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INCEPTION OF SALTATION MODE OF SEDIMENT TRANSPORT

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Summary: *The bed load transport usually occurs in three consecutive modes: rolling, saltation, and suspension. The start of rolling motion is investigated widely, and different models are constructed and calibrated using available experiments. There are available few experiments and models for threshold between rolling and saltation. Herein the threshold of saltation in turbulent flow was determined and presented in common variables introduced by Shields (1936). The additional dimensionless parameter characterizing the bed surface formation was introduced. For the condition of the threshold there was chosen the equality of the submerged gravitational force and the lift force on a bed particle. For the low-studied shear induced lift force there were used data on well studied Magnus force on a rotating ball moving translationally in calm water. The critical shear stress as function of bed particle Reynolds number was determined analytically and reported in graphical form for different values of the bed parameter. The obtained results agree with available experiments on saltation start.*

1. Introduction

Saltation is the predominant mode of bed load transport in rivers and channels. During saltation particles hop up from the channel bed and follow ballistic like trajectory till the next bounce with bed. The bed load transport occurs not always in mode of saltation. The preceding modes are sliding and rolling of the particles over bed. Determining the flow parameters corresponding to threshold of the saltation mode of the particles movement is the aim of this paper.

With aim to determine the moment of initial bed load movement Shields (1936) examined the forces acting on a grain resting on the bed of a water stream. He considered the horizontal resisting force to be proportional to $\left(\rho-\rho_f\right)gd^3$, where ρ is the density of the grain immersed in the fluid of density ρ_f, g is gravitational acceleration, and d is a mean diameter of the grain. The drag force exerted by the flow on a grain was taken to be proportional to $\rho_f d^2 u^2$, where u is the fluid velocity. The velocity was assumed according to Nikuradse (1933) to be given by the logarithmic law:

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$$\frac{u}{u_*} = 5.75 \lg \frac{y}{d} + f_1\left(Re_*\right), \tag{1}$$

where $Re_* = \dfrac{u_* d}{\nu}$ is the particle Reynolds number based on shear velocity u_*, ν is the fluid kinematic viscosity, and y is the elevation above the datum somewhere below the bed surface. For the elevation corresponding to the grain starting motion, $\dfrac{y}{d}$ was assumed to be a constant of the order of magnitude 1, giving $\dfrac{u}{u_*} = f_2\left(Re_*\right)$. Substituting this into the drag force, equating drag and resistance forces as corresponding to initiation of motion, and re-arranging,

$$\vartheta_0 = \frac{u_*^2 \rho_f}{\left(\rho - \rho_f\right) g d} = \vartheta_0\left(Re_*\right), \tag{2}$$

where ϑ_0 is dimensionless threshold shear stress, also known as Shield stress. Dependence (2) can be derived also from dimensional considerations only, see van Rijn (1984), for instance. The variation of ϑ_0 with Re_*, corresponding to the initial moment of motion, was determined experimentally in channel with flat bed formed of almost equal grains, see Fig. 1.

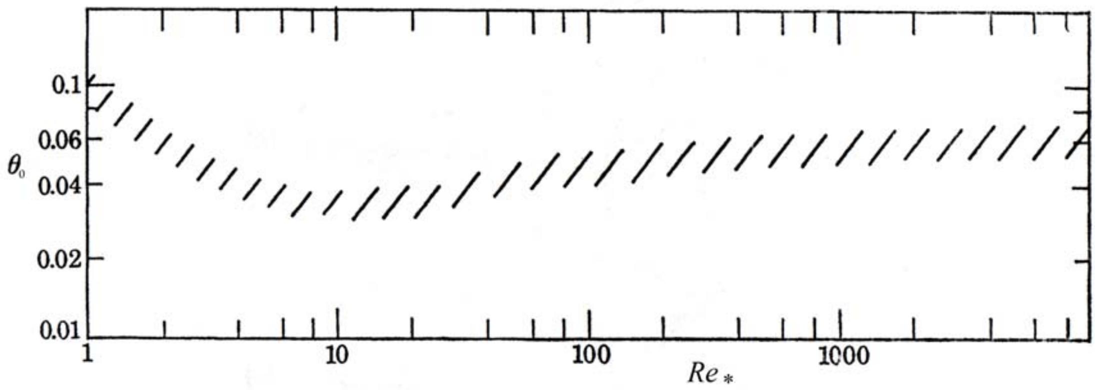

Fig. 1: The Shields diagram: the shaded region defines the threshold ϑ_0, above which grains move.

Yang (1973) developed the model for starting of grains movement considering the drag, the lift, and the submerged gravitational forces. The model was calibrated using numerous experimental data reported in several papers of other authors. As a result a relationship was proposed determining critical average flow velocity related to terminal fall velocity as a function of grain shear Reynolds number.

Fenton & Abbott (1977) measured the dimensionless threshold stress ϑ_0 and its dependence on grain protrusion, which was found to be very marked. However, after examining the experimental results, they concluded that the Shields diagram implicitly contains variation with relative protrusion of the grains.

Yalin & Karahan (1979) studied experimentally the threshold of spherical grains movement in a channel. Their investigation was devoted mostly to the laminar regime $Re_* < 25$. As a result they corrected the Shields diagram in that region, stating $\vartheta_0 = 0.1 Re_*^{-0.3}$ for $Re_* < 1$ instead of approximation to Shield's curve $\vartheta_0 = 0.1 Re_*^{-1}$ at the same Re_*.

Ling (1995) considered the balance of forces and moments on a grain resting on a bed in order to estimate the threshold of grain movement. He distinguished two thresholds – rolling threshold for incipient motion and lifting threshold. The condition of second threshold was the equality of lift force and submerged gravitational force on a laying grain. However, the lack of data on lift coefficient compelled him to use the Saffman (1965) shear lift and Rubinov & Keller (1961) spin induced lift, that are valid for creeping flows with Reynolds numbers based on velocity, angular velocity and velocity gradient, being much less than unity. According to Ling (1995), for $20 \le Re_* \le 500$ the lifting threshold occurred at $\vartheta_0 \approx 0.1$.

Chepil (1958) noted that lift on hemispherical surface projections, similar to grains resting on a surface in a wind stream, is substantial. He concluded that lift must be recognised together with drag in determining an equilibrium or critical condition between the soil grains and the moving fluid at the threshold of movement of the grains.

According to experiments of Ancey et al. (2002), conducted for about $4 < Re_* < 350$, the threshold between saltating motion and rolling occurs at $\vartheta_0 = 0.3$.

First, sliding and rolling occurs, then saltation and then suspension, governed by turbulent fluctuations. Before the turbulent fluctuations are big enough, the upward momentum, acquired by a particle, comes only from change in momentum during collision and from lift force due to rotation and shearing of flow. Due to the nature of origins of upward momentum, we suppose that the main factor responsible for starting of saltation is the lift force. This hypothesis can be summarized in the following equation, determining the instant of saltation start, expressing the equality of the lift force and the submerged gravitational force on a particle:

$$F_L = F_g, \tag{3}$$

If a particle starts saltation due to the lift force from immobile position on the bed it will not cease it under the same flow conditions, see Ancey et al. (2002). Thus condition (3) describes the start of saltation and slightly overestimates the shear stress ϑ_0 corresponding to saltation stop.

For correct analysis of the flow and determining parameters corresponding to the start of saltation, it is indispensable to know the bed structure and the relative position and elevation of the bed grain, that is about to start saltating motion.

Sekine and Kikkawa (1992) observe that in natural rivers, bed load transport often occurs over a bed composed of grains of similar size. Sliding or rolling modes are severely limited, due to irregularity of bed, and the dominant mode of bed-load transport is saltation. The irregularity of grain size thus can be neglected. Saltation ceases when a particle is captured in occasional bed depressions. A captured particle undergoes an in situ oscillating or vibrating motion until its energy is dissipated.

According to van Rijn (1984), Sumer & Deigaard (1980), Song et al, (1994), the virtual bed level, associated with the velocity profile, is 0.25*d* below the top of the bed particles. Van

Rijn supposes that the particle that is about to start movement rests on a bed surface of closely packed particles. Thus the elevation of the particle above the bed level is 0.6*d*. This schematization evidently can not represent the movements of all the bed particles; however it can simulate the start of motion for some classes of beds.

As indicate Fenton & Abbot (1977), the bed configuration may vary considerably from densely and accurately packed bed grains particle to particle, up to grains packed so randomly that some odd grains protrude above the average bed level to almost a complete grain diameter. That validates the bed model used, see Fig. 2.

Ling (1995) also accepts the bed formation model similar to that of van Rijn and according to his scheme, the resting bed grain that is about to start movement is 0.5d above the virtual bed level. For estimation of the lift force Ling uses the data of Saffman (1965) and Rubinov and Keller (1961). However, Saffman and Rubinov & Keller results are valid only for small Reynolds numbers, and predict the lift force substantially higher than that corresponding to Reynolds numbers considered. Hence, he can underestimate the bed shear stress required to lift the particle.

In present paper we decided to use the same approach as Ling (1995) but to account for the values of lift coefficient that are valid for higher Reynolds numbers.

2. Model

Consider a bed formed of uniform particles. The particle that is about to start saltation (test particle) rests on the tops of the bed particles, see Fig. 2. The shape of particles is spherical. The flow of the stream is turbulent with velocity distribution logarithmic law

$$\frac{u}{u_*} = 2.5 \ln \frac{y}{y_0}, \tag{4}$$

valid for $Re_* > 20$; where y_0 is the roughness parameter equal $k_s/30$, $k_s = \xi d$ is the equivalent sand roughness of Nikuradse's experiments on rough pipes. According to van Rijn (1984), dimensionless parameter ξ should be set equal to about 2 or 3. The level $y = 0$ is the virtual bed level.

The elevation of the centre of the test particle above virtual bed level is $y = \zeta d$. In paper of Ling (1995), the dimensionless parameter ζ was 0.5, according to van Rijn, it equals 0.6. Parameters ξ and ζ due to their ambiguity remain much of freedom and in reasonable ranges will be varied herein.

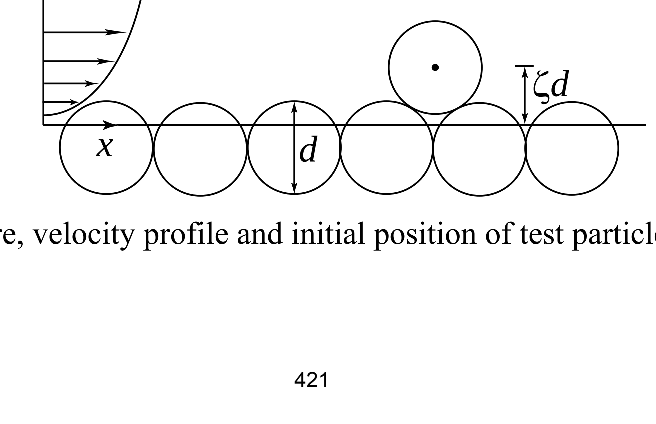

Fig. 2: Bed structure, velocity profile and initial position of test particle.

The condition of the particle start of saltation is accepted to be the equality of submerged gravitational force and lift force due to fluid shearing exerted on immobile grain resting on the bed surface:

$$F_L = F_g, \tag{5}$$

where

$$F_g = g\Omega\left(\rho - \rho_f\right), \tag{6}$$

Ω is the volume of the test particle. Data on the lift force on a sphere due to shear flow is very scarce. For the lift force there was taken the expression for the Magnus force, which is relatively widely investigated:

$$F_L \approx F_M = C_M \Omega \rho_f \omega u. \tag{7}$$

The angular velocity ω was replaced by $\frac{1}{2}\left|\mathrm{rot}\,\vec{u}\right| = \frac{1}{2}\frac{du}{dy}$, what corresponds to the angular velocity of infinitesimal fluid particle in the centre of test grain in case if the grain is absent. C_M is a dimensionless Magnus force coefficient and its value in known for $0.1 < \Gamma < 10$, $0.5 < Re < 140000$, where $\Gamma = \frac{\omega d}{2u}$ is a spin parameter, and $Re = \frac{ud}{\nu}$ is the particle Reynolds number based on actual particle velocity.

From (4) we calculate

$$\omega = \frac{5}{4}\frac{u_*}{y} = \frac{5}{4}\frac{u_*}{\zeta d}. \tag{8}$$

Substituting (4), (6), and (7) into (5) we obtain

$$g\left(\rho - \rho_f\right) = C_M \rho_f \frac{25}{8}\frac{u_*^2}{\zeta d}\ln\frac{30\zeta}{\xi}, \tag{9}$$

or

$$\vartheta_0 = \frac{1}{C_M}\frac{8\zeta}{25}\frac{1}{\ln\left(30\zeta/\xi\right)}. \tag{10}$$

The dependence $C_M\left(Re,\Gamma\right)$ is obtained by fitting of the Magnus coefficient data retrieved from Maccoll (1928), Barkla & Auchterlonie (1971), Tsuji et al. (1985), Tanaka et al. (1990), Naumov et al. (1993), Oesterle & Dinh (1998), Changfu et al. (2003), using the least square method with a simple function, see Fig. 3 ,4. The average fitting error is 25%.

$$C_M = \frac{1 + cRe}{a + bRe}, \quad a = a_1 + a_2\Gamma + a_3\Gamma^2,$$
$$b = \frac{b_1 + \Gamma}{b_2 + b_3\Gamma}, \quad c = \frac{1}{c_1 + c_2\Gamma}, \tag{11}$$

where

$$a_1 = 1.333, \quad a_2 = -0.061, \quad a_3 = 0.029,$$
$$b_1 = 5.9, \quad b_2 = 38, \quad b_3 = 4.6, \tag{12}$$
$$c_1 = 25, \quad c_2 = 21.$$

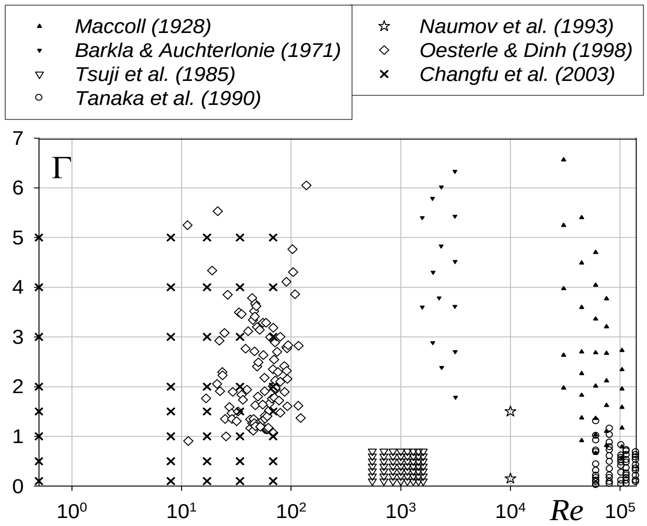

Fig. 3: The map $Re \times \Gamma$ of available experiments.

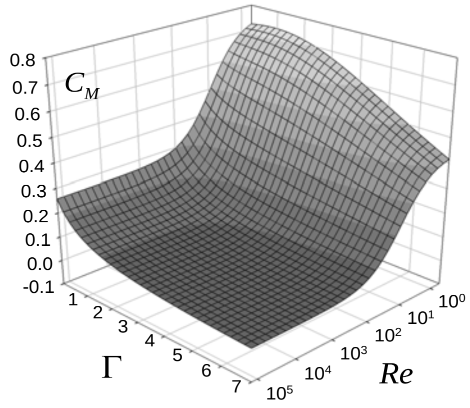

Fig. 4: The Magnus force coefficient.

Using formulae (4) and (8) we express Re and Γ in terms of Re_*, ζ, ξ:

$$Re = 2.5 \ln\left(30\zeta/\xi\right) Re_*, \tag{13}$$

$$\Gamma = \left(\zeta \ln\left(30\zeta/\xi\right)\right)^{-1}. \tag{14}$$

Now function $\vartheta_0\left(Re_*\right)$ can be plotted for different parameters ζ and ξ, and compared with existing experimental evidence.

3. Results

The curves expressing (10) for different ξ are presented at Fig. 5. Parameter ζ almost does not affect the criterion for the saltation threshold, when varying in reasonable ranges $0.5 \le \zeta \le 0.7$. Parameter ξ, connected with logarithmic velocity profile, can vary in higher ranges. According to experiments of Schlichting (1960), depending on bed configuration: $0.2 \le \xi \le 4$. In his experiments, when the spheres, forming bed surface, were put closer together, ξ increased substantially.

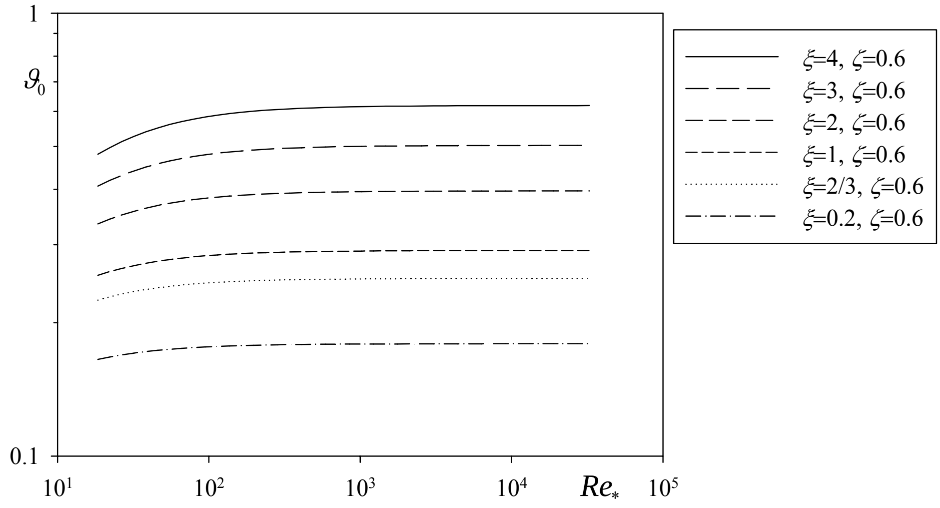

Fig. 5: Calculated threshold shear stress of saltation beginning for different values of parameter ξ, $20 \le Re_* \le 30000$.

Ling (1995) supposes that $\xi = 3$ corresponds to an average situation. Wiberg & Smith (1987) used $\xi = 1$ as a standard case. Parameter ξ and the bed configuration, that it describes, should also depend on shape and material of the bed particles, its adhesive properties due to the presence of silt and clay. Thus we come to the conclusion that ξ should be considered as additional parameter that determines the threshold of saltation.

The presented dependence corresponds to saltation start, and the values of ϑ_0 are somewhat higher than that corresponding for the saltation stop. The obtained results do not differ much from experimental results of Ancey et al. (2002): $\vartheta_0 = 0.3$, where parameter ξ was determined from analysis of velocity profile, and equalled $2/3$. The discrepancy between our results and that of Ling (1995), who calculated $\vartheta_0 \approx 0.1$ assuming $\xi = 3$, is probably due to his using improper values of Magnus force coefficient, that correspond to very low Reynolds numbers.

Errors arise from use of data on Magnus force for smooth spheres whereas sand grains are mostly not smooth and often have their shape far from spherical. Also errors arise from use of rotation induced Magnus force data for shear induced lift force. However, the agreement of calculated threshold of saltation with the experiments of Ancey et al. (2002) inspires with the using of Magnus force instead of low-studied shear lift for instance in saltation modelling, when applied according to adduced scheme.

4. Conclusions

The shear stress corresponding to threshold between rolling and saltation modes of bed load transport was determined analytically for turbulent flows with Reynolds numbers $20 \le Re_* \le 30000$. The equality of lift force and submerged gravitational force on a grain resting on the bed surface was chosen as the criterion for start of saltating motion. For calculation of lift force there were used the data on Magnus force on a rotating ball moving translationally in calm water. In expression for Magnus force (7), the vorticity of ambient average flow in centre of the considered grain was used for the angular velocity of rotating ball. The data on Magnus force was retrieved from multiple articles describing results of experiments and numerical simulations. The Magnus force coefficient $C_M\left(Re,\Gamma\right)$ was fitted with a simple function using the least square method.

The plots $\vartheta_0\left(Re_*\right)$ were reported for different values of parameter ξ that describes the bed surface formation and influences the flow velocity profile. Agreement with the experimental data confirms that the lift force is the most responsible for the particle saltation start, and that the Magnus force data can be used for modelling lift force on a particle in shear flow.

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Notation

C_M - dimensionless Magnus force coefficient;

d - diameter of bed grains;

F_g - submerged gravitational force;

F_L - lift force;

F_M - Magnus force;

g - gravitational acceleration;

k_s - equivalent sand roughness;

$Re = \dfrac{ud}{\nu}$ - particle Reynolds number based on particle slip velocity;

$Re_* = \dfrac{u_* d}{\nu}$ - particle Reynolds number based on shear velocity;

u - fluid velocity;

u_* - shear velocity;

y - elevation of moving bed particle above virtual bed level;

y_0 - roughness parameter;

$\Gamma = \dfrac{\omega d}{2u}$ - spin parameter;

ν - fluid kinematic viscosity;

ϑ_0 - dimensionless threshold shear stress;

ρ - the density of bed grains;

ρ_f - fluid density;

ω - angular velocity;

Ω - volume of bed particle;

ξ - dimensionless sand roughness;

ζ - dimensionless elevation of bed particle above bed level;

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