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DAMAGE MODELS USED FOR MODELLING OF SEQUENTIAL CASTING PROCEDURE OF FOUNDATION SLABS

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Summary: *New modifications of SIFEL program improved simulation of the real process of the casting procedure. It was implemented and successfully used numerical solution of B3 model using a continuous retardation spectrum method. A new solver of time dependent problems was implemented and used. The solver contains Newton-Raphson iteration method which significantly improved results. A scalar damage model was definitely modified to respect time variability of Young modulus and tensile strength. Advanced models of anisotropic damage were theoretically derived and implemented. Mentioned methods and models were used for the 2D model slab which was cast in several layers. The behavior was solved as coupled problem.*

1. Introduction

The foundation of structures on foundation slabs is very common case. Foundation slabs can have a significant thickness in case of large structures and thus the problem with their concreting arises due to the hydration heat generation. It is necessary to use a sufficiently apposite numerical model for the verification of the casting process. That was the reason for developing the suitable tools and models for checking of casting procedure in the SIFEL program.

Casting procedure of the foundation slab can be solved as a coupled thermo-hydro-mechanical problem in this software. The Künzel and Kiessl`s models are at disposal for the simulation of the heat and moisture transport processes. The Bazant's B3 model can be used for the description of concrete creep and shrinkage, damage can be modelled by the scalar isotropic damage model or some more advanced anisotropic damage model. The subsoil under the foundation slabs had to be modelled by the system of spring supports due to the

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complexity of the computation process. The stiffness of the springs can evolve nonlinearly depending on settlement of the foundation slab. Another possibility of the subsoil model represents using of the full 2D or 3D subsoil model, but this approach would increase the computation time too.

2. Modelling of mechanical behaviour in case of sequential casting procedure

Several important modifications of SIFEL program were performed on basis of experiences from the previous analyses of coupled problems. These modifications improved simulation of the real process of the casting procedure. First of all, the code was extended by the implementation of the sequential construction using time controlled switching on/off of particular degrees of freedom. A new solver of time dependent problems was implemented and used. The solver contains Newton-Raphson iteration method which significantly improved results. A scalar damage model was definitely modified to respect time variability of Young modulus and tensile strength. Advanced models of anisotropic damage were theoretically derived and implemented. Mentioned methods were used for a model of cranked foundation slab which was cast in three layers. The behavior was solved as a coupled problem in which the mechanical behavior was assumed together with heat and moisture conduction and their interactions.

It was shown that the results hardly depend on precise model of the real process of casting procedure. Used creep model B3 is too complicated and it was not possible to use zero or almost zero Young modulus for non-cast parts. This is why the SIFEL code was extended by the time controlled switching on/off of particular structure pieces. This was realized by the time controlled change of nodal code numbers. Above approach can simulate the sequential construction in the best possible way because the non-cast parts of structure are not assumed during the computational process.

Concrete exhibits very complicated mechanical behavior and many material models were developed to describe it. But most of them can capture only one or several attributes of this behavior and this leads to combination of these models.

The decomposition of the total strain in the material point to the particular components is the base of the computations with several different material models describing different attributes of the mechanical behavior. In described cases, the following decomposition of the total strain is used

$$\varepsilon_{tot} = \varepsilon_e + \varepsilon_d + \varepsilon_{cr} + \varepsilon_{shr} + \varepsilon_{ft}, \quad (1)$$

where:

ε_{tot} is a total strain

ε_e is an elastic component of strain

ε_d is a strain component due to damage of concrete

ε_{cr} is a creep component of strain

ε_{shr} is a shrinkage component of strain

ε_{ft} is a free temperature strain

Creep and shrinkage effects of concrete together with free temperature strains have significant effect to the total strain which is decisive for evolution of cracks. The B3 model can be used for creep and shrinkage modeling, details about this model can be found in (Bazant et. al, 1984). Both effects are included in it and they are temperature dependent. The original numerical solution of B3 model was based on Dirichlet series and it was found too slow for purposes of the given coupled problem and that is why the continuous retardation spectrum method was implemented. Detailed description of the above method can be found in Bazant & Xi (1995).

2.1 Scalar isotropic damage model

Significant number of material models were developed describing damage of concrete. Taking into account that the computing coupled problems is extremely time consuming, the simple scalar isotropic model have been used. In this model, the damage description is very simplified, but it can be used as an indicator, whether a structure will be damaged and where it will be damaged eventually. The model is dependent on sizes of mesh elements and it dissipates different energy for the various mesh elements density. This was the reason for the rewriting of the model and the variable softening modulus technique was employed, which partially suppress this fault. The method consists in that the strain due to damage is assumed in the following form

$$\varepsilon_{\mathrm{d}} = \varepsilon - \varepsilon_{el} = \frac{w}{h}, \tag{2}$$

where w is the crack opening and h is a characteristic size of element and ε is defined as follows

$$\varepsilon = \varepsilon_{tot} - (\varepsilon_{cr} + \varepsilon_{shr} + \varepsilon_{ft}) \tag{3}$$

During damage evolution, the stress can be formulated depending on the crack opening w

$$\sigma = f_t \exp\left(\frac{w}{w_f}\right), \tag{4}$$

where f_t denotes tensile strength of concrete and w_f is the initial crack opening which is treated as a material parameter. The stress can be written for scalar isotropic damage in the following form

$$\sigma = (1-\omega) D_{el} \varepsilon \, . \tag{5}$$

Equations (3), (4) and (5) can be combined and assuming ε_{el} as

$$\varepsilon_{el} = \frac{\sigma}{E} \tag{6}$$

the resulting nonlinear equation can be written for the damage parameter ω.

$$(1-\omega) E \varepsilon = f_t \exp\left(-\frac{\omega h \varepsilon}{w_f}\right) \tag{7}$$

Equation (7) was derived for the uniaxial stress state assumption and it is necessary to replace strain ε by the equivalent strain ε_{eq} for other cases. Equation (7) is nonlinear and it can be solved by Newton method. Detailed description of the variable strain softening technique can be found in Jirásek (1998).

2.2 Anisotropic damage model

The scalar isotropic damage model can be used for case of uniaxial tension quite successfully. The computation of this model is very fast and this was the reason for its using in the given coupled problem. The model has only one damage parameter ω and that is why damage evolution in one direction reduces stiffness in rest directions. It introduces certain inaccuracy especially for changing to a 3D model. Employing much more effective method for B3 model (continuous retardation spectrum) creates space for application of a more robust damage model.

An anisotropic damage model represents one possible way of damage modelling. The base of this model was described in Papa & Taliercio (1996). Following damage parameter and two independent damage tensors were established:

d- volumetric damage parameter introduced only for tension

$\mathbf{D}^t$ - damage tensor for damage induced by tensile strains

$\mathbf{D}^c$ - damage tensor for damage induced by compressive strains

The model assumes splitting of the elastic strain tensor into its tensile and compressive parts

$$\mathbf{e} = \mathbf{e}^t + \mathbf{e}^c \tag{8}$$

where $\mathbf{e}^t$ (respectively $\mathbf{e}^c$) is a strain tensor having the same positive (respectively negative) eigenvalues as $\mathbf{e}$ and vanishing remaining eigenvalues. They can be expressed as:

$$e^t_{ij} = \sum_{\alpha=I}^{III} \langle e_\alpha \rangle n_i^\alpha n_j^\alpha ; \quad e^c_{ij} = -\sum_{\alpha=I}^{III} \langle - e_\alpha \rangle n_i^\alpha n_j^\alpha \tag{9}$$

where the index α denotes principal direction, $\langle \cdot \rangle$ are Macauley brackets and n_i^α are components of eigenvectors of $\mathbf{e}$. The model is derived form the elastic potential which was slightly modified in the volumetric part compared to the original proposed in Papa & Taliercio (1996). The following expression is obtained assuming above equations for Helmholtz free energy

$$\begin{aligned} \rho\psi_{el} = \frac{3}{2}\left[(3K-2G)\left(\varepsilon_{vol}^2 - d\langle \varepsilon_{vol} \rangle^2\right)\right] + G\left(\mathbf{1}-\mathbf{D}^t\right)^{1/2}\mathbf{e}^t\left(\mathbf{1}-\mathbf{D}^t\right)^{1/2} : \mathbf{1} \\ + G\left(\mathbf{1}-\mathbf{D}^c\right)^{1/2}\mathbf{e}^c\left(\mathbf{1}-\mathbf{D}^c\right)^{1/2} : \mathbf{1} \end{aligned} \tag{10}$$

where K is bulk modulus, G is shear modulus, ε_{vol} is the volumetric strain and $\mathbf{1}$ is the second order identity tensor. Damage driving forces conjugated to d, $\mathbf{D}^t$ and $\mathbf{D}^c$ can be derived from Equation (10) by its derivation with respect to damage parameters:

$$\mathbf{Y}^t = -\frac{\partial(\rho\psi_{el})}{\partial \mathbf{D}^t} = G\,\mathbf{e}^t\mathbf{e}^t,\ \mathbf{Y}^c = -\frac{\partial(\rho\psi_{el})}{\partial \mathbf{D}^c} = G\,\mathbf{e}^c\mathbf{e}^c,\ y = -\frac{\partial(\rho\psi_{el})}{\partial d} = \frac{9}{2}K\langle \varepsilon_{vol} \rangle^2 \tag{11}$$

The tensors $\mathbf{Y}^t$ and $\mathbf{Y}^c$ have the same principal directions as $\mathbf{e}$ and by consequence, both $\mathbf{D}^t$ and $\mathbf{D}^c$ also have the same directions. These forces have the form of potential energy of the appropriate part of the strain tensor i.e. deviatoric and volumetric respectively. Evolution of principle damage components of tensors $\mathbf{D}^t$ and $\mathbf{D}^c$ is controlled by load functions which are defined by Equation (12).

$$f_\alpha^\beta = \left(1 - D_\alpha^\beta\right)\left[1 + A_\beta \left(Y_\alpha^\beta / E - Y_0^\beta\right)^{B_\beta}\right] - 1 \leq 0 \tag{12}$$

where the index α denotes principal direction, the index β can be either t or c, and A, B are material parameters ($A \geq 0, B \geq 1$), E is the Young modulus and Y_0 is a threshold value of the given non-dimensional damage driving force.

$$f = \left(1 - d\right)\left[1 + a\left(y / E - y_0\right)^b\right] - 1 \leq 0 \tag{13}$$

Similarly, evolution of volumetric damage is controlled by the load function described by Equation (13), where a, b are material parameters and y_0 is a threshold value of the non-dimensional volumetric damage driving force. Introduced load functions define elastic domains for their negative values and no evolution of damage occurs in these domains. When the given damage force is on boundaries of domain ($f = 0$), damage components evolves. Note that the damage components can be increased only and their values have to be in range <0, 1>, where 0 means no damage and 1 means full damage.

Components of principal stresses σ_α can be obtained from Equation (10) by derivation with respect to $\mathbf{e}$ and the transformation to principal directions:

$$\sigma_\alpha = \left[(3K - 2G)(1 - d\,H(\varepsilon_{vol}))\right]\varepsilon_{vol} + 2G\left[(1 - H(\varepsilon_\alpha))D_\alpha^t - H(-\varepsilon_\alpha)D_\alpha^c\right]\varepsilon_\alpha \tag{14}$$

where $H(\cdot)$ is Heaviside function.

Correct energy dissipation depending on the element size can be derived similarly as for the scalar isotropic damage model. Dissipation is defined by

$$\mathcal{D} = \int_0^\infty \mathbf{Y}\,\dot{\mathbf{D}}\,dt \tag{15}$$

and the dissipation can be related with fracture energy $\mathcal{G}_f$

$$\mathcal{G}_f = h\mathcal{D} \tag{16}$$

where h is a characteristic element length. Rewriting Equation (15) for one principal damage component and substituting time derivations from Equation (14) leads to the integral

$$\mathcal{D} = \int_{Y_0}^\infty Y \frac{A\,B\left(Y - Y_0\right)^{B-1}}{E\left[1 + A\left(Y - Y_0\right)^B\right]^2}\,dY \tag{17}$$

After integration, the final relation is obtained depending on fracture energy, element size and material parameters A, B for particular principal directions and tension or compression

$$\left[\left(\frac{G_f}{h}-\frac{Y_0}{E}\right)\frac{BE\sin\left(\frac{\pi}{B}\right)}{\pi}\right]^{-B} = A \tag{18}$$

3. Results form performed analyses

Thermo-hydro-mechanical coupled problems are extremely time consuming and that was the reason for using only 2D problem although the program can solve also the 3D problems and all material models are derived for 3D too. Computations are slowed down not only by the complex material models but also by necessity of a relatively fine mesh and short time steps. The solution of the given problem takes about one day on a computer with Pentium 4 on 3.2 GHz frequency. Approximately, two days of structure real existence are computed during this time. The numerical analysis was concerned with modelling of the foundation slab of a commercial building in Prague-Těšnov. The slab is 1 m thick and it is cast with step 1.3 m. Particular slab spans are 14.8 and 15 m, shrinkage bands are left on the boundaries and their width is 1.35 m. The slab is reinforced with 12ØV25/m both in longitudinal and transversal directions. Dimensions and shape of the finite element mesh is depicted in Fig. 1.

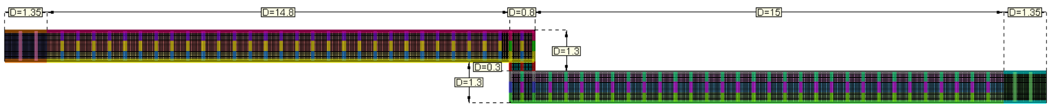

Fig. 1: Dimensions of the model and finite element mesh – overall view.

The mesh had to be fine on boundaries, which was caused by increasing of temperature and humidity gradients in these places and by the damage occurrence too. The slab was cast in three layers and it was watered by three days and covered by PE sheet after the casting finish. The plane-strain model of the slab was assumed. The computer simulation begins at 1 hour after the casting finish of the first layer. In the performed thermo-hydro-mechanical analysis, Künzel-Kiessl model was used for modelling of transport processes, B3 model and scalar isotropic damage model were used for the description of the mechanical behaviour. The slab was supported by spring supports on the bottom and stiffness of springs on the edges was increased in order to capture of subsoil behaviour. The slab was loaded by dead weight and thermal boundary conditions were applied. These thermal conditions simulated average temperature in June and they were obtained by the long-term measurements in the given locality. Casting of particular slab spans was performed in three layers which were sequentially turned on in the model by 1 hour from the casting finish.

Figures 2-5 capture damage distribution and they were obtained for time 15 hours from the casting. Fig. 2. captures overall view on the bottom level of the foundation slab and a major damaged zone can be seen at the bottom. Its detail is also in Fig.4. Minor damaged zones are depicted in Fig. 3., which represents detail of damaged zone at the top, and Fig. 5, which represents damaged zone at the right corner. Time history of damage parameters evolution is depicted in Fig. 6.

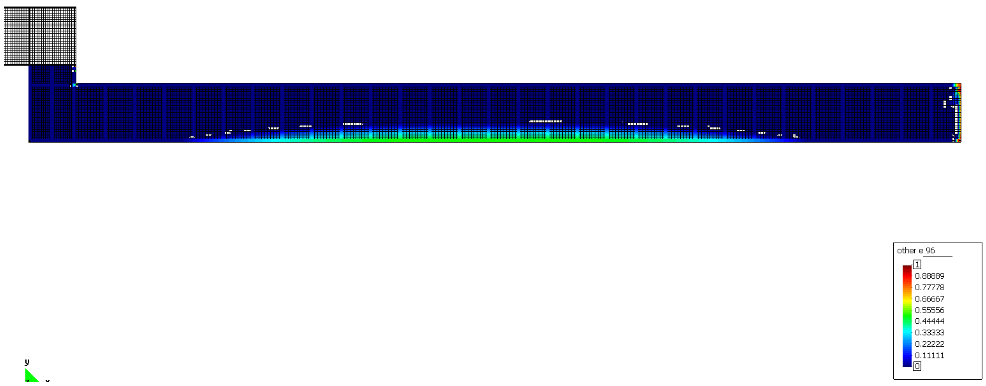

Fig. 2: Distribution of the damage parameter ω for three layers of concrete after 15 hours from the casting.

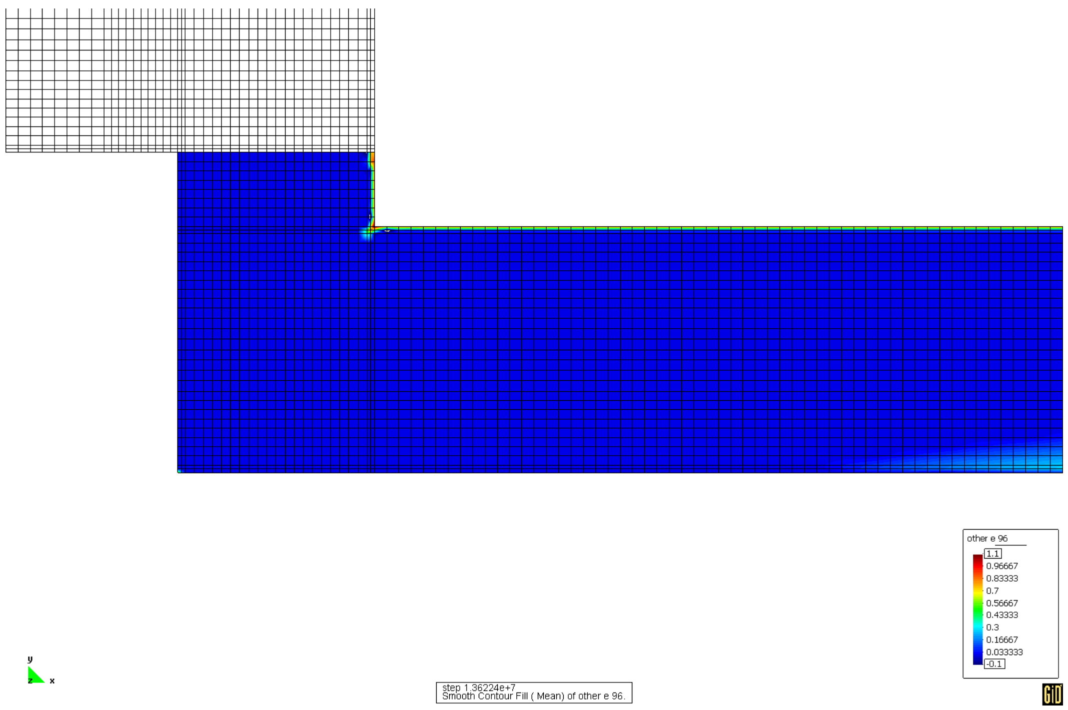

Fig. 3: Distribution of damage parameter ω for three layers of concrete in the top left corner of the slab after 15 hours from the casting.

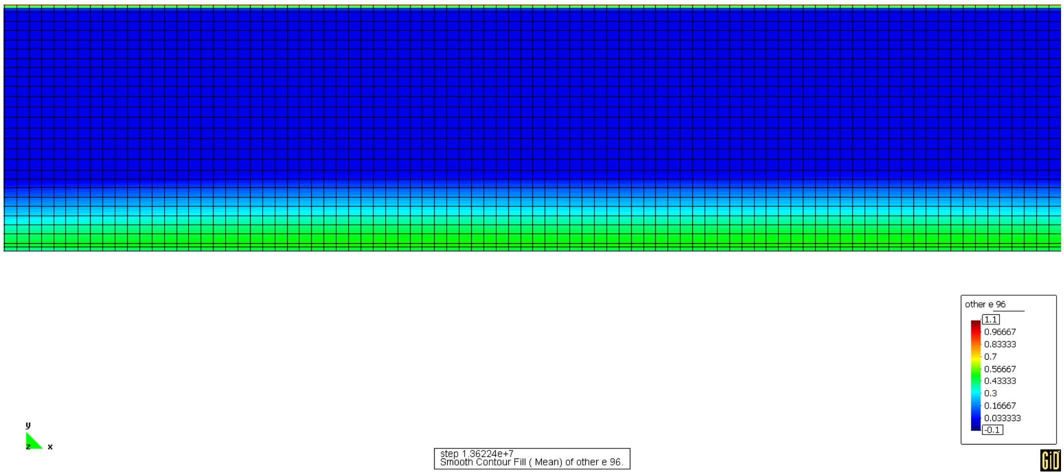

Fig. 4: Distribution of the damage parameter ω for three layers of concrete in the middle of the slab after 15 hours from the casting.

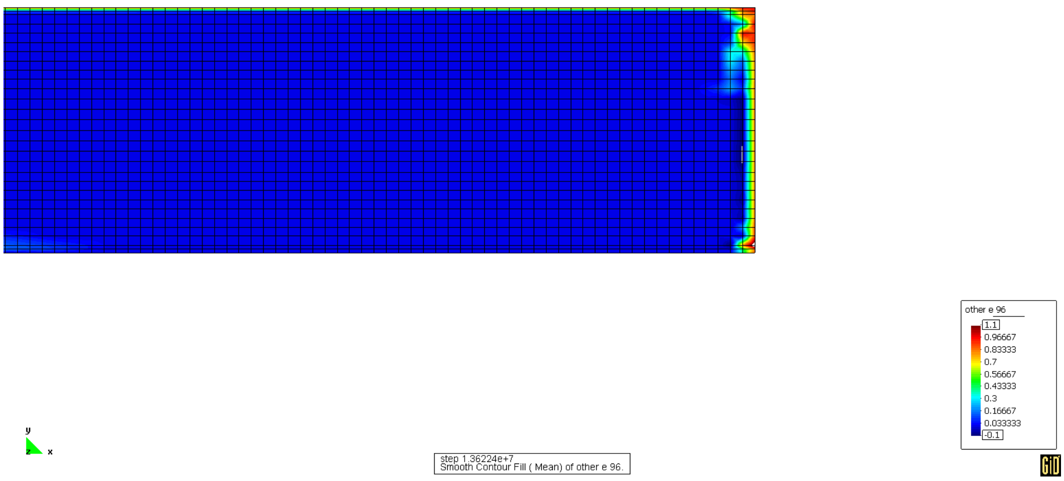

Fig. 5: Distribution of the damage parameter ω for three layers of concrete in the top right corner after 15 hours from the casting.

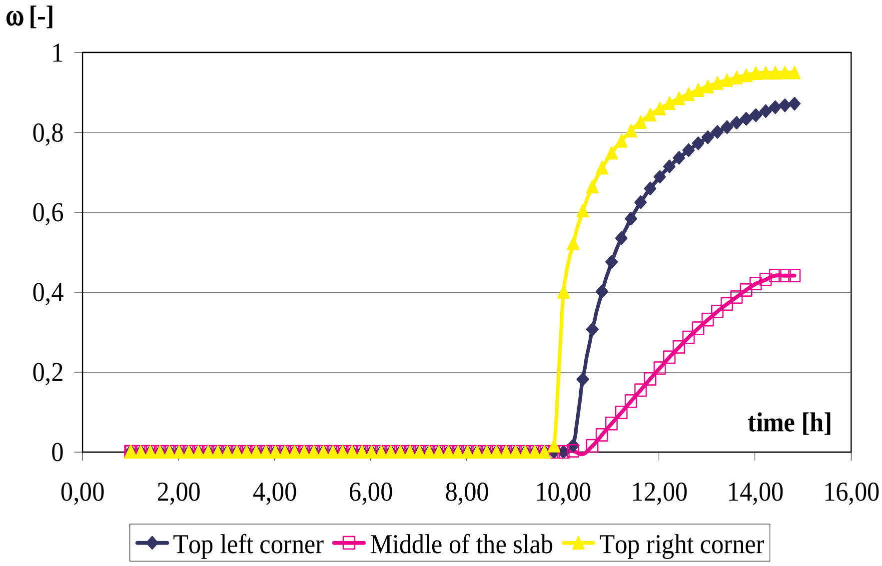

Fig. 6: History of evolution of the damage parameter ω for three characteristic domains of the slab.

4. Conclusions

Results of the analysis confirmed that the precise modelling of the sequential construction is very important for damage parameter evolution. Resulting distribution of the damage parameter can be viewed in Fig. 2. The distribution corresponds approximately to time 15 hours from the casting finish of the first bottom layer of the low slab. In Fig. 4, it can be seen on the detailed view that the bottom layer is damaged to 20 cm depth. Maximal damage parameter value is about 0.4. This damage is caused by the hydration process in the top layer which is delayed compared to bottom layers. During the peak of hydration heat generation in the top layer, the slab is deformed by the reason of non-uniform heating and it has tendency to deflect up. The bottom layer is than damaged in the middle as a result of dead weight load.

Climate conditions are another important factor causing damage. It can be recognized in Fig. 3 that all top surface of the slab is damaged but only to the low depth. This damage is caused by drying shrinkage which is amplified by applied climate conditions. The top right corner and the slab front were last significantly damaged areas. Damage was caused by shear stresses and it can be seen in Fig. 5.

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