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3D NUMERICAL MODEL OF AN ELLIPSOIDAL PARTICLE COLLISION WITH A ROUGH BED

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Summary: *The paper deals with 3D numerical model of an ellipsoidal particle collision with rough bed. The stochastic method of the contact point is based on the concept of contact zone. The translational and rotational particle motions are taken into account. The two – parameters (restitution and friction coefficients) system of the impulse equations are used. The dependences of values of the particle translational and angular velocities immediately after the collision on the values immediately before the collision are obtained for two collision patterns: impact with slip and impact without slip. The model can be used, particularly, as the part of the ellipsoidal particle saltation in channel with rough bed.*

1. Introduction

The numerical models of the particle saltation in the channel with a rough bed suppose usually a spherical shape of the particle (e.g., Nino & Garcia, 1994; Lukerchenko *et al.*, 2004, Lukerchenko *et al.*, 2006). This way simplifies the problem, but does not allow the studying some important details of the phenomenon.

It was shown by experiments of Nino & Garcia (1998) dealing with a saltation of sand in an open channel that "elongated shapes colliding with the bed at the proper orientation seem to result in higher particle angular velocity than in more spherical shapes". This property can be modelled only under the condition that the particle shape is non-spherical.

2D numerical model of the ellipsoidal particle saltation and of the ellipsoidal particle collision with rough bed were developed in Lukerchenko et al., 2005(1), 2005(2). The present work is devoted to the development of the 3D numerical model of the ellipsoidal particle collision with a rough fixed bed.

The shape of the particle is an ellipsoid with semi-axes a, b and c. This shape allows using the same formulas for elongated particles and for particles of nearly spherical shape and investigating the effect of the particle elongation on the saltation parameters.

Two stages can be distinguished in the particle saltation process: the particle movement in the stream of water and the particle-bed collision. The second stage, i.e. the random process of the particle impact and rebound from the bed is studied in the present paper. A numerical stochastic model of particle-bed collision was developed.

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The particle translational and rotational velocity components immediately after the collision were expressed as a function of the particle translational and angular velocity components immediately before the collision for two collision patterns: impact with slip and impact without slip. The pattern depends appreciably on the particle translational and angular velocity before the collision.

2. Contact zone

The contact point is the point of the particle surface that is in direct contact with the bed during the particle-bed collision. Its position has a random nature. The contact zone is the set of all possible contact points.

The particle translational velocity $\bar{v}\left(v_x, v_y, v_z\right)$ and angular velocity $\bar{\omega}\left(\omega_x, \omega_y, \omega_z\right)$ as well as the coordinates of the ellipsoidal particle centre of gravity O immediately before the collision (see Fig. 1) are assumed to be known from the calculation of the particle motion in the channel.

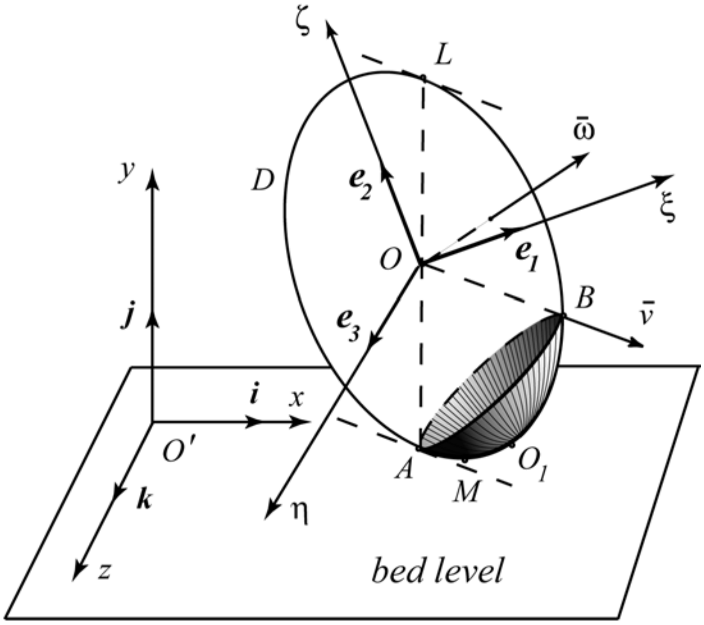

Fig.1. Coordinates of the ellipsoidal particle in channel

Let us suppose that the moving particle contacts the channel bed at one point during the collision (it follows from the assumption of the bed geometry regularity). The particle-bed contact at the point B *c*orresponds with a central impact and the contact at the point A corresponds with a tangential impact.

In the moment of collision points on the segment ADL are in the particle shadow. The probability of the contact for points on the segment is very small; the contact is possible only for a very large bed roughness. Let us suppose that the points of the ellipsoidal segment AO_1B are the points in which the particle can contact the bed, i.e. this segment is the contact zone. The contact point for each collision is chosen from the contact zone as random variable using a random-number generator.

3. Definition of the co-ordinate systems and particle position

Let us consider two orthogonal systems of coordinates (see Fig.1). The first of them, O'xyz (system 1), is firmly connected with the channel bed. The second one, Oξζη (system 2), is connected with the moving ellipsoidal particle and is movable. The position of any ellipsoidal particle in the channel can be unambiguously determined by the coordinates of its centre and by the Euler's angles. The vector $\boldsymbol{R_o}(x_o, y_o, z_o)$ is the radius – vector of the point O in the system O'xyz.

The base vectors and coordinates of the two systems are expressed using the Euler's angles φ, ψ, θ by following formulas

$$\begin{cases} \vec{e}_1 = (\cos\varphi\cos\psi - \sin\psi\cos\theta\sin\varphi)\vec{i} + (\cos\psi\sin\varphi + \sin\psi\cos\theta\cos\varphi)\vec{j} + (\sin\psi\sin\theta)\vec{k} \\ \vec{e}_2 = (-\cos\varphi\sin\psi - \cos\psi\cos\theta\sin\varphi)\vec{i} + (-\sin\psi\sin\varphi + \cos\psi\cos\theta\cos\varphi)\vec{j} + (\cos\psi\sin\theta)\vec{k} \\ \vec{e}_3 = (\sin\theta\sin\varphi)\vec{i} + (-\sin\theta\cos\varphi)\vec{j} + (\cos\theta)\vec{k} \end{cases}; \quad (1)$$

$$\begin{cases} \vec{i} = (\cos\varphi\cos\psi - \sin\psi\cos\theta\sin\varphi)\vec{e}_1 + (-\cos\varphi\sin\psi - \cos\psi\cos\theta\sin\varphi)\vec{e}_2 + (\sin\theta\sin\varphi)\vec{e}_3 \\ \vec{j} = (\cos\psi\sin\varphi + \sin\psi\cos\theta\cos\varphi)\vec{e}_1 + (-\sin\psi\sin\varphi + \cos\psi\cos\theta\cos\varphi)\vec{e}_2 + (-\sin\theta\cos\varphi)\vec{e}_3 \\ \vec{k} = (\sin\psi\sin\theta)\vec{e}_1 + (\cos\psi\sin\theta)\vec{e}_2 + (\cos\theta)\vec{e}_3 \end{cases}; \quad (2)$$

$$\begin{cases} \xi = (\cos\varphi\cos\psi - \sin\psi\cos\theta\sin\varphi)(x - x_O) + (\cos\psi\sin\varphi + \sin\psi\cos\theta\cos\varphi)(y - y_O) + (\sin\psi\sin\theta)(z - z_O) \\ \zeta = (-\cos\varphi\sin\psi - \cos\psi\cos\theta\sin\varphi)(x - x_O) + (-\sin\psi\sin\varphi + \cos\psi\cos\theta\cos\varphi)(y - y_O) + (\cos\psi\sin\theta)(z - z_O) \\ \eta = (\sin\theta\sin\varphi)(x - x_O) + (-\sin\theta\cos\varphi)(y - y_O) + (\cos\theta)(z - z_O) \end{cases} \quad (3)$$

$$\begin{cases} x = (\cos\varphi\cos\psi - \sin\psi\cos\theta\sin\varphi)\xi + (-\cos\varphi\sin\psi - \cos\psi\cos\theta\sin\varphi)\zeta + (\sin\theta\sin\varphi)\eta + x_O \\ y = (\cos\psi\sin\varphi + \sin\psi\cos\theta\cos\varphi)\xi + (-\sin\psi\sin\varphi + \cos\psi\cos\theta\cos\varphi)\zeta + (-\sin\theta\cos\varphi)\eta + y_O \\ z = (\sin\psi\sin\theta)\xi + (\cos\psi\sin\theta)\zeta + (\cos\theta)\eta + z_O \end{cases}. \quad (4)$$

Let us denote the coordinates of the base vector $\boldsymbol{j}$ in the system 2 as

$$j_\xi = (\cos\psi\sin\varphi + \sin\psi\cos\theta\cos\varphi); \quad j_\zeta = (-\sin\psi\sin\varphi + \cos\psi\cos\theta\cos\varphi); \quad j_\eta = -\sin\theta\cos\varphi.$$

4. The condition of the particle – bed collision

The bed roughness is k_s (see Fig. 2). The particle contact with the bed is defined by the following way. The random generator chooses the value k^*_s in the range (0; k_s). During each step of calculation of the particle motion in fluid, at first the lowest point M of the ellipsoidal particle (see Fig.1), i. e. the point with minimal value of the coordinate y, is determined. For y_M fulfilling the condition $y_M \le k_s^*$, the particle-bed collision is calculated. The coordinates of the lowest point M of the ellipsoidal particle must comply with the following two conditions:

(1) the point M belongs to the particle surface

$$\frac{\xi_M^{\ 2}}{a^2} + \frac{\zeta_M^{\ 2}}{b^2} + \frac{\eta_M^{\ 2}}{c^2} - 1 = 0, \quad (5)$$

where a, b and c are the semi-axes of the ellipsoid, and

(2) the external normal $\boldsymbol{n}$ to the particle surface in the point M and coordinate vector $\boldsymbol{j}$ are anti-parallel vectors: $\boldsymbol{n} = - k\boldsymbol{j}$, $k > 0$.

The equation of the tangent plane to the ellipsoid in the point M is

$$(\xi - \xi_M)\frac{2\xi_M}{a^2} + (\zeta - \zeta_M)\frac{2\zeta_M}{b^2} + (\eta - \eta_M)\frac{2\eta_M}{c^2} = 0,$$

the coordinates of the vector ***n*** are $\left(\frac{2\xi_M}{a^2}, \frac{2\zeta_M}{b^2}, \frac{2\eta_M}{c^2}\right)$. The condition ***n*** = - *k* ***j*** using Eq. (2) can be written for the components of the vector as

$$\begin{cases} \xi_M = -\frac{ka^2}{2}(\cos\psi\sin\varphi + \sin\psi\cos\theta\cos\varphi); \quad \zeta_M = -\frac{kb^2}{2}(-\sin\psi\sin\varphi + \cos\psi\cos\theta\cos\varphi); \\ \eta_M = -\frac{kc^2}{2}(-\sin\theta\cos\varphi) \end{cases}$$

The coordinates of the sought point M are $M = (\xi_M, \zeta_M, \eta_M) = \frac{-\left(j_\xi a^2, j_\zeta b^2, j_\eta c^2\right)}{\sqrt{a^2 j_\xi^2 + b^2 j_\zeta^2 + c^2 j_\eta^2}}$

Then taking into account Eq. (4) the condition of the collision can be written

$$y_M = \frac{-\left(j_\xi^2 a^2 + j_\zeta^2 b^2 + j_\eta^2 c^2\right)}{\sqrt{a^2 j_\xi^2 + b^2 j_\zeta^2 + c^2 j_\eta^2}} + y_0 \le k_s^*. \tag{6}$$

5. The contact zone

For modelling of the particle – bed collision it is necessary to calculate coordinates of the contact point. The contact zone is bounded by the point *A* and *B* (see Fig.1). The coordinates of the point *A* must satisfy the three conditions.

(1) The point *A* is the point of the ellipsoid surface, see Eq. (5)

$$\frac{\xi_A^{\ 2}}{a^2} + \frac{\zeta_A^{\ 2}}{b^2} + \frac{\eta_A^{\ 2}}{c^2} - 1 = 0, \tag{7}$$

(2) The normal vector $\vec{n}\left(\frac{2\xi_A}{a^2}, \frac{2\zeta_A}{b^2}, \frac{2\eta_A}{c^2}\right)$ to the tangent plane at the point *A* is perpendicular to the velocity vector: $(\vec{n} \cdot \vec{V}) = 0$, or

$$V_\xi \frac{2\xi_A}{a^2} + V_\zeta \frac{2\zeta_A}{b^2} + V_\eta \frac{2\eta_A}{c^2} = 0, \tag{8}$$

and this normal vector is located in the plane of the velocity vector and the coordinate vector ***j*** $\left(\vec{n}, \vec{V}, \vec{j}\right) = 0$ or

$$\frac{\xi_A}{a^2}\left(j_\eta V_\zeta - j_\zeta V_\eta\right) + \frac{\zeta_A}{b^2}\left(j_\xi V_\eta - j_\eta V_\xi\right) + \frac{\eta_A}{c^2}\left(j_\zeta V_\xi - j_\xi V_\zeta\right) = 0. \tag{9}$$

The system of Eqs. (7)-(9) has two solutions that correspond to two point A_1 *and* A_2, located symmetrically on the ellipsoid surface

$$A_1 = \frac{-(q, p, 1)}{\sqrt{\left(\frac{q^2}{a^2} + \frac{p^2}{b^2} + \frac{1}{c^2}\right)}}; \quad A_2 = \frac{(q, p, 1)}{\sqrt{\left(\frac{q^2}{a^2} + \frac{p^2}{b^2} + \frac{1}{c^2}\right)}}, \tag{10}$$

where

$$p=\frac{\dfrac{V_\eta\left(V_\zeta j_\eta-V_\eta j_\zeta\right)}{V_\xi c^2}-\dfrac{\left(V_\xi j_\zeta-V_\zeta j_\xi\right)}{c^2}}{\dfrac{\left(V_\eta j_\xi-V_\xi j_\eta\right)}{b^2}-\dfrac{V_\zeta\left(V_\zeta j_\eta-V_\eta j_\zeta\right)}{V_\xi b^2}},\qquad q=-\frac{a^2}{V_\xi}\left(V_\zeta\frac{p}{b^2}+V_\eta\frac{1}{c^2}\right).$$

The correct point A is the point, which is closer to the bed, i.e. the point with smaller coordinate y.

The calculation of the point B consists of two steps. During the first step the intersection point B_1 of the velocity vector with the ellipsoid surface is calculated. Because the coordinate y_{B1} of the contact point cannot overcome the bed roughness k_s, during the second step the coordinate y_{B1} of the point B_1 is compared with the bed roughness value k_s (see Fig.2). If $y_{B1} \le k_s$, the intersection point B_1 is considered to be the point B. If $y_{B1} > k_s$, the point B is transferred to the level k_s.

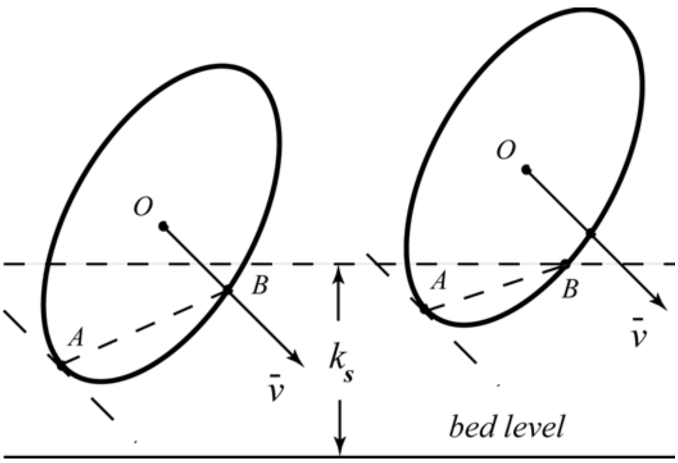

Fig. 2. Calculation of the point B

The coordinates of the intersection point B_1 must satisfy the following conditions

$$\frac{\xi_{B_1}^{\ 2}}{a^2}+\frac{\zeta_{B_1}^{\ 2}}{b^2}+\frac{\eta_{B_1}^{\ 2}}{c^2}-1=0\,, \tag{11}$$

$$\overline{O}\overline{B}_1=k\overline{V},\quad k>0, \qquad => \ \xi_{B_1}=kV_\xi\,,\quad \zeta_{B_1}=kV_\zeta\,,\quad \eta_{B_1}=kV_\eta;\quad k>0. \tag{12}$$

Since

$$y_{A_1}\left(\xi_{A_1},\zeta_{A_1},\eta_{A_1}\right)=\frac{-(qj_\xi+pj_\zeta+j_\eta)}{\sqrt{\left(\dfrac{q^2}{a^2}+\dfrac{p^2}{b^2}+\dfrac{1}{c^2}\right)}}+y_O\,;\qquad y_{A_2}\left(\xi_{A_2},\zeta_{A_2},\eta_{A_2}\right)=\frac{qj_\xi+pj_\zeta+j_\eta}{\sqrt{\left(\dfrac{q^2}{a^2}+\dfrac{p^2}{b^2}+\dfrac{1}{c^2}\right)}}+y_O\,,$$

the coordinates of the point A must satisfy the condition

$$y_A\left(\xi_A,\zeta_A,\eta_A\right)=\min\left[y_{A_1}\left(\xi_{A_1},\zeta_{A_1},\eta_{A_1}\right),y_{A_2}\left(\xi_{A_2},\zeta_{A_2},\eta_{A_2}\right)\right].$$

The solution of the system of Eqs. (11) – (12) is unequivocal

$$B_1\frac{(V_\xi,V_\zeta,V_\eta)}{\sqrt{\dfrac{V_\xi^2}{a^2}+\dfrac{V_\zeta^2}{b^2}+\dfrac{V_\eta^2}{c^2}}}\,. \tag{13}$$

The coordinate y of the point B_1 is

$$y_{B_1}\left(\xi_{B_1},\zeta_{B_1},\eta_{B_1}\right)=\frac{j_\xi V_\xi+j_\zeta V+j_\eta V_\eta}{\sqrt{\dfrac{V_\xi^{\ 2}}{a^2}+\dfrac{V_\zeta^{\ 2}}{b^2}+\dfrac{V_\eta^{\ 2}}{c^2}}}+y_O\,.$$

If $y_{B_1}\left(\xi_{B_1},\zeta_{B_1},\eta_{B_1}\right)\le k_s$, the point B_1 is the sought after point B, otherwise the coordinates of the point B must satisfy the following conditions

$$\frac{\xi_B^{\ 2}}{a^2}+\frac{\zeta_B^{\ 2}}{b^2}+\frac{\eta_B^{\ 2}}{c^2}-1=0\,, \qquad (14)$$

$$y_B\left(\xi_B,\zeta_B,\eta_B\right)=j_\xi\xi_B+j_\zeta\zeta_B+j_\eta\eta_B+y_O=k_s\,, \qquad (15)$$

$$\left(\overrightarrow{OB},\vec{V},\overrightarrow{OA}\right)=0\,.$$

The last condition means that the points A and B and the velocity vector are in the same plane given by the equation

$$\xi_B\left(\eta_A V_\zeta-\zeta_A V_\eta\right)+\zeta_B\left(\xi_A V_\eta-\eta_A V_\xi\right)+\eta_B\left(\zeta_A V_\xi-\xi_A V_\zeta\right)=0\,. \qquad (16)$$

The system of equations (14)-(16) has two solutions

$$\xi_B=(\kappa\alpha+\chi)\eta_B+\kappa\beta;\quad \zeta_B=\alpha\eta_B+\beta;\quad \eta_B^{(1,2)}=\frac{-b^*\pm\sqrt{b^{*2}-4a^*c^*}}{2a^*}, \qquad (17)$$

where

$$a^*=\left(\frac{(\kappa\alpha+\chi)^2}{a^2}+\frac{\alpha^2}{b^2}+\frac{1}{c^2}\right);\quad b^*=\left(\frac{2\kappa\beta(\kappa\alpha+\beta)}{a^2}+\frac{2\alpha\beta}{b^2}\right);\quad c^*=\left(\frac{\kappa^2\beta^2}{a^2}+\frac{\beta^2}{b^2}-1\right);$$

$$\alpha=-\frac{j_\xi\chi+j_\eta}{j_\xi\kappa+j_\zeta};\quad \beta=-\frac{y_0-k_s}{j_\xi k+j_\zeta};\qquad \kappa=\frac{V_\xi\eta_A-V_\eta\xi_A}{V_\zeta\eta_A-V_\eta\zeta_A};\quad \chi=\frac{V_\zeta\xi_A-V_\xi\zeta_A}{V_\zeta\eta_A-V_\eta\zeta_A}.$$

These solutions give two points $B_{(1)}(\xi_B,\zeta_B,\eta_B^{(1)})$ and $B_{(2)}(\xi_B,\zeta_B,\eta_B^{(2)})$, disposed on different sides of the point A. For the sought after point B the angle between the vector $\overrightarrow{OB}$ and velocity vector is smaller. Let us consider the vectors $\overrightarrow{OB}_{(1)}(\xi_B,\zeta_B,\eta_B^{(1)})$ and $\overrightarrow{OB}_{(2)}(\xi_B,\zeta_B,\eta_B^{(2)})$, angles between them and the velocity vector

$$\cos\delta_1=\frac{\left(\overrightarrow{OB}_{(1)},\vec{V}\right)}{\left|\overrightarrow{OB}_{(1)}\right|\left|\vec{V}\right|}=\frac{\xi_B V_\xi+\zeta_B V_\zeta+\eta_B^{(1)}V_\eta}{\sqrt{\xi_B^2+\zeta_B^2+\left(\eta_B^{(1)}\right)^2}\sqrt{V_\xi^2+V_\zeta^2+V_\eta^2}},$$

$$\cos\delta_2=\frac{\left(\overrightarrow{OB}_{(2)},\vec{V}\right)}{\left|\overrightarrow{OB}_{(2)}\right|\left|\vec{V}\right|}=\frac{\xi_B V_\xi+\zeta_B V_\zeta+\eta_B^{(2)}V_\eta}{\sqrt{\xi_B^2+\zeta_B^2+\left(\eta_B^{(2)}\right)^2}\sqrt{V_\xi^2+V_\zeta^2+V_\eta^2}}.$$

If $\cos\delta_1>\cos\delta_2$, then the point $B_{(1)}$ is the sought otherwise point B, else the point B is point $B_{(2)}$.

Let us now calculate the secant plane which passes through the point A and B and which intersects the ellipsoid. Its normal is in the plane of the vectors $\vec{V}$ and $\vec{j}$, which equation is

$$K'\xi+L'\zeta+M'\eta+N'=0,$$

where K', L', M', N' are constants. The particle centre O does not belong to the secant plane, hence $N'\neq 0$ and the plane equation can be written as

$$K\xi+L\zeta+M\eta+1=0,$$

and the system of equations for the definition K, L and M can be written

$$K\xi_A + L\zeta_A + M\eta_A + 1 = 0\,, \tag{18}$$

$$K\xi_B + L\zeta_B + M\eta_B + 1 = 0\,, \tag{19}$$

$$\left(\vec{n}, \vec{j}, \vec{V}\right) = 0\,.$$

The last equation in detail is

$$K\left(j_\zeta V_\eta - j_\eta V_\zeta\right) + L\left(j_\eta V_\xi - j_\xi V_\eta\right) + M\left(j_\xi V_\zeta - j_\zeta V_\xi\right) = 0\,. \tag{20}$$

The solution of the system of Eqs. (18), (19), (20) is

$$K = \frac{\Delta_K}{\Delta}; \quad L = \frac{\Delta_L}{\Delta}; \quad M = \frac{\Delta_M}{\Delta}\,, \tag{21}$$

where

$$\Delta = \begin{vmatrix} \xi_A & \zeta_A & \eta_A \\ \xi_B & \zeta_B & \eta_B \\ j_\zeta V_\eta - j_\eta V_\zeta; & j_\eta V_\xi - j_\xi V_\eta; & j_\xi V_\zeta - j_\zeta V_\xi \end{vmatrix}; \quad \Delta_K = \begin{vmatrix} -1 & \zeta_A & \eta_A \\ -1 & \zeta_B & \eta_B \\ 0 & j_\eta V_\xi - j_\xi V_\eta; & j_\xi V_\zeta - j_\zeta V_\xi \end{vmatrix};$$

$$\Delta_L = \begin{vmatrix} \xi_A & -1 & \eta_A \\ \xi_B & -1 & \eta_B \\ j_\zeta V_\eta - j_\eta V_\zeta; & 0 & j_\xi V_\zeta - j_\zeta V_\xi \end{vmatrix}; \quad \Delta_M = \begin{vmatrix} \xi_A & \zeta_A & -1 \\ \xi_B & \zeta_B & -1 \\ j_\zeta V_\eta - j_\eta V_\zeta; & j_\eta V_\xi - j_\xi V_\eta; & 0 \end{vmatrix}.$$

The secant plane dissects the ellipsoid surface to two regions. The lower one is the contact zone of the particle with the bed.

Since the particle – bed collision is the random process, also the contact point is a random variable, and it is chosen from the contact zone using the random number generator. The coordinates of the point at the ellipsoid surface can be written as

$$\left\{ \xi^* = a\cos\psi^*\cos\varphi^*; \quad \zeta^* = b\sin\psi^*; \quad \eta^* = c\cos\psi^*\sin\varphi^*\,, \right.$$

where $\psi^* \in [0, 2\pi), \quad \varphi^* \in [0, \pi)$.

The parameters ψ^* and φ^* are chosen from these ranges, respectively, using the random-number generator. Then it is verified that the obtained point belong to the contact zone. In this case, the found point and the point O are disposed on different sides of the secant plane or the found point belongs to the secant plane. Therefore,

$$sign\, f(C(\xi^*, \zeta^*, \eta^*)) \neq sign\, f(O(0,0,0)),$$

where

$$f(C(\xi^*, \zeta^*, \eta^*)) = \alpha_1\xi^* + \zeta^* + \alpha_2\eta^* + \alpha_3,\ f(O(0,0,0) = \alpha_3,$$

i.e.

$$(\alpha_1\xi^* + \zeta^* + \alpha_2\eta^* + \alpha_3) * \alpha_3 \leq 0. \tag{22}$$

6. The collision coordinate system

Let us define the collision coordinate system $C\tau n\sigma$, in which the impulse equations can be written in the simplest form. The coordinate system $C\tau n\sigma$ is right and orthogonal with the origin in the contact point C. The axis Cn is normal to the ellipsoid surface in the point C and directed inward. The axis $C\tau$ is in the plane of the velocity vector and the axis Cn. The unit base vectors of the coordinate system $C\tau n\sigma$ are

$$\bar{n}(\xi_n, \zeta_n, \eta_n) = \frac{-(\xi_C, \varsigma_C, \eta_C)}{a^2\sqrt{\left(\frac{\xi_C}{a^2}\right)^2 + \left(\frac{\zeta_C}{b^2}\right)^2 + \left(\frac{\eta_C}{c^2}\right)^2}},$$

$$\bar{\sigma}((\xi_n, \zeta_n, \eta_n) = \frac{\left(\zeta_n V_\eta - \eta_n V_\zeta ; \eta_n V_\xi - \xi_n V_\eta ; \xi_n V_\zeta - \zeta_n V_\eta\right)}{\sqrt{\xi_\sigma^{*2} + \zeta_\sigma^{*2} + \eta_\sigma^{*2}}},$$

$$\bar{\tau}(\xi_\tau, \zeta_\tau, \eta_\tau) = (\zeta_\sigma \eta_n - \zeta_n \eta_\sigma, \xi_n \eta_\sigma - \xi_\sigma \eta_n, \xi_\sigma \zeta_n - \xi_n \varsigma_\sigma),$$

where $\xi_\sigma^* = \zeta_n V_\eta - \eta_n V_\zeta$; $\zeta_\sigma^* = \eta_n V_\xi - \xi_n V_\eta$; $\eta_\sigma^* = \xi_n V_\zeta - \zeta_n V_\eta$.

The relation between base vectors is

$$\bar{n} = \xi_n \bar{e}_1 + \zeta_n \bar{e}_2 + \eta_n \bar{e}_3; \quad \bar{\tau} = \xi_\tau \bar{e}_1 + \zeta_\tau \bar{e}_2 + \eta_\tau \bar{e}_3; \quad \bar{\sigma} = \xi_\sigma \bar{e}_1 + \zeta_\sigma \bar{e}_2 + \eta_\sigma \bar{e}_3; \tag{23}$$

$$\bar{e}_1 = \xi_n \bar{n} + \xi_\tau \bar{\tau} + \xi_\sigma \bar{\sigma}; \quad \bar{e}_2 = \zeta_n \bar{n} + \zeta_\tau \bar{\tau} + \zeta_\sigma \bar{\sigma}; \quad \bar{e}_3 = \eta_n \bar{n} + \eta_\tau \bar{\tau} + \eta_\sigma \bar{\sigma}. \tag{24}$$

It is valid for the coordinates

$$\begin{cases} n = \xi_n(\xi - \xi_C) + \zeta_n(\zeta - \zeta_C) + \eta_n(\eta - \eta_C); \\ \tau = \xi_\tau(\xi - \xi_C) + \zeta_\tau(\zeta - \zeta_C) + \eta_\tau(\eta - \eta_C); \\ \sigma = \xi_\sigma(\xi - \xi_C) + \zeta_\sigma(\zeta - \zeta_C) + \eta_\sigma(\eta - \eta_C); \end{cases} \quad \begin{cases} \xi = \xi_n n + \xi_\tau \tau + \xi_\sigma \sigma + \xi_C; \\ \zeta = \zeta_n n + \zeta_\tau \tau + \zeta_\sigma \sigma + \zeta_C; \\ \eta = \eta_n n + \eta_\tau \tau + \eta_\sigma \sigma + \eta_C. \end{cases} \tag{25, 26}$$

7. The collision calculation

Let us consider the second Newton's law and the equation of the torque in view:

$$m\, d\bar{V} = \bar{F}\, dt, \tag{27}$$

$$d\,\bar{L} = \bar{M} dt = \left[\bar{r}_C \times \bar{F}\, dt\right] \tag{28}$$

where m is the particle mass, $d\bar{V}$ is the differential of the particle velocity, $\bar{F}$ is the impact force, with which the bed acts on the particle, dt is differential of time. After the integration of Eqs. (27) and (28):

$$m\left(\bar{V}^+ - \bar{V}^-\right) = \bar{I}, \tag{29}$$

$$\bar{L}^+ - \bar{L}^- = \left[\bar{r}_C \times \bar{I}\right], \tag{30}$$

where $\bar{V}^-$ is the particle velocity immediately before the collision, $\bar{V}^+$ is the particle velocity immediately after the collision, $\bar{I}$ is the impulse of the force $\bar{F}$. The Eqs. (29), (30) give in the projections to the coordinate axes $\bar{n}, \bar{\tau}, \bar{\sigma}$:

$$m\left(V_n^+ - V_n^-\right) = I_n; \quad m\left(V_\tau^+ - V_\tau^-\right) = I_\tau; \quad m\left(V_\sigma^+ - V_\sigma^-\right) = I_\sigma = 0. \tag{31}$$

$$J_n\left(\omega_n^+ - \omega_n^-\right) = -r_{C\sigma} I_\tau; \quad J_\tau\left(\omega_\tau^+ - \omega_\tau^-\right) = r_{C\sigma} I_n; \quad J_\sigma\left(\omega_\sigma^+ - \omega_\sigma^-\right) = r_{Cn} I_\tau - r_{C\tau} I_n. \tag{32}$$

The condition $I_\sigma = 0$ is valid in the collision coordinate system. The equation for the normal component of velocity is

$$V_n^+ = -e V_n^-, \tag{33}$$

where e is the restitution coefficient.

The moments of inertia of ellipsoid relative to the axes parallel to the co-ordinate axes n, τ, σ , which passes to the ellipsoid centre are J_n, J_τ, J_σ. The moments of inertia of an ellipsoid relative its principal axes are

$$A = J_\xi = \frac{1}{5}m\left(b^2 + c^2\right); \quad B = J_\zeta = \frac{1}{5}m\left(a^2 + c^2\right); \quad C = J_\eta = \frac{1}{5}m\left(a^2 + b^2\right).$$

and the moment of inertia relative to an arbitrary axis Δ passed the body centre of mass can be calculated using the formula

$$J_\Delta = A^2 \cos^2 \alpha + B^2 \cos^2 \beta + C^2 \cos^2 \gamma ,$$

where cos α, cos β, cos γ are the direction cosines of the axis Δ. Therefore for the J_n, J_τ, J_σ

$$J_n = A^2\xi_n^2 + B^2\zeta_n^2 + C^2\eta_n^2; \quad J_\tau = A^2\xi_\tau^2 + B^2\zeta_\tau^2 + C^2\eta_\tau^2; \quad J_\sigma = A^2\xi_\sigma^2 + B^2\zeta_\sigma^2 + C^2\eta_\sigma^2;$$

Two different patterns of particle collision can exist according to the collision conditions:

- impact with slip, i.e. the particle slips along the surface of the channel bed during the collision process;
- impact without slip, i.e. the tangential component of the contact point velocity (see Eq. (34,2)) vanishes before the particle rebound, Lukerchenko *et al*. (2006) .

The impact with slip is possible, if the impulse acting on the particle during the collision is

$$|I_\tau| = fI_n, \tag{34}$$

where the friction coefficient f is an empirical constant and the Eq. (34) closes the system of Eqs. (31), (32), (33).

The impact without slip is possible, if the impulse acting on the particle during the collision is $|I_\tau| < fI_n$ and the tangential component of the contact point velocity vanishes

$$V_{C\tau}^+ = V_\tau^+ + \left(r_n\omega_\sigma^+ - r_\sigma\omega_n^+\right) = 0 \tag{35}$$

and Eq. (35) closes the system of Eqs. (31), (32), (33).

The system of *Eqs*. (31), (32), (33) and (34) or (35) consist of eight equations for eight unknown variables V_n^+, V_τ^+, V_σ^+, ω_n^+, ω_τ^+, ω_σ^+, I_n, I_τ. The solutions of them are

$$\begin{cases} V_n^+ = -eV_n^-; \quad V_\tau^+ = V_\tau^- + f(e+1)V_n^-; \quad V_\sigma^+ = V_\sigma^-; \quad I_n = -(e+1)mV_n^-; \quad I_\tau = f(e+1)mV_n^-; \\ \omega_n^+ = \omega_n^- - \dfrac{f(e+1)mr_{C\sigma}V_n^-}{J_n}; \quad \omega_\tau^+ = \omega_\tau^- - \dfrac{(e+1)mr_{C\sigma}V_n^-}{J_\tau}; \quad \omega_\sigma^+ = \omega_\sigma^- + \dfrac{(e+1)mV_n^-}{J_\sigma}\left(fr_{C\sigma} + r_{C\tau}\right) \end{cases}$$

.

$$\begin{cases} V_n^+ = -eV_n^-; \quad V_\tau^+ = V_\tau^- - \dfrac{J_n}{mr_{C\sigma}}\left(\dfrac{A^*}{B^*} - \omega_n^-\right); \quad V_\sigma^+ = V_\sigma^-; \quad \omega_n^+ = \dfrac{A^*}{B^*}; \quad \omega_\tau^+ = \omega_\tau^- - \dfrac{(e+1)mr_{C\sigma}V_n^-}{J_\tau}; \\ \omega_\sigma^+ = \left(\dfrac{r_{C\sigma}}{r_{Cn}} + \dfrac{J_n}{mr_{Cn}r_{C\sigma}}\right)\dfrac{A^*}{B^*} - \left(\dfrac{J_n\omega_n^-}{mr_{Cn}r_{C\sigma}} + \dfrac{V_\tau^-}{r_{Cn}}\right); \quad I_n = -(e+1)mV_n^-; \quad I_\tau = -\dfrac{J_n}{r_{C\sigma}}\left(\dfrac{A^*}{B^*} - \omega_n^-\right), \end{cases}$$

where

$$A^* = \frac{J_n r_{Cn} \omega_n^-}{r_{C\sigma}} + \frac{J_n J_\sigma \omega_n^-}{m r_{Cn} r_{C\sigma}} + \frac{J_\sigma V_\tau^-}{r_{Cn}} + m(e+1) r_{C\tau} V_n^- + J_\sigma \omega_\sigma^- ; \qquad B^* = \frac{J_\sigma r_{C\sigma}}{r_{Cn}} + \frac{J_n J_\sigma}{m r_{C\sigma} r_{Cn}} + \frac{J_n r_{Cn}}{r_{C\sigma}} .$$

The obtained vectors of translational and angular particle velocities immediately after the collision must be calculated in the coordinate system *O'xyz* and can be used as initial conditions for the next particle trajectory calculation.

8. Conclusions

3D numerical model of the ellipsoidal particle collision with the channel rough bed is developed. The concept of the contact zone, i.e. the set of the particle surface points, which can be in contact with the bed during the collision, is used. Since the particle-bed collision is the random process, the contact point is chosen from the contact zone as random variable using the random number generator. Then the values of the translational and angular particle velocity immediately after collision is calculated based on these values just before the collision using the impulse equations.

The presented model, which allows determination of the influence of the particle oblongness on the saltation parameters, can be used as the part of 3D numerical model of the ellipsoidal particle saltation in the channel with rough bed..

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