field were calculated for the cases of liquid at a height from 0 to 1000 mm with step 200 mm. The task is symmetrical hence the velocities are calculated only on one half - plane. The vectors of velocities on the beam surface correspond to the chosen shape of vibration. Software performing and all calculations were done in program system MATLAB.

For the software performing are used curvy – linear co – ordinates. Transformation relations are presented in the appendix. For the solution of the Navier – Stokes eq. is used Finite Volume Method (FVM) and for the continuity eq. Finite Difference Method. (FDM) This combination was chosen to achieve the best numerical stability of numerical solution. It is necessary to note that all possible combination between FVM and FDM methods. Also it is necessary to note, that the both methods are used as collocations.

The Bezier body is used for the approximation of the geometrical configuration and for the also for the approximation of velocity and pressure solution. The expressions for an application see the appendix.

6. Conclusions

In this contribution is presented, new approach to the composition of local added modal matrices of mass and damping of liquid. They are assumed and presented, two models of liquids, ideal and real. For ideal liquid, the tensor of reversible forces is not included. It is possible to use, the presented approach to the solution, for the continuum with large displacement and large constrains.

The general algorithm for numerical analysis is as follows:

1. Whole range in frequency or time domain is divided into finite number of steps.
2. It is provided the modal behavior analysis of individual continuum (without liquid) for finite number of steps of geometry configuration.
3. Analysis of individual liquid with the boundary conditions which are given by the chosen eigen shape of vibration. This step is repeated until the finite number of eigen values is achieving. For each step, the velocity and pressure field for given continuum position, is obtained.
4. On behalf of velocity and pressure field on continuum surface are calculated the added matrices of liquid influence. Also the global added modal matrices for given shape of vibration and given vibrating position of continuum are composed.
5. Interpolation analysis of individual continuum with including the global matrices from analysis of individual liquid (step 4) is used during numerical solution.

They are evident the following conclusions from the comparison the experimental and calculation analysis (see figures. 7 and 8):

1. Relatively good agreement between the experiment and numerical solution is evident for the form of dependence of eigen value on the height of water.
2. For the both shapes are evident lower values for the numerical solution. It can be caused by the following reasons:
   a. The Fourier transformation is valid for the periodic signal. In our case the measurement signal was transient with the variable frequency of excitation. The Fourier transformation is not right for this analysis. Next experiment can be done by smaller angular acceleration and smaller force of excitation.
   b. It was evident from experimental analysis, that the eigen frequency in the two directions are a little bit different and both are bounded.
   c. Mathematical and computational model is not completed for calculation the added mass and damping. The functions $\boldsymbol{\alpha}_2$ and $\boldsymbol{\beta}_3$ are not included for the calculation of added effects.

Acknowledgement

This research is sponsored by grant GACR No. 101/06/0152.

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Appendix

Base functions

$$u_1(x) = 1 - \frac{3x^2}{L_E^2} + \frac{2x^3}{L_E^3}$$

$$u_2(x) = x - \frac{2x^2}{L_E} + \frac{x^3}{L_E^2}$$

$$u_3(x) = \frac{2x^2}{L_E^2} - \frac{2x^3}{L_E^3}$$

$$u_4(x) = -\frac{x^2}{L_E} + \frac{x^3}{L_E^2}$$

Transformation relations from the Cartesian to the curvy – linear co - ordinates

$$c_i = a^k \frac{\partial x_i}{\partial u^k}$$

$$\frac{\partial c_k}{\partial x_k} = \frac{\partial a^k}{\partial u^k} + a^k \frac{\partial^2 x_i}{\partial u^k \partial u^l} \frac{\partial u^l}{\partial x_i}$$

$$\frac{\partial p}{\partial x_i} = \frac{\partial p}{\partial u^m} \frac{\partial u^m}{\partial x_i}$$

$$\frac{\partial^2 c_i}{\partial x_j \partial x_k} = \left(\frac{\partial^2 a^l}{\partial u^m \partial u^n} \frac{\partial u^m}{\partial x_j} \frac{\partial u^n}{\partial x_k} \right) \frac{\partial x_i}{\partial u^l} +$$
$$+ \left(\frac{\partial a^l}{\partial u^m} \frac{\partial u^m}{\partial x_j} \right) \frac{\partial^2 x_i}{\partial u^n \partial u^l} \frac{\partial u^n}{\partial x_k} +$$
$$+ \left(\frac{\partial a^l}{\partial u^m} \frac{\partial u^m}{\partial x_k} \right) \frac{\partial^2 x_i}{\partial u^n \partial u^l} \frac{\partial u^n}{\partial x_j} +$$
$$+ a^l \frac{\partial^3 x_i}{\partial u^n \partial u^l \partial u^o} \frac{\partial u^n}{\partial x_j} \frac{\partial u^o}{\partial x_k}$$
$$\frac{\partial^2 c_i}{\partial x_k \partial x_k} = \left(\frac{\partial^2 a^l}{\partial u^m \partial u^n} \frac{\partial u^m}{\partial x_k} \frac{\partial u^n}{\partial x_k} \right) \frac{\partial x_i}{\partial u^l} +$$
$$+ 2 \left(\frac{\partial a^l}{\partial u^m} \frac{\partial u^m}{\partial x_k} \right) \frac{\partial^2 x_i}{\partial u^n \partial u^l} \frac{\partial u^n}{\partial x_k} +$$
$$+ a^l \frac{\partial^3 x_i}{\partial u^n \partial u^l \partial u^o} \frac{\partial u^n}{\partial x_k} \frac{\partial u^o}{\partial x_k}$$
$$\frac{\partial^2 c_k}{\partial x_i \partial x_k} = \left(\frac{\partial^2 a^l}{\partial u^m \partial u^n} \frac{\partial u^m}{\partial x_i} \frac{\partial u^n}{\partial x_k} \right) \frac{\partial x_k}{\partial u^l} +$$
$$+ \left(\frac{\partial a^l}{\partial u^m} \frac{\partial u^m}{\partial x_i} \right) \frac{\partial^2 x_k}{\partial u^n \partial u^l} \frac{\partial u^n}{\partial x_k} +$$
$$+ \left(\frac{\partial a^l}{\partial u^m} \frac{\partial u^m}{\partial x_k} \right) \frac{\partial^2 x_k}{\partial u^n \partial u^l} \frac{\partial u^n}{\partial x_i} +$$
$$+ a^l \frac{\partial^3 x_k}{\partial u^n \partial u^l \partial u^o} \frac{\partial u^n}{\partial x_i} \frac{\partial u^o}{\partial x_k}$$

or

$$\frac{\partial^2 c_k}{\partial x_i \partial x_k} = \left(\frac{\partial^2 a^l}{\partial u^m \partial u^l} \frac{\partial u^m}{\partial x_i} \right) +$$
$$+ \left(\frac{\partial a^l}{\partial u^m} \frac{\partial u^m}{\partial x_i} \right) \frac{\partial^2 x_k}{\partial u^n \partial u^l} \frac{\partial u^n}{\partial x_k} +$$
$$+ \left(\frac{\partial a^l}{\partial u^m} \frac{\partial u^m}{\partial x_k} \right) \frac{\partial^2 x_k}{\partial u^n \partial u^l} \frac{\partial u^n}{\partial x_i} +$$
$$+ a^l \frac{\partial^3 x_k}{\partial u^n \partial u^l \partial u^o} \frac{\partial u^n}{\partial x_i} \frac{\partial u^o}{\partial x_k}$$

Relations for the normal and rational Bézier 3D body application

$$\mathbf{r}(u,v,w)=\mathbf{r}=\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{p}\mathbf{r}_{ijk}B_i^n(u)B_j^m(v)B_k^p(w)$$

$$\mathbf{r}(u,v,w)=\mathbf{r}=\frac{\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{p}\omega_{ijk}\mathbf{r}_{ijk}B_i^n(u)B_j^m(v)B_k^p(w)}{\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{p}\omega_{ijk}B_i^n(u)B_j^m(v)B_k^p(w)}$$

Designation $jme=\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{p}\omega_{ijk}B_i^n(u)B_j^m(v)B_k^p(w)$

The first derivation $\frac{\partial\mathbf{r}}{\partial u}$

$$\frac{\partial\mathbf{r}}{\partial u}=n\left[\sum_{i=1}^{n}\sum_{j=0}^{m}\sum_{k=0}^{p}\left[\left(\mathbf{r}_{ijk}-\mathbf{r}_{i-1jk}\right)\right]B_{i-1}^{n-1}(u)B_j^m(v)B_k^p(w)\right]$$

$$\frac{\partial\mathbf{r}}{\partial u}=\frac{n}{jme}\left[\begin{array}{l}\sum_{i=1}^{n}\sum_{j=0}^{m}\sum_{k=0}^{p}\left(\omega_{ijk}\mathbf{r}_{ijk}-\omega_{i-1jk}\mathbf{r}_{i-1jk}\right)B_{i-1}^{n-1}(u)B_j^m(v)B_k^p(w)-\\-\mathbf{r}\sum_{i=1}^{n}\sum_{j=0}^{m}\sum_{k=0}^{p}\left(\omega_{ijk}-\omega_{i-1jk}\right)B_{i-1}^{n-1}(u)B_j^m(v)B_k^p(w)\end{array}\right]$$

The second derivation $\frac{\partial^2\mathbf{r}}{\partial u^2}$

$$\frac{\partial^2\mathbf{r}}{\partial^2 u}=n(n-1)\left[\sum_{i=2}^{n}\sum_{j=0}^{m}\sum_{k=0}^{p}\left[\left(\mathbf{r}_{ijk}-2\mathbf{r}_{i-1jk}+\mathbf{r}_{i-2jk}\right)\right]B_{i-2}^{n-2}(u)B_j^m(v)B_k^p(w)\right]$$

$$\frac{\partial^2\mathbf{r}}{\partial^2 u}=\frac{n(n-1)}{jme}\left[\begin{array}{l}\sum_{i=2}^{n}\sum_{j=0}^{m}\sum_{k=0}^{p}\left[\left(\omega_{ijk}\mathbf{r}_{ijk}-2\omega_{i-1jk}\mathbf{r}_{i-1jk}+\omega_{i-2jk}\mathbf{r}_{i-2jk}\right)\right]B_{i-2}^{n-2}(u)B_j^m(v)B_k^p(w)-\\-\mathbf{r}\sum_{i=2}^{n}\sum_{j=0}^{m}\sum_{k=0}^{p}\left[\left(\omega_{ijk}-2\omega_{i-1jk}+\omega_{i-2jk}\right)\right]B_{i-2}^{n-2}(u)B_j^m(v)B_k^p(w)\end{array}\right]$$

The second derivation $\frac{\partial^2\mathbf{r}}{\partial u\partial v}$

$$\frac{\partial^2\mathbf{r}}{\partial u\partial v}=nm\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=0}^{p}\left[\left(\mathbf{r}_{ijk}-\mathbf{r}_{i-1jk}\right)-\left(\mathbf{r}_{ij-1k}-\mathbf{r}_{i-1j-1k}\right)\right]B_{i-1}^{n-1}(u)B_{j-1}^{m-1}(v)B_k^p(w)\right]$$

$$\frac{\partial^2\mathbf{r}}{\partial u\partial v}=\frac{nm}{jme}\left[\begin{array}{l}\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=0}^{p}\left[\left(\omega_{ijk}\mathbf{r}_{ijk}-\omega_{i-1jk}\mathbf{r}_{i-1jk}\right)-\begin{pmatrix}\omega_{ij-1k}\mathbf{r}_{ij-1k}-\\-\omega_{i-1j-1k}\mathbf{r}_{i-1j-1k}\end{pmatrix}\right]B_{i-1}^{n-1}(u)B_{j-1}^{m-1}(v)B_k^p(w)-\\-\mathbf{r}\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=0}^{p}\left[\left(\omega_{ijk}-\omega_{i-1jk}\right)-\left(\omega_{ij-1k}-\omega_{i-1j-1k}\right)\right]B_{i-1}^{n-1}(u)B_{j-1}^{m-1}(v)B_k^p(w)\end{array}\right]$$