

**PRELIMINARY STUDY ON THE EFFECT OF THE WALL  
ROUGHNESS AND FREE STREAM TURBULENCE ON THE  
BOUNDARY LAYER DEVELOPMENT****P. Jonáš<sup>\*</sup>, O. Mazur<sup>\*\*</sup>, V. Uruba<sup>\*\*\*</sup>****1. Introduction**

The effects of the roughness elements distributed over a wall (WR) and of the free stream turbulence (FST) on laminar turbulent boundary layer transition are known long ago. In general they both accelerate the transition process under otherwise equal conditions and thus affect the boundary layer development into turbulent boundary layer. Experimental investigations of the effects in question are beneficial even if they are individually acting. So far the authors are not aware of the investigation of the joint action of the mentioned influences even though they may be important in many engineering devices.

The flow in every boundary layer undergoes the same phases of development: laminar structure, loss of stability, transitional flow and fully turbulent structure, from its onset  $x = 0$  up to the state of self similar turbulent boundary layer provided that it does not separate from the surface. External flow disturbances - random perturbations (e.g. velocity disturbances, acoustic waves) and scratches, protrusions or indentations on surface control the start and termination of the individual phases. It is beneficial briefly remind some generally known knowledge on the effect of surface roughness and free stream turbulence on boundary layer.

Scratches, protrusions or indentations or some roughness elements distributions on a surface cause local pressure distributions (local flow separations) resulting in local form drags which act as a component of tangential forces exerted on the surface together with the viscous wall shear stress  $m\partial U/\partial y$ . Technically, the roughness elements act as vorticity sources and thus they attenuate the viscous damping in the proximity of wall. The roughness only has an effect on the boundary layer characteristics when the *admissible roughness*  $s_l$  is exceeded. Then it accelerates the laminar turbulent transition so that under equal boundary conditions the transition starts at smaller Reynolds number on rough surface than on a smooth wall. The critical Reynolds number defined with the displacement thickness  $d_l$  ( $Re_l = 950 \div 1200$ ) in

---

<sup>\*</sup> RNDr. Pavel Jonáš, DrSc.: Institute of Thermomechanics, v.v.i. AS CR, Dolejškova 5, 182 00 Praha 8; tel.: +420 266 052 025, fax: +420 286 584 695, e-mail: [jonas@it.cas.cz](mailto:jonas@it.cas.cz)

<sup>\*\*</sup> Oton Mazur, prom. fyzik: Institute of Thermomechanics, v.v.i. AS CR, Dolejškova 5, 182 00 Praha 8; tel.: +420 266 052 031, fax: +420 286 584 695, e-mail: [mazur@it.cas.cz](mailto:mazur@it.cas.cz)

<sup>\*\*\*</sup> Doc. Ing. Václav Uruba, CSc.: Institute of Thermomechanics, v.v.i. AS CR, Dolejškova 5, 182 00 Praha 8; tel.: +420 286 588 547, fax: +420 286 584 695, e-mail: [uruba@it.cas.cz](mailto:uruba@it.cas.cz)

boundary layer developing on smooth flat plate is reduced up to about 40 percent with the surface roughness length  $s = 0.8 d_v$ . (see e.g. Schlichting & Gersten, 2000).

Nikuradse's investigations of flows in pipes of various roughness are usually starting point of the surface roughness effect analysis, e.g. Pope (2000), Rotta (1962) and (1972), Schlichting & Gersten (2000). The knowledge is important of the values  $s_1$  and  $s_2$  of the roughness element representative length  $s$  below which the flow is developing as on a *hydraulically smooth* surface  $s < s_1$  and above which,  $s > s_2$ , further increase of  $s$  does not cause additional qualitative changes of the flow, the surface is *completely rough*. Subsequently, two limiting cases of the roughness effect on turbulent boundary layer can be considered. The first one is if the representative length  $s$  is very small in comparison with the length  $d_v$ ,

$$s = d_v \rightarrow \frac{dU}{dy} = \frac{u_t}{y} \Phi_1 \left( \frac{y}{d_v} \right). \quad (1)$$

For large values of Reynolds number, the function  $\Phi_1$  tends asymptotically to a constant  $\Phi_1 : 1/k$  and previous equation integrates to the law of the wall (the log-law) for the (hydraulically) *smooth surface* ( $s u_t / n \leq 5$ ) with the von Karmán constant  $k = 0.41$  and with the universal constant of integration  $B = 5.2$

$$u^+ = \frac{U}{u_t} = \frac{1}{k} \ln y^+ + B; \quad y^+ = \frac{y u_t}{n} \quad (2)$$

The second limiting case relates to very large ratio  $s/d_v$  associated with large Reynolds number of the flow over roughness grains. Then the drags on grains prevail over viscous stresses and quantities including viscosity are not more relevant

$$s \gg d_v \rightarrow \frac{dU}{dy} = \frac{u_t}{y} \Phi_R \left( \frac{y}{s} \right), \quad (3)$$

here  $\Phi_R$  is a universal function that tends asymptotically to the value  $1/k$  with  $y/s \rightarrow \infty$ . Integrating the previous equation we derive the log law for the *completely rough* surface ( $s u_t / n \geq 70$ )

$$u^+ = \frac{1}{k} \ln \left( \frac{y}{s} \right) + B_R \quad (4)$$

with  $B_R = 8.5$  for completely rough surface.

*Transitional roughness* region ranges between the extreme cases

$$5 \leq s^+ = \frac{s}{d_v} = \frac{s u_t}{n} \leq 70. \quad (5)$$

Then having in mind that the flow is affected by roughness only near the surface and farther from the surface the action of viscosity and roughness on flow disappears, we can derive the log-law for transitional roughness of surface with the constant of integration which is a function of the roughness parameter  $s^+$

$$u^+ = \frac{1}{k} \ln \left( \frac{y}{s} \right) + B(s^+) = \frac{1}{k} \ln y^+ + B(s^+) - \frac{1}{k} \ln s^+. \quad (6)$$

Comparing this equation with that one valid for the smooth surface we receive

$$B(s^+) = B + \frac{1}{k} \ln s^+ \quad (7)$$

and similarly the comparison with the case of fully rough surface leads to

$$B(s^+) = B_R. \quad (8)$$

The roughness only has an effect on the drag when the *admissible roughness*  $s_l$  is exceeded. Then it accelerates the laminar turbulent transition so that under equal boundary conditions the transition starts at smaller Reynolds number on rough surface than on a smooth wall. The critical Reynolds number defined with the displacement thickness  $d_l$  ( $Re_l = 950 \div 1200$ ) in boundary layer developing on smooth flat plate is reduced up to about 40 percent with the surface roughness length  $s = 0.8 d_l$ , e.g. Schlichting & Gersten (2000).

The effect of the free stream turbulence (FST) level  $Tu$  on the location of transition onset is known as very important and was investigated long ago, since Schubauer and Skramstad in forties of 20th century, e.g. Schubauer & Klebanoff (1955). Later the effect of the turbulence length scale possibly the dissipation length parameter  $L_e$  of the FST on the start of boundary layer by-pass transition was also proved

$$Iu = \sqrt{\overline{u^2}} / \bar{U}_e; \quad L_e = -(\overline{u^2})^{3/2} / \bar{U}_e \frac{\partial(\overline{u^2})}{\partial x}. \quad (9)$$

The transition from laminar to turbulent flow structure depends on the specific type of flow and on the type of the acting disturbances that influence the process. Let us focus on the boundary layer on a flat plate at zero incidences. First of all, let us suppose a smooth surface and an external flow without disturbances. Initially the boundary layer is laminar and growing with the distance  $x$  from the leading edge. The Blasius solution describes the development of boundary layer parameters like the increase of the boundary layer thickness, e.g. the displacement thickness  $d_l$  with the distance from the origin of boundary layer ( $x = 0$ , leading edge of the plate)

$$d_l = 1,72 \frac{x}{\sqrt{x U_e / \nu}}, \quad (10)$$

where  $U_e$  and  $\nu$  are the outer stream velocity and kinematic viscosity. Reynolds number characterizes the state of the boundary layer evolution e.g. defined with the displacement thickness as the representative length.

$$\text{Re}_1 = \frac{d_1(x)U_e}{u} \quad (11)$$

Random perturbations (e.g. velocity disturbances from the external flow, acoustic waves, scratches, protrusions or indentations on surface) generate two dimensional instability waves, TS waves downstream from the vicinity of the leading edge. The occurring random disturbances are suppressed by the action of viscosity till arriving the value of the indifference Reynolds number  $(\text{Re}_1)_{\text{ind}} = 520$  (minimal coordinate  $\text{Re}_1$  of points on the curve of neutral stability, e.g. Schlichting & Gersten, 2000). Then TS waves begin to be amplified. At first only very narrow band of wave lengths/frequencies is unstable and can be amplified. The amplification follows in accordance with the linear stability theory (exponential amplification) until the amplitudes of the instability waves exceed the magnitude of about  $(0.01 - 0.02)$  of  $U_e$ . Then non-linear amplification with generation of 3D disturbances begins. Next the high frequency secondary instability suddenly comes, characterised: by the breakdown of the wave development, with  $\Lambda$ -vortex structure formation and with the gradual increase of the energy of fluctuating motions inside the layer. But still the boundary layer integral parameters (e.g. shape factor, wall shear stress) remain close to those one corresponding to laminar boundary layer. Finally the wavy structures are decaying and  $\Lambda$ -vortices are replaced by turbulent spots formation. The flow is highly intermittent. The boundary layer integral parameters and the energy of fluctuations are gradually joining the levels usual in a turbulent boundary layer. The transition process is finishing in the distance  $x_{\text{crit}}$  that corresponds to the critical Reynolds number  $(\text{Re}_1)_{\text{crit}} = 950 \div 1200$ .

Higher level of external flow disturbances  $Iu_e > 0.6 \div 1.0$  percent causes “by-pass” transition” with a mechanism converting outer flow turbulence into eigen boundary layer oscillations e.g. Morkovin (1969) and (1993). According to Greg et al. (1991), the stability of boundary layer flow remains the same as in the case of the low external disturbances. But now, the external disturbances continuously penetrating into the layer amplify the production of turbulent spots and thus accelerate the onset of transition and shorten the length of the transition region. Similarly the local wakes produced by roughness grains on the surface act as a source of innumerable disturbances.

The aim of the contribution is to compare consequences of the individual action and the joint action of the uniform roughness of the flat plate surface and homogeneous close to isotropy free stream turbulence on development of the zero pressure gradient boundary layer.

## 2. Experimental facility

The flat plate boundary layer is investigated experimentally in the close circuit wind tunnel IT AS CR, Prague  $(0.5 \times 0.9) \text{ m}^2$ . The boundary layer develops on an aerodynamically smooth plate (2.75 m long and 0.9 m wide) made from a laminated wood-chip board 25 mm thick in the primary configuration. The shape of very thin (2 mm) leading edge has been designed and examined by Kosorygin et al. (1982). The scheme of the working section is shown in Figure 1.

Covering the primary plate with the sandpaper on a thin plywood plate (7 mm thick) modifies the wall roughness; available are grits 60, 80 and 100 respectively. The rough plate

leading edge has an elliptic shape ( $a \times b = 60 \times 20 \text{ mm}^2$ ) which covers the primary leading edge.

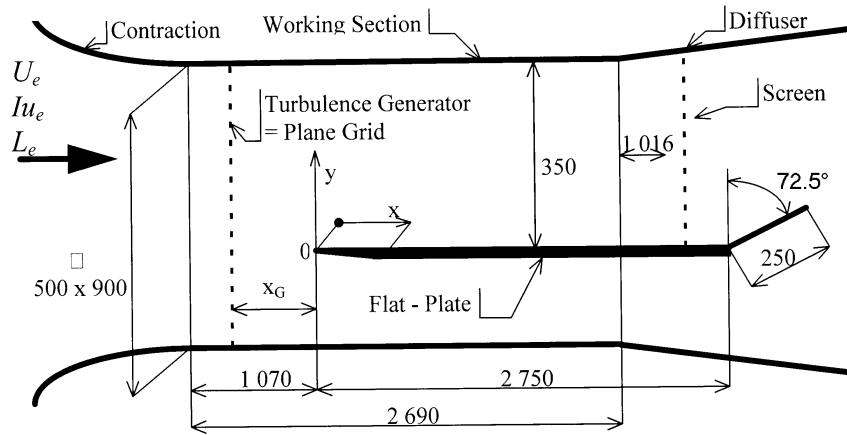


Figure 1. Working section of the wind tunnel ( $0.5 \times 0.9 \text{ m}^2$ ).

The maximum size of grains on sandpaper was chosen as the representative length of roughness  $s$ . It was repeatedly measured by means of a micrometer and then averaged with the results:

$$\begin{aligned} s \text{ (grits 60)} &= (0.435 \pm 0.014) \text{ mm}; \\ s \text{ (grits 80)} &= (0.343 \pm 0.009) \text{ mm}; \\ s \text{ (grits 120)} &= (0.215 \pm 0.007) \text{ mm}; \end{aligned} \quad (12)$$

Presented preliminary results relates to the smooth surface and the wall roughness grits **80** only.

Turbulence features of the external flow were controlled by means of square mesh ( $M$ ) plane grids – screens with cylindrical rods ( $D$ ) across the external flow in the distance  $x_G$  upstream of the leading edge ( $x = 0$ ) of the plate with investigated boundary layer. The distance  $x_G$  was adjusted sufficiently long,  $x_G < 0$  ;  $-x_G/M > 20$ , as to secure homogeneous, close to isotropy, turbulence downstream from the leading edge. The turbulence generators used were chosen from the family of grid generators developed in the Institute of Thermomechanics AS CR, Prague. For more details of the experimental facility and turbulence generators see e.g. Jonáš (1989) and Jonáš et al. (2000).

Grid No.	$D$ [mm]	$M$ [mm]	$x_G/M$	$Iu$ [%]	$L_e$ [mm]
1	3	20	22.7	3	7.0
4	6	20	36.9	3	16.2
5	10	35	34.6	3	30.5
8A	1.65	5.75	136	1	5.7
8B	1.65	5.75	33.7	3	3.0
8C	1.65	5.75	21.7	5	2.2

Table 1. Turbulence generators

### 3. Measurement technique

Representative pressure  $q_r$  [Pa] and the local dynamic pressure  $q'(x,y)$  [Pa] were simultaneously measured as to avoid errors caused by small and slow variations of the external flow velocity  $U_e$ .

$$q(x, y) = 0.5rU^2(x, y) = \frac{q'(x, y)\bar{q}_r}{q_r}; \quad \bar{q}_r = \text{Mean}(q_r). \quad (13)$$

Pitot-static tube ( $\Phi = 6$  mm) connected with the pressure transducer OMEGA Techn. Ltd., (max 1.2 kPa;  $\pm 0.25\%$  FS) was used for the measurement of the representative pressure  $q_r$  and at the same time for measurement absolute static pressure  $P$  [Pa] by means of the pressure transducer Druck DPI 145 (max 100 kPa;  $\pm 0.005\%$  FS).

The couple of the flattened Pitot probe ( $0.18 \times 2.95 \text{ mm}^2$ ) and round nosed static pressure probe ( $\Phi = 0.18$  mm) connected with the pressure transducer BARATRON (special order on high accuracy, max 1 kPa;  $\pm 0.02\%$  of reading above 20 Pa) was used for the measurement of

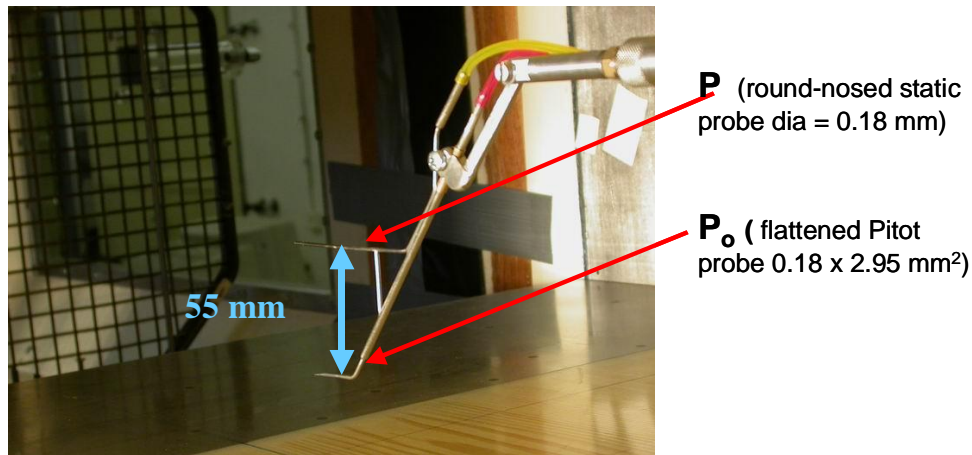


Figure 2. Photo of the working section with the couple of the Pitot and static pressure probe, with the grid turbulence generator and with flat plate.

the local dynamic pressure  $q'(x,y) = P_o - P$ .

Thermometer Pt 100 connected to the Data Acquisition/Switch Unit HP 34970A was applied for the flow temperature  $t$  [°C] measurement.

Output signals proportional to the mean values of  $P$ ,  $q_r$  and  $t$  are read by means of the unit HP 34970A just after start of measurement-observation in the point  $[x, y, 0]$  ( $z = 0$  is the plane of vertical symmetry of flow). Afterwards the simultaneous reading and 30 s averaging of signals proportional to  $q_r$  and  $q'(x,y)$  follows. After the end of the reading the data are recorded in a personal computer.

As to reach high measurement accuracy they were executed:

- § Carefully calibration of pressure transducers against the transducer BARATRON (till 1 kPa) and if necessary against the micro-manometer AVA Göttingen (Betz type) for the pressure differences till 4 kPa;
- § Reading and the record of the zero-readings before any observation.
- § Correction of the total pressure  $P_o$  readings after MacMillan, see Tropea et al. (2007). Estimates of upper limits of relative measurement errors derived from the accuracy of applied devices and with the regard to the scatter of repeated observations are following

$$\frac{\Delta q_r}{q_r} \leq \pm 0.02 \text{ at } U_r \cong 5 \text{ m/s}; \quad \frac{\Delta q}{q} \leq \pm 0.02 \text{ at } U(x,y) \leq 0.6 \text{ m/s}; \quad \Delta P; \pm 5 \text{ Pa}. \quad (14)$$

The absolute error of the local dynamic pressure at higher local velocity  $U(x,y)$  remains constant, about  $\pm 0.005$  Pa, i.e. on the level at  $U \approx 0.6$  m/s.

#### 4. Errors estimates continuation and evaluation methods

The representative velocity  $U_r$  was held in average on 5 m/s in the course of all here presented measurements. From the performed error analysis follow estimates of relative probable errors:

$$\begin{aligned} \text{mean velocity: } & \frac{\Delta U_r}{U_r}; 0.015; \quad \frac{\Delta U}{U(x,y)} \approx 0.01, \\ \text{displacement (i=1) and momentum (i=2) thickness: } & \frac{\Delta d_i}{d_i} \approx 0.015, \\ \text{shape factor } & \frac{\Delta H_{12}}{H_{12}} \approx 0.03. \end{aligned} \quad (15)$$

Wall shear stress  $\tau_w(x)$  [Pa] was evaluated from mean velocity profiles  $U(x,y)$  either from the slope interpolated very near the surface or from the interpolation of log-law.

The example of the first procedure is shown in Figure 3. It is based on the very satisfactory measurement accuracy of the Pitot probe shift from the starting position with the probe nose in contact with the surface. The observed distance from the wall  $y'$  of the probe

nose is measured with an accurate cathetometer. Unfortunately the dead travel of the probe traverser and elastic deflection of the probe nose are sources of uncertainty where the *ideal* touch with wall arises. For that reason the interpolation must determine the effective position of wall  $y'_0$

$$U(y) = a + b y' = b(y' - y'_0) = b y; \quad 0 < y' \leq 10^{-3} \text{ m}. \quad (16)$$

Figure 3 is an example of such interpolation. This procedure works quite satisfactory in laminar layers and boundary layers of moderate thickness not far from the transition start. The error of calculated  $t_w$  is less 5 percent.

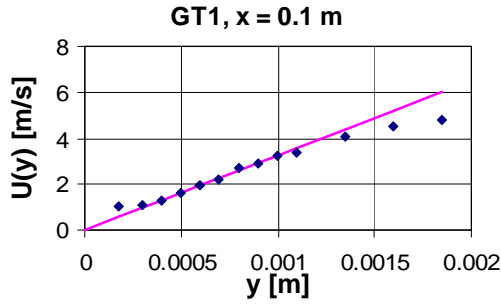


Figure 3. Example of the mean velocity profile downstream the grid turbulence generator GT1; interpolation near the smooth surface.

profiles upstream.

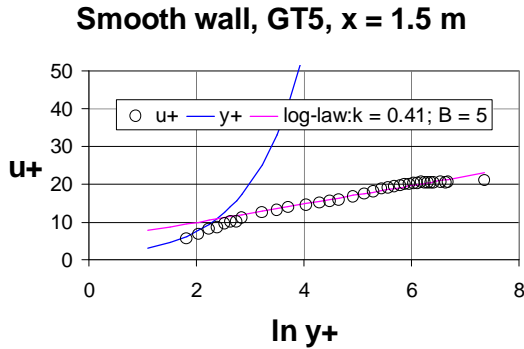


Figure 4. Example of the semi-logarithmic plot of the mean velocity profile after interpolation in overlap layer.

Significant uncertainties in the determination of  $t_w$  occur during developed process of laminar turbulent transition and as well in case of turbulent boundary layer on rough surface. The preliminary evaluation was done with using assumptions:

§ At least one point of the mean velocity profile arises in the viscous sub-layer

$$y^+ = \frac{u_t y}{\nu} \leq 5; \quad u_t = \sqrt{\frac{t_w}{\rho}} \quad (17)$$

§ The actual value of  $y'_0$  (20) equals to the average one determined from

The log-law (2) interpolation was applied for the evaluation of  $y'_0, u_t, B$  from points in the *overlap* (*logarithmic*) layer in the region where a fully developed turbulent boundary layer occurs. The task: search  $y'_0, u_t, B$  so as reach the best statistical pertinence:

$$\begin{aligned} \frac{U(y)}{U_e} &= a_0 + a_1 \ln \left( \frac{(y' + y'_0) U_e}{\nu} \right), \\ a_0 &= \frac{1}{k} \frac{u_t}{U_e} \ln \left( \frac{u_t}{U_e} \right) + B \frac{u_t}{U_e}, \\ a_1 &= \frac{1}{k} \frac{u_t}{U_e}, \\ u^+ &= \frac{1}{k} \ln y^+ + B; \quad k = 0.41. \end{aligned} \quad (18)$$

Having in mind the above mentioned and still existing uncertainties in the accuracy of the wall friction determination, the analysis of results is based on integral characteristics –



displacement thickness  $d_1$ , momentum thickness  $d_2$  and shape parameter  $H_{12} = d_1/d_2$ . The necessary integrations were done using the trapezium rule.

## 5. Results

As an introduction of presentation and analysis of results a note on classification of surface roughness in the investigated boundary layers with the regard to criteria (2), (4) and (5). All measured mean velocity profiles are giving the values of the roughness parameter in the limits

$$5.6 \leq s^+ = \frac{s}{d_v} = \frac{s u_t}{n} \leq 6.2 \text{ on the surface covered with sandpaper grits } 80. \quad (19)$$

The case on lower limit of transitional roughness was investigated.

The shape factor value is an elemental transparent indicator of the state of development a boundary layer on plate with zero pressure gradient. According to the Blasius solution its value is 2.6 everywhere with laminar flow structure and decreases to values from 1 to 1.5 in turbulent boundary layer (value depends on the Reynolds number).

The effect of the wall roughness on the shape factor distribution vs Reynolds number defined with the displacement thickness is shown in Figures 5 and 6. The turbulence level  $Iu$  and dissipation length parameter  $Le$  in the leading edge plane are quoted in captions.

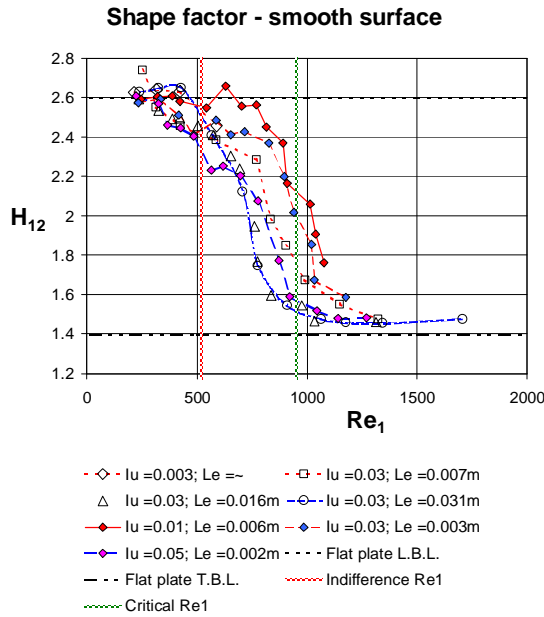


Figure 5.

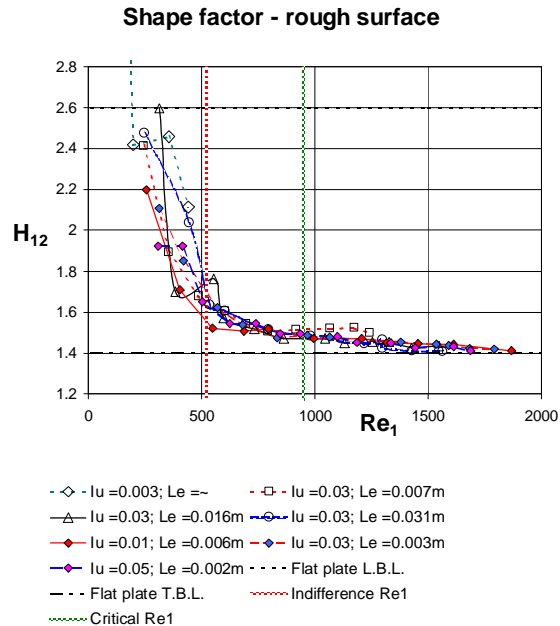


Figure 6.

The distributions in Figure 5 confirm that the length scale of turbulence in the incoming flow influences the beginning and extent of the transition region in a boundary layer on the smooth flat plate. It is worth make a note, that a 3 percent intensity accompanied with the

large length scale (0.031 m) affects the layer likewise turbulence with 5 percent intensity but of an order lower length scale.

The distributions shown in Figure 6 indicate that there is necessary some passage for the boundary layer development to turbulence even though the surface is considerably rough. A distinct effect is evident of the external flow turbulence upstream from the place where Reynolds number reaches the value of about indifference Reynolds number. It looks like that the turbulence length scale affects the transition to turbulence on rough surface contrary-wise than on smooth surface.

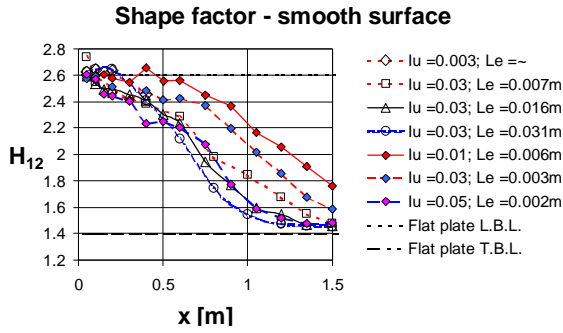


Figure 7.

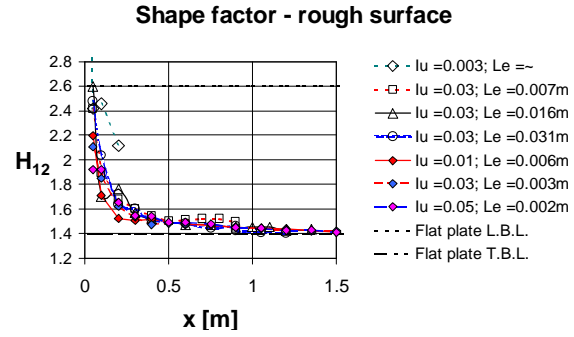


Figure 8.

Another view on the development of the observed boundary layer gives the plots of the parameter  $H_{12}$  vs the distance from the leading edge  $x$  shown in Figure 7 and Figure 8.

The actions of external turbulence characteristics on the boundary layer on smooth surface are from Figure 7 more apparent than in Figure 5. According to Figure 8, the distance  $x$  from the leading edge is the most important factor for the boundary layer development on the flat plate with transitional roughness of the surface.

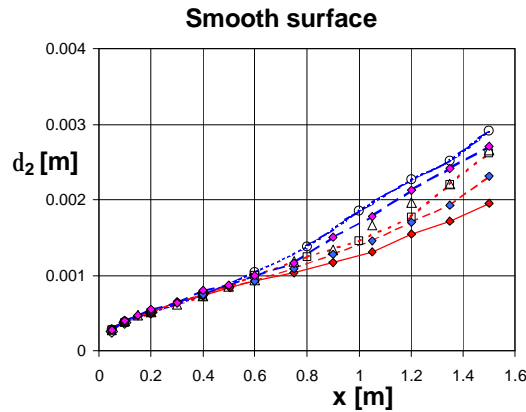


Figure 9.

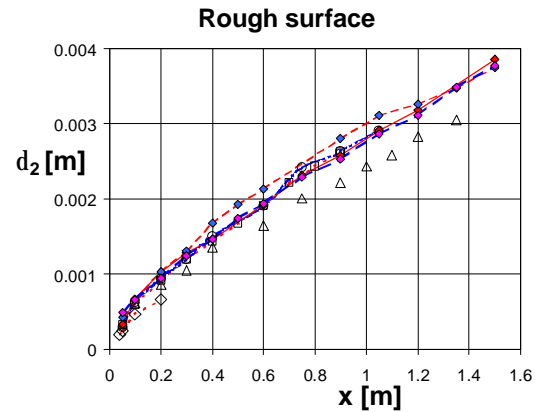


Figure 10.

The momentum integral equation yields that drag coefficient of a 2D-plate of length  $L$  is proportional to the momentum thickness

$$C_D = 2 \frac{d_2}{L}. \quad (20)$$

The momentum thickness is therefore a measure of the friction force acting on the surface from the leading edge up to the section  $x$  and thus useful for a qualitative check of wall

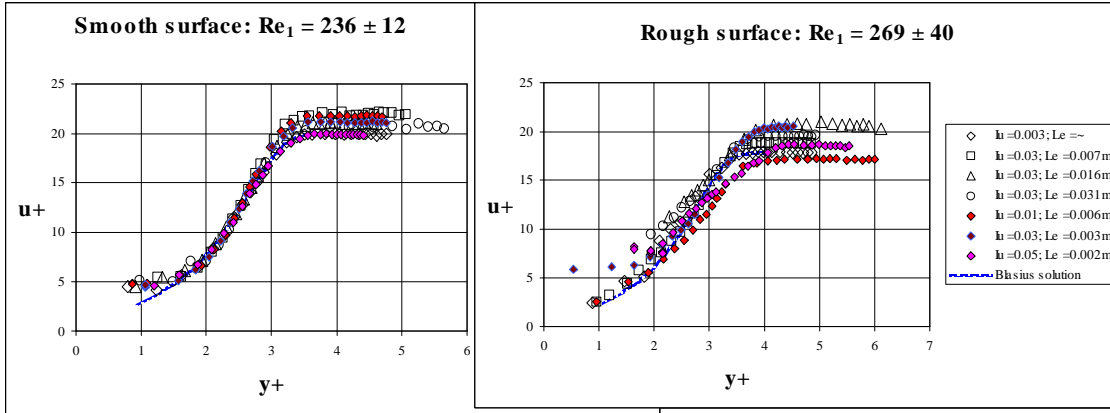


Figure 11.

Figure 12.

friction development. In Figures 9 and 10 are plotted distributions of the momentum thickness; captions are the same as in previous figures.

Distributions concerning to the rough surface does not display a laminar part.

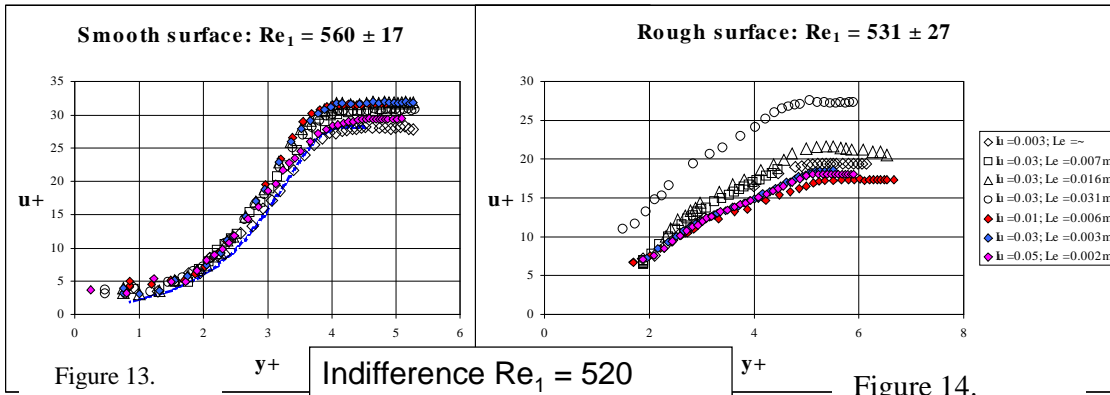


Figure 13.

Figure 14.

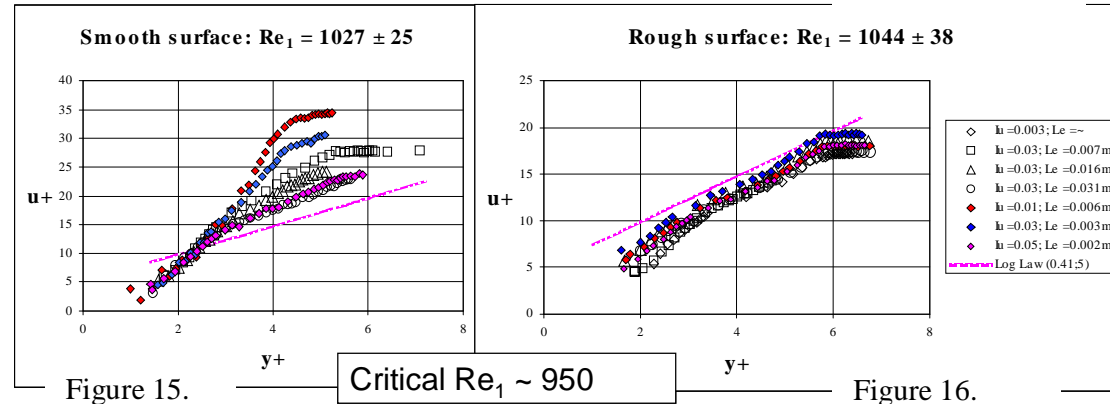


Figure 15.

Figure 16.

A series of semilogarithmic plot of velocity profiles illustrates differences between boundary layers on hydraulically smooth flat plate and boundary layers on flat plate with transitional roughness of the surface at otherwise same boundary conditions. Demonstrated are profiles very near the leading edge,  $x = 0.05$  m in Figures 11 and 12, profiles a little farther downstream from the location with the indifference Reynolds number, Figures 13 and 14, profiles near the critical Reynolds number, Figures 15 and 16 and finally profiles measured at the downstream end of the investigated working section region  $x = 1.5$  m.

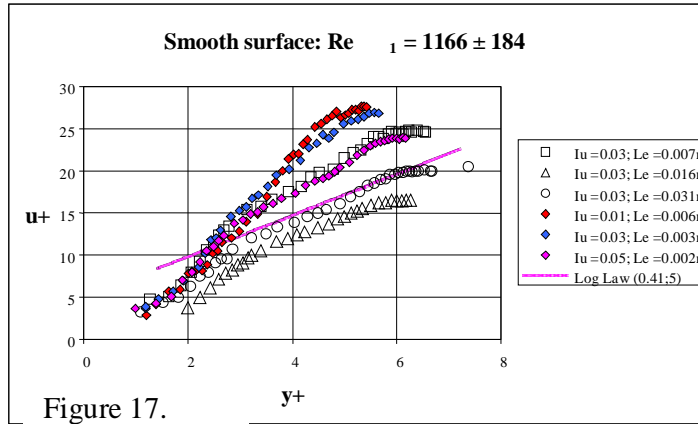


Figure 17.

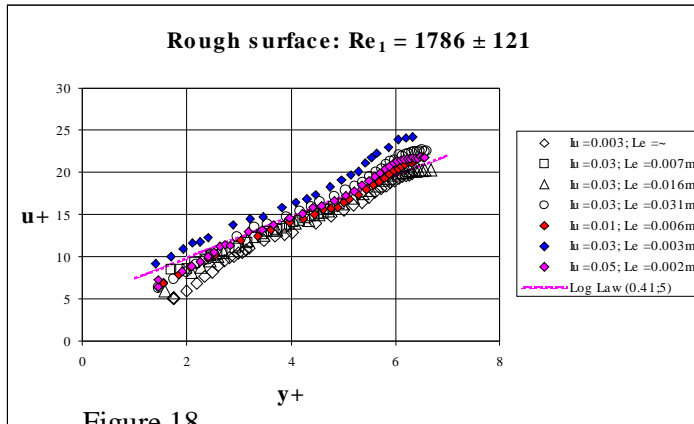


Figure 18.

## 6. Conclusions

Developments of the flat-plate boundary layer were investigated on both the smooth surface and the rough one in artificially turbulized external flow. The surface roughness was of transient category.

*Laminar* velocity profiles were observed near the leading edge even though rough surface and external flow turbulence accelerate transition.

The effect of both turbulence characteristics  $Iu$  and  $Le$  is evident from the  $H_{I2}$  distributions:

- beyond the location of critical  $Re_1$  with the smooth surface;

- till the location of indifference  $Re_1$  on rough surface.

Further downstream from the critical  $Re_1$  the effect of roughness predominates, transition terminates earlier than in boundary layer on smooth plate and distributions  $H_{12}$  within various  $(Iu, L_e)$  merge into one.

The constant of integration in the Log-Law depends on  $(Iu, L_e)$  even if it appears that transition has been completed.

## 7. Acknowledgement

This work was supported by the Grant Agency of the AS CR (A200760614) and by the Grant Agency of the Czech Republic (103060461).

## 8. References

- Grek, G.R., Kozlov, V.V. & Ramazanov, M.P. (1991) Boundary layer stability investigation at increased outer stream turbulence level. *Sibir. Phys.-Tech. J.*, 6, 106, pp. 117-125. (in Russian).
- Jonáš, P. (1989) Control of free stream turbulence by means of passive devices, in: *Proc. Int.Sem. Problems of modeling in wind-tunnels* (I TPM SO AN SSSR, Novosibirsk, Vol. 2, pp. 160-174.
- Jonas P., Mazur O. & Uruba V. (2000) On the receptivity of the bz/pass transition to the length scale of the outer stream turbulence. *Eur.J.Mech. B Fluids* 19, 707-722.
- Kosorygin, V. S., Levchenko, V. Ja. & Polyakov, N. F. (1982) *On problem of the origin of waves in a laminar boundary layer*. Preprints ITPM, No. 12-82, ITPM SO AN SSSR, Novosibirsk (in Russian).
- Morkovin, M.V. (1969) On the many faces of transition. In: *Viscous Drag Reduction*, (ed. C.S. Wells), Plenum, New York, pp. 1-31.
- Morkovin, M.V. (1993) Bypass transition research: issues and philosophy. Instabilities and Turbulence. In: *Engineering Flows* (D.E. Ashpis, T.B. Gatski & R. Hirsh) Kluwer, Amsterdam, pp. 3-30.
- Pope, S. B. (2000) *Turbulent flows*. Cambridge University Press, Cambridge.
- Rotta, J. C. (1962) Turbulent boundary layer in incompressible flow, in: *Progress in Aeronautical Sciences* (A. Ferri, D. Küchemann & L. H. G. Sterne eds.) Pergamon Press, Oxford, pp. 1-220.
- Rotta, J.C. (1972) *Turbulente Strömungen*. B.G. Teubner, Stuttgart.
- Schlichting, H. & Gersten, K. (2000) *Boundary-Layer Theory*. Springer, Berlin.
- Schubauer, G.B. & Klebanoff, P.S. (1955) Contribution on the mechanics of boundary layer transition. *NACA Technical Note*, TN 3489.
- Tropea, C., Yarin, A. L. & Foss, J. F. (Eds.) (2007) *Springer Handbook of Experimental Fluid mechanics*. Springer-Verlag Berlin.