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TRIAXIAL PLASTICITY BASED MODEL FOR CONCRETE

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Summary: *This paper describes the development of nonlinear constitutive model for concrete and its implementation in finite element package OOFEM. The model uses non-associated plasticity formulation with corresponding hardening and softening laws. The application of the model is illustrated on nonlinear analysis of reinforced concrete column. Obtained results were compared to experimental data and to numerical analysis using micro-plane model.*

1. Introduction

Concrete is the one of the most common materials used in civil engineering. Despite the fact that many structures have been designed with the aid of quite simple relations, reliable modeling of structural response requires the development of advanced constitutive models that would be able to describe behavior in inelastic range.

The constitutive model is based on the triaxial model developed at the University of Colorado at Boulder by Kang and Willam (1999). The model formulated within the framework of non-associated plasticity theory includes material nonlinearities, pressure sensitivity, inelastic dilatancy, deviatoric evolution, brittle-ductile transition, strain softening, and limitation of hardening in equitriaxial compression. The model uses single yield surface, represented in the principal stress space as curvilinear, triple-symmetric cone that opens in the negative direction of hydrostatic axis. The yield and loading surfaces are functions of so-called Haigh-Westergaard coordinates. Hardening and softening laws depend on two variables – on equivalent plastic strain and equivalent fracture tensile strain.

This material model was implemented in the framework of OOFEM code (Patzák, 2007). OOFEM is an object-oriented finite element solver distributed under GNU general public license. The software already contains models for nonlinear analysis of concrete, such as the Microplane M4 model. However, existing models are computationally highly demanding. The new implemented model is supposed to offer a more efficient alternative.

To illustrate practical application of the model, the results of the nonlinear analysis of an eccentrically compressed column is presented.. This example was intentionally selected in

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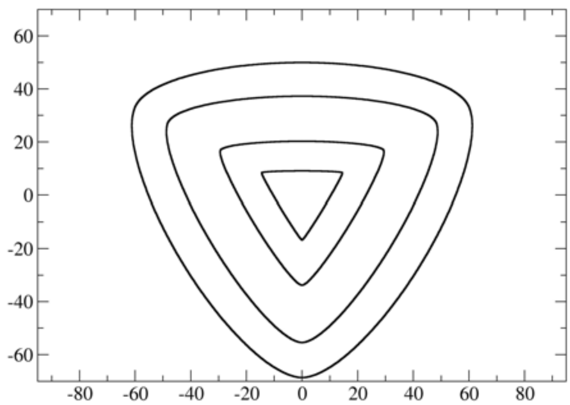
order to compare obtained results with available experimental results and with numerical results obtained using microplane-model (Němeček et al., 2005).

2. Formulation of the model

In order to describe the nonlinear behavior of concrete, the elastoplastic formulation is used. The model is based on a closed definition of the yielding surface in principal stress space. The surface is smooth except the vertex in equitriaxial tension. In order to involve different failure mode in tension and compression the evolution of surface depends mostly on the hydrostatic part of stress, which controls the transition between brittle and ductile failure.

Deviatoric sections evolve from a round triangle to circular shape with increasing confinement (Fig.1). In order to describe the failure surface in deviatoric plane, the third invariant of stress has to be taken into account. The yield surface is in triaxial compression closed by smooth cap, suddenly opening up when the hardening surface reaches the triaxial failure envelope. On the tension side the surface is limited by the so-called transition point. When this point is reached, the material starts to soften.



(Fig. 1: Evolution of deviatoric sections)

The yield surface is defined in Haigh-Westergaard coordinates, which are also invariant in stress space and represent there cylindrical coordinates with hydrostatic axis.

To avoid excessive inelastic dilatancy, it is necessary to use non-associated flow theory. The gradient of yield function is replaced by the gradient of so-called loading function, whose gradient determines the direction of plastic flow.

The yield function consists of sum of three terms: the description of the yield surface itself (F_{fail}), hardening term (F_{hard}) and softening term (F_{soft}).

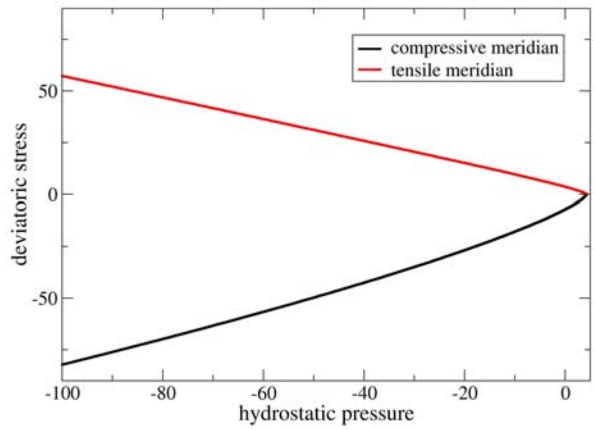
$$F\left(\xi,\rho,\theta;k,c\right)=F_{fail}+F_{hard}+F_{soft}=0\,, \tag{1}$$

where ξ, ρ, θ are the Haigh-Westergaard coordinates, k and c are the hardening and softening parameters.

Failure envelope represents in stress space a curvilinear triple-symmetric cone, which opens in the negative direction of hydrostatic axis (Fig. 2).

$$F(\xi,\rho,\theta)_{fail}=\frac{\rho r(\theta,e)}{f_c}-\frac{\rho_1}{f_c}\left(\frac{\xi-\xi_0}{\xi_1-\xi_0}\right)^{\alpha}, \tag{2}$$

Radial distance from hydrostatic axis $r(\theta,e)$ with eccentricity parameter e establish the position of the yield surface in deviatoric plane, as already mentioned. In hydrostatic direction the surface is limited by equitriaxial tensile strength ξ_0 and the compressive limiter ξ_1. The value of ρ_1 is determined from the condition, that compressive meridian should pass through the point of uniaxial compression. Parameter f_c denotes the compressive strength.



(Fig. 2: Meridional sections)

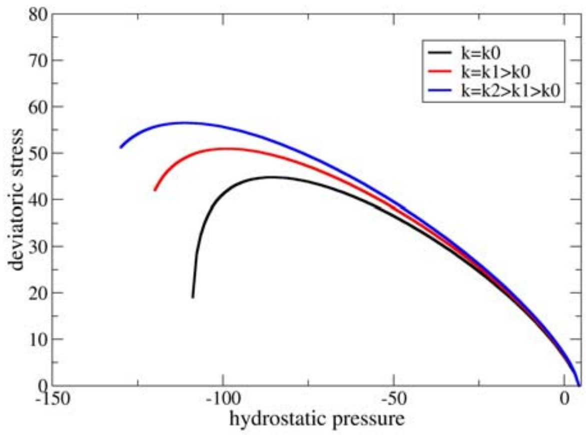
Evolution of hardening is included through the second term of the yield function (Fig. 3). It depends on hydrostatic pressure ξ and parameter *k*.

$$F(\xi,k)_{hard}=-\frac{\rho_1}{f_c}\left[\left(\frac{\xi-\xi_0}{\xi_1-\xi_0}\right)^{\beta(k)}-1\right]. \tag{3}$$

The hardening parameter *k* is a function of the equivalent plastic strain and the hydrostatic pressure ξ, where the effect of increasing ductility under increasing confinement can be involved. It takes the value

$$k_0\le k\le 1. \tag{4}$$

When *k* reaches the value of 1, the exponent β becomes 0, the F_{hard} term vanishes and the yield surface is identical to failure envelope.



(Fig. 3: Evolution of yield surface with hardening)

When hydrostatic pressure ξ is greater than so-called transition point ξ_c and the stress path reaches the failure envelope $k=1$, the failure envelope starts to soften

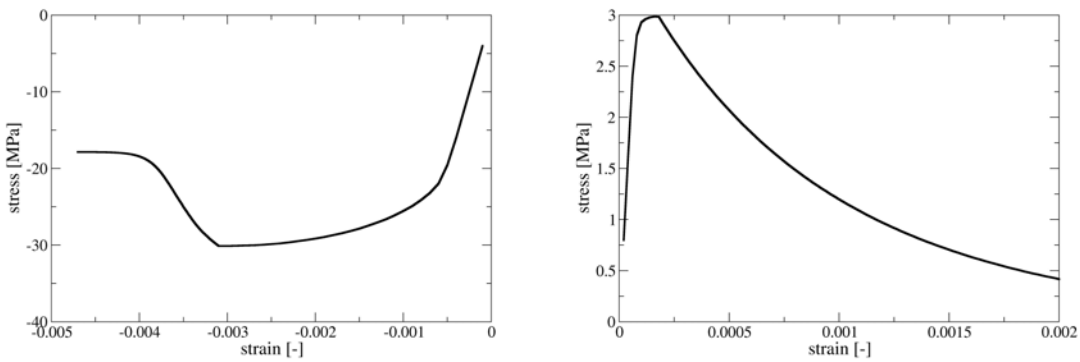
$$F(\xi,c)_{soft} = \frac{\rho_1}{f_c}(1-c)\left(\frac{\xi_0}{\xi_0-\xi_1}\right)^{\alpha}\left(\frac{\xi_c-\xi}{\xi}\right)^2 \tag{5}$$

The softening parameter c is at the beginning equal 1. Softening process is driven by equivalent tensile strain. In order to achieve objectivity with respect to the element size, softening law depend also on characteristic length of an element, to ensure correct energy dissipation.

3. Impelementation

The material model was implemented into the finite element code OOFEM, developed at Department of Mechanics. OOFEM is a general purpose finite element solver with an object-oriented architecture. The implementation into the code consists of the introduction of two new classes representing new material model and associated status for storing history of variables. The class representing the material model has been derived from MPlasticMaterial class, which is a common class for all plasticity models. This allows taking advantage from reusing existing general purpose implementation of stress return and stiffness matrix evaluation. As a result, only the methods that evaluate yield and loading functions and their derivatives have to be implemented. Because of complex definition of the failure criteria and complicated relationship between Haigh-Westergaard coordinates and values of stress, the MAPLE software has been used for checking und computing complex expressions of derivatives.

The new model was verified on simple examples (Fig. 4), that included results published in the original paper (Kang and Willam, 1999) and with other results from literature.

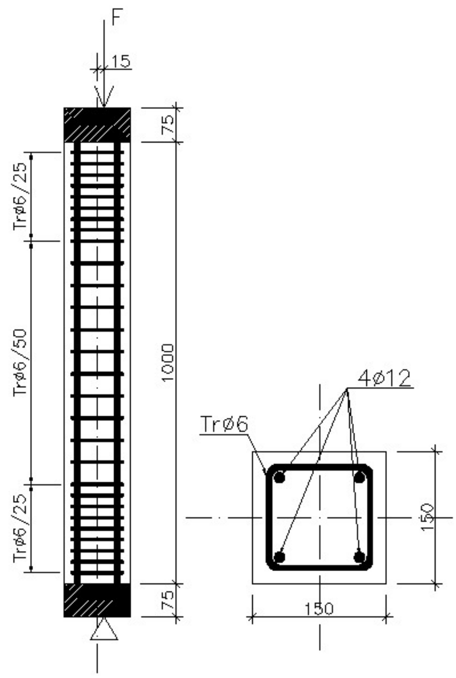


(Fig. 4: Stress-strain response in compression and tension)

4. Application of the model

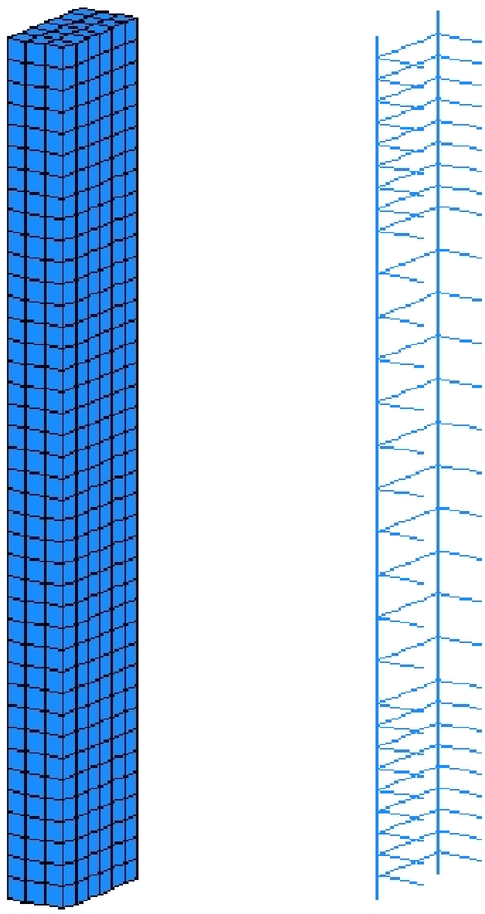
The implemented model was applied for a limit analysis of eccentrically compressed column. The experiments have been performed by Němeček et.al (2005), in the same work a numerical analysis using microplane model has been done. The character of failure, as well as column strength was studied for different transversal reinforcement settings. The M4-microplane model was chosen for the numerical analysis that time.

The analyzed column with square cross section (150x150mm) had a length of 1000mm. It was reinforced with 4 bars of 12mm in longitudinal direction. Transversal reinforcement was formed by closed stirrups with diameter 6mm. The column was loaded with compressive load with eccentricity of 15mm (Fig. 5).



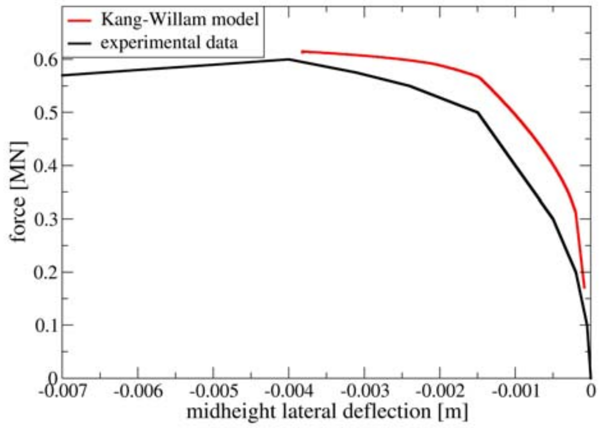
(Fig. 5: Column geometry)

To make the analysis less time-consuming, the computation was performed only for half of the column with appropriate boundary conditions with respect to the problem symmetry. The structured mesh made of brick elements with linear interpolation has been used. The reinforcement has been included using beam elements for longitudinal reinforcement and truss element for stirrups, with plasticity-based model for steel (Fig. 6). The use of hanging nodes allowed inserting reinforcement into column mesh conveniently.



(Fig. 6: Mesh used for concrete and reinforcement)

A nonlinear static analysis was controlled using prescribed displacement history of the loaded point. The obtained results are compared to the experimental results (Fig. 7). The overall agreement is very good, illustrating the capabilities of the model.



(Fig. 7: Results comparison)

5. Conclusions

A new nonlinear triaxial model has been implemented in the OOFEM software. This model has been verified on simple examples for different loading regimes and applied on the analysis of eccentrically compressed column.

Acknowledgement

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