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APPLICATION OF MICROPOLAR CONTINUUM TO CRITICAL STATE MATERIAL MODELS

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Introduction

Elasto-plastic material models based on the concept of critical state represent a useful tool for simulating the behavior of soil bodies when implemented in finite element method. The idea of the soil description which predicts either hardening or softening in dependence on the actual porosity was first formulated by Hvorslev (1937) and further developed by Roscoe & Burland (1968) who introduced the modified Cam clay model. The ability of the model to soften, i.e. to decrease stress with increasing deformation, requires additional modification of the standard FEM algorithms when analyzing the post failure response. First, the Newton-Raphson solver of the global system of equilibrium equations has to be replaced with an algorithm enabling to decrease the applied load such as the arc-length method. Second, the mesh dependent strain localization has to be regularized by introducing the internal length of the material which indirectly controls the width of the shear bands.

Constitutive law

One of the natural ways how to introduce the internal length is to implement so called micropolar continuum which uses three additional rotational degrees of freedom. Under 2D plane strain conditions the enhanced vectors of stress and strain are written in the form

$$\sigma = \{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yx}, \mu_{xz}/l_c, \mu_{yz}/l_c\}^T \tag{1}$$

$$\varepsilon = \{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xy}, \varepsilon_{yx}, \kappa_{xz} l_c, \kappa_{yz} l_c\}^T \tag{2}$$

where μ denotes couple forces acting on the material element, κ represents microcurvatures and l_c stands for internal length. The stress measure J_2 is redefined by the relation (de Borst, 1993)

$$J_2 = \frac{1}{4} s_{ij} s_{ij} + \frac{1}{4} s_{ij} s_{ji} + \frac{1}{2} \mu_{ij} \mu_{ij} \tag{3}$$

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where s are components of the deviatoric part of the stress tensor. This allows for keeping the governing equations of the theory of J_2-plasticity unchanged. For the generalized Cam clay model the definition of third stress invariant parameter, the Lode angle, is required in the form

$$\sin 3\theta = -\frac{3\sqrt{3} I_{3s}^{sym}}{2 (J_2^{sym})^{\frac{3}{2}}} \tag{4}$$

with I_{3s}^{sym} and J_2^{sym} being the second and third invariant of symmetric part of deviatoric stress tensor. The enhanced elastic operator is derived from the generalized Lame's formulation

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + (\mu + \mu_c) \varepsilon_{ij} + (\mu - \mu_c) \varepsilon_{ji} \tag{5}$$

$$\mu_{ij} = 2 \mu l_c^2 \kappa_{ij} \tag{6}$$

where μ_c is additional material parameter called Cosserat shear modulus. The stress update algorithm on the material level is driven by Newton-Raphson method which ensures the elastic law, consistency condition and hardening law to be satisfied at the end of every load step (Groen, 1997).

Results and conclusions

A simple 2D plane strain problem of biaxial compression was solved using modified and generalized Cam clay models employing both the standard and the micropolar continuum. The macroscopic load-displacement curve computed with different values of internal length is compared to the results obtained for standard continuum. The examination of the influence of the element size on the overall mechanical response demonstrates the regularization capability of the proposed method which, only with minor changes, follows the traditional concept of J_2 flow theory.

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