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TO THE INFLUENCE OF NONLINEAR DAMPING ON THE BIFURCATION PHENOMENA IN GEAR MESH OF ONE BRANCH OF POWER FLOW OF THE PSEUDOPLANETARY GEAR SYSTEM

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Summary: *The aim of this contribution is the analysis of damping properties both the material of gear mechanics in the mesh and the lubricating oil film in the tooth space at the tooth profile contact bounces into the area of the technological gear backlash. The damping influence over gear mesh stability is pursued on the special case of the simulation model of the system with split power flow for the selected frequency range of the resonance characteristics. The nonlinear damping in gear mesh and in gear system is concerned significantly in the amplitude progress, greatness and phase shift of relative motion towards stiffness function alternatively towards its modify form in gear mesh.*

1. Introduction

The nonlinear dynamics of the time-heteronomous parametric systems has formed especially in internal dynamics of these in past few decades the extra high-actual branch, above all by the high-speed differential of pseudoplanetary transmission systems with kinematic couplings. The damping in the gear mesh both in the normal or inverse mesh and in the phase of the tooth profiles contact bounce by the impact effects forms here the important problems.

The damping in gear mesh and in gear system is concerned significantly in the amplitude progress, greatness and phase shift of relative motion towards stiffness function alternatively towards its modify form in gear mesh. In consequence of these and another actions rise above resonance characteristics certain singular locations with jump amplitude course.

Deeper and more accurate dynamic research particularly in the aeronautical transmissive systems in light high-speed turbopropelled units forms a basis of their operational reliability and safety.

The forces and dynamic effects which occur in the gear mesh of kinematic pairs are not only a basis for their quantitative i.e. strength dimensioning, but forms a basis for qualitative

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tribology analysis in complicated gear meshes (rolling – sliding) of kinematic pairs of cog wheels.

One of the topical problems of the tribology in the dynamics of high-speed light gear systems is among others by theory documented determination of “real” carrying width of gearing in comparison with “constructional” one. This is given by the theoretical width of carrying oily film in gear mesh within the constant pressure what is enlarged of areas where the pressure diminishes into static one in the tooth face.

One of the main factors which influence this problem are dynamic forces in gear mesh of kinematic pairs e.g. in the planetary gear systems by the normal gear mesh.

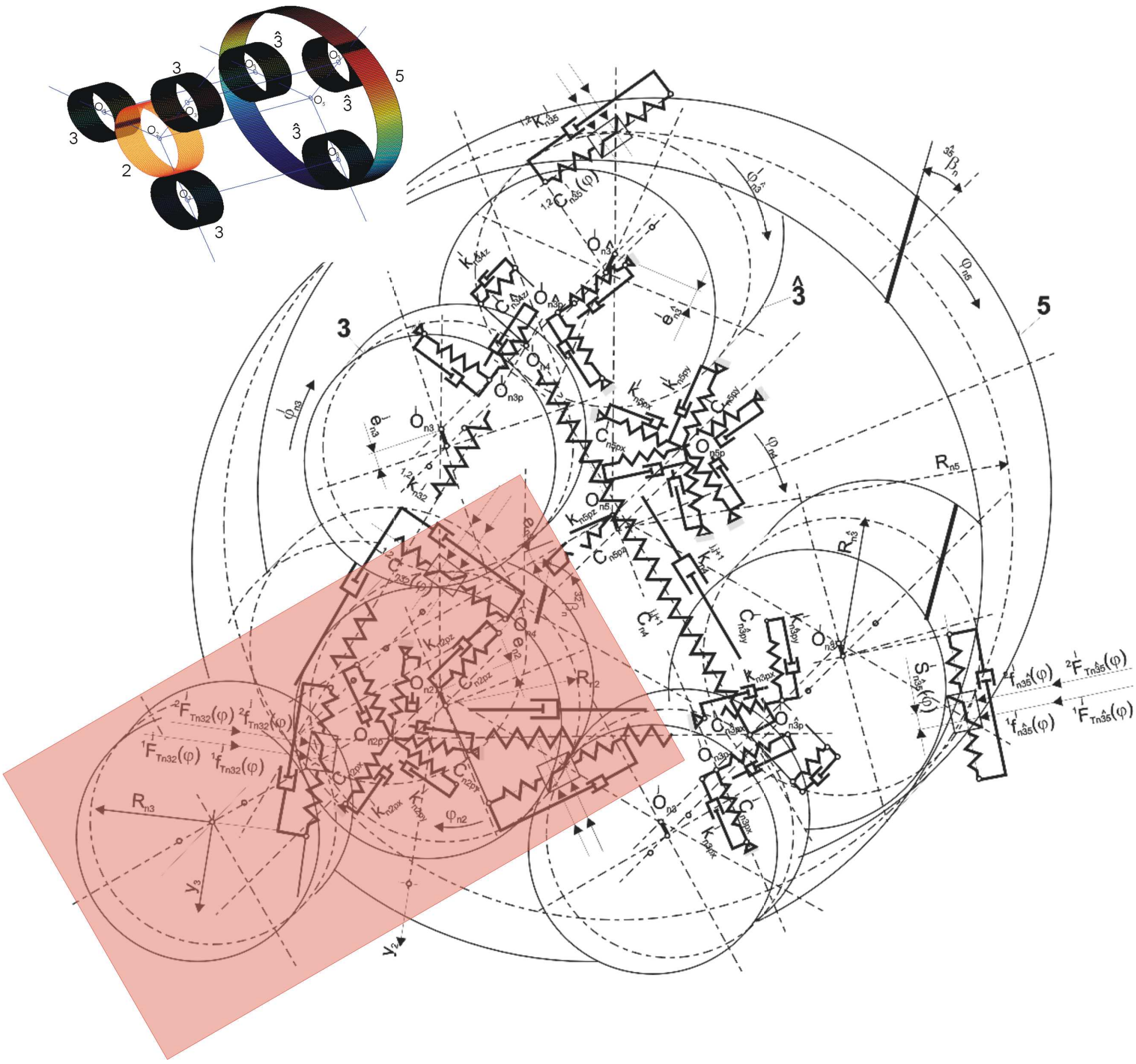

Fig.1 – The substitutive mathematical – physical model of kinematic pair of gears (in the red area) of common differential planetary gear system with double planet wheels.

2. Mathematical-physical model and solution methodology of dynamic problem

The contribution reassumes onto hitherto published works [4],[5],[6] and [7] and also here goes out from the solution of special case of improved discrete mathematical-physical model of kinematic pair of cog wheels from one branch of pseudoplanetary systems, s. Fig.1, which represents system with six degrees of freedom.

The motion in such special case (of common model, Fig.1) of pair wheels with spur gearing leads by mass discretisation as well as the existence influence of different weak – analytical and strong – non-analytical nonlinearities such as e.g. influence of technological gear backlash and next by the parametric exciting sources on the solution of six nonlinear deterministic ordinary differential equations with time variable coefficients [1]

$$\boldsymbol{M}\boldsymbol{v}'' + {}_1\boldsymbol{K}(\beta,\delta_i,H)\boldsymbol{v}' + \sum_{K_1>1} {}_{K_1}\boldsymbol{K}(D,D_i,H)\left|\boldsymbol{w}'(\boldsymbol{v}')\right|^{K_1} sgn(\boldsymbol{w}'(\boldsymbol{v}'))$$
$$+ {}_1\boldsymbol{C}(\varepsilon,\kappa,Y_n,U_n,V_n,H,\tau)\boldsymbol{v} + \sum_{K>1} {}_K\boldsymbol{C}(\varepsilon,\kappa,I_n,H,\tau)\boldsymbol{w}^K(\boldsymbol{v}) = \boldsymbol{F}(a_n,b_n,\overline{\varphi},H,\tau). \quad (1)$$

Here $\boldsymbol{v}$ means generally the m-dimensional vector (m = 6) of displacement of system vibration, $\boldsymbol{w}^K(\boldsymbol{v})$ K-th power of vector $\boldsymbol{v}$, which is defined by expression $\boldsymbol{w}^K(\boldsymbol{v}) = \boldsymbol{D}(\boldsymbol{w}(\boldsymbol{v})\boldsymbol{w}^{K-1}(\boldsymbol{v}))$. $\mathbf{D}(\mathbf{w}(\mathbf{v}))$ denote the diagonal matrix, whose elements at the main diagonal are comprised by elements of vector $\boldsymbol{w}(\boldsymbol{v}) \equiv \boldsymbol{v}$. Furthermore $\boldsymbol{M}$ is the matrix of mass and inertia forces, ${}_1\boldsymbol{K}$ and ${}_{K_1}\boldsymbol{K}$ are the matrix of linear and nonlinear damping forces, ${}_1\boldsymbol{C}$ and ${}_K\boldsymbol{C}$ are the matrix of linear and nonlinear reversible forces and $\boldsymbol{F}(\tau)$ is the vector of non-potential external excitation with components a_n, b_n and with the phase angle $\overline{\varphi}$. H is the Heaviside´s function, which allows to describe the motions – contact bounces – due to strongly non-analytical nonlinearities, for example due to technological tooth backlash $s(\tau)$. Corresponding linear and nonlinear coefficients of damping are denoted by β,δ_i D,D_i linear parametric stiffness function by the symbols Y_n,U_n,V_n and nonlinear parametric functions, so-called parametric nonlinearities, by the symbol I_n. ε and κ are the coefficients of mesh duration and amplitude modulation of stiffness function ${}_1\mathbf{C}$. Derivative by non-dimensional time τ are denoted by dashes, $\tau = \omega_c t$, ω_c … mesh frequency, t … time.
The measure of the dynamic load in the gear mesh is represented by the dynamic force

$$F_{dyn} = {}_1C(\tau)y(\tau), \quad (2)$$

where $y(\tau)$ is the relative motion in gear mesh in the course of the mesh line.

The relative motion as the measure of dynamic loading in the gear mesh, i.e. in the course of mesh line, can be described for the generally elastic supported system with bearing motions $\{y_{3,2}; z_{3,2}\}$ of the gear pairs 3,2 by respecting so-called run-out of pitch circles, which are modelled by eccentricities $e_{3,2}$, in the form [1],[3]

$$y(\tau) = R_{b3}\varphi_3 + R_{b2}\varphi_2 + y_3 - y_2 + e_3 \sin\varphi_3 - e_2 \sin(\Delta - \varphi_2) + {}^{1,2}f(\tau), \quad (3)$$

where ${}^{1,2}f(\tau)$ is the deflect function, or the deviation of the cog side form from the ideal involute, Δ is the phase angle of angular displacement between eccentricities $e_{3,2}$ and $R_{b3,2}$ are radii of basic circles.

The analytical form of the resulting stiffness function of spur gearing in mesh $C(\tau)$,see eq.(1) and eq.(2), can be expressed for $\varepsilon \in \langle 1;2 \rangle$ by Fourier´s series in real time t in form [1], [3]

$$C(t) = C_s + \frac{C_{max}(1-\kappa)}{2} \sum_{n=1}^{\infty} \frac{4}{\pi n} (-1)^n \sin n[(\varepsilon - 2)\pi] \cos n\omega_c t, \tag{4}$$

where the mean stiffness C_s is defined by

$$C_s = \kappa C_{max} + \frac{C_{max}(1-\kappa)}{2}[1 + (2\varepsilon - 3)]. \tag{5}$$

The symbol $\kappa = C_{\min} C_{\max}{}^{-1}$ represent the amplitude modulation of resulting stiffness function in gear mesh, $C_{\min}, C_{\max}$ are minimal and maximal values of stiffness in gear mesh and ε is coefficient of mesh duration, which indicates how many teeth pairs is at any one time in mesh at mesh line. In extreme cases, for example $\varepsilon = 1$, is during the mesh time at mesh line only one teeth pair, in the case $\varepsilon = 2$ are two pairs of teeth whole time in mesh.

In these cases verges the parametric system i.e. in the stiffness with time heteronomous system on the system with constant coefficients. The intermediate values ε determine the proportion of the change of the number of teeth pairs in the gear mesh at the mesh line. In the Fourier´s series (4) ε determines the time proportion the change of the minimal and maximal resulting stiffness C_{min}, C_{max} during the gear mesh. This fact markedly affects the dynamics of system and is connected with the size of amplitude of relative motion in gear mesh. That is influenced by time duration of mesh on that which potential stiffness level of appropriate reversible force by the given frequency tuning. In the stiffness of teeth is respected in the next application only the stiffness of the separate cogs and their fixation into a solid half-space, discs are considered absolutely solid.

On the basis of the carried analytical analysis of the weakly and strongly nonlinear parametric integrodifferential problem with the solving cores in the form of splitting Green´s resolvents [2], in that the solved differential boundary-value problem was transformed, concurs now the numerical solution of the given problem. For the numerical analysis of the dynamic phenomena with impact effects of the cited system of the kinematic pairs of spur gears was carried out the methodology of solution by means of the simulation model of this system in the MATLAB/Simulink. [3]

The next factor which influences qualitatively and quantitatively the course of *y(t)* is the friction in gear mesh or frictional forces in the motion rolling – sliding of kinematic pair – gearing. They induce the variance of originally considered constant preload $M = M_3 - (-M_2) = konst.$ on $M_v = M \pm \Delta M_T$, where ΔM_T is the additional moment from friction forces.

The friction forces in the gear mesh constitute the separate chapter in the frame of tribological process of lubrication. The lubricant between frictional faces of cog profiles in gear mesh suppresses the friction and thereby the wear of frictional faces and add to the energy efficiency of transmission. The theory of the EHD lubrication in gear mesh of the *finite* width at the line contact under the rolling and sliding contact is on the present-day one of more developed areas, because the constructional width of gearing is not the same as the carrying width of the lubricant film. The high pressure in the gear mesh is suppressed in the gearing margin in the static pressure. The gear profiles of kinematic pairs pursue complicated motion rolling - sliding namely sliding resulting as from the gear mesh geometry so from the relative motions of elastic bearings, i.e. motions caused as by the elasticity of bearings so by the wheel run-out. Purely rolling motion in gear mesh occurs only in the pitch point on the mesh line by absolutely solid bearing of wheels. In this contribution is applied for qualitative complying with friction force $F_T(t)$ as the zero approximation only very estimative theory of Coulomb´s friction

$$\begin{aligned} F_T(t) = -f_T\{ y(t)H1 + [y(t)+s(t)]H2 \}C(t)\gamma(t)sign\{ \delta^0_{e_2+e_3} \\ + [e_3\dot{\varphi}_3 \sin\varphi_3 - e_2\dot{\varphi}_2 \sin(\Delta - \varphi_2)] \}sign[\delta^0_{e_2+e_3} + (\dot{z}_3 - \dot{z}_2)], \end{aligned} \tag{6}$$

where f_T is the coefficient of dry friction, $\gamma(t)$ is the function with values 0.5;1;0;-1;-0.5 with regard to momentaneous position of gear mesh at mesh line of gearing, that corresponding with the resulting stiffness function and with the sense of action of friction force (s. [3]) and $\delta^0_{e_2+e_3}$ is the Kronecker´s symbol.

The analysis of dynamic features of solved special case of general non-linear parametric, i.e. time heteronomous, system with kinematic couplings – spur gears is in this contribution aimed to the investigation of reasons of forms of resonance characteristics of given mathematical-physical model both by

a) the conservative system in gear mesh* and

b) non-conservative system.

By reason that in such complicated parts of transmissive systems, e.g. in the gear mesh "rolling – sliding", are still unknown neither approximate data about damping properties or about damping patterns both in gear mesh and also in the connection with fixation into gear rim and discs including hubs like unit, will be this damping simulated by means of different functional relation both in the area of gear material at normal or inverse mesh incl. corresponding parts of gear rim and discs, and the influence of viscous damping in the area of technological gear backlash, i.e. the influence of oils in the phase of teeth profile contact loss. The lightening holes in discs of wheels also markedly influence the damping, similarly the viscous damping of lubricant mediums is dependent on their temperature etc.

In this study will be the influence of damping or the damping forces in system of motion equations (1) of given model of mechanic system represented – modelled by the terms in form

$$_1\boldsymbol{K}(\beta,\delta_i,H)\boldsymbol{v}' + \sum_{K_1>1} {}_{K_1}\boldsymbol{K}(D,D_i,H)\left|\boldsymbol{w}'(\boldsymbol{v}')\right|^{K_1} sgn(\boldsymbol{w}'(\boldsymbol{v}')) \quad for\ K_1 = 2{,}3. \tag{7}$$

* Under the definition „conservative system“ will we **here** suppose the system what is depicted in Fig.1 and described by the system of motion equations (1) without terms contained in eq. (7) what describes the linear and nonlinear damping forces. The friction force in gear mesh presents certain internal exciting component of system. In the contribution it is described by eq. (6) and creates also certain dissipation of energy in frame of definition of conservative system. Despite of we keep it in the contribution because induces changes – alternations of sense of friction which are given not only by means of gear mesh geometry at the passing of mesh by the pitch point on the mesh line, but also by every change of relative motions of wheels with elastic bearings, accordingly as certain exciting motion source in system.

The particular combinations of linear and nonlinear damping in gear mesh and in teeth backlash *s(t)* by the impact effects are given in Tab.1.

Tab. 1. Combinations of damping in gear mesh.

	conser-vative system in gear mesh	linear damping in gear mesh - L			quadratic damping in gear mesh - Kv			cubic damping in gear mesh - Ku			Combinations of damping in gear mesh											
											L + Kv			L + Ku			L + Kv + Ku			Kv + Ku		
k_1	0	×		×							×		×	×		×	×		×			
k_2	0				×		×				×		×				×		×	×		×
k_3	0							×		×				×		×	×		×	×		×
k_{1m}	0		×	×								×	×		×	×		×	×			
k_{2m}	0					×	×					×	×					×	×		×	×
k_{3m}	0								×	×					×	×		×	×		×	×
Symbol of marked solution in figures	○	□	•	×	□	•	×	□	•	×	□	•	×	□	•	×	□	•	×	□	•	×
Variation	a	b			c			d			e			f			g			h		

Note: $k_{1,2,3}$ … material damping in gear mesh; $k_{1m,2m,3m}$ … viscous damping of lubricant mediums in the tooth backlash (without contemplation of temperature influence;1 – linear, 2 – quadratic, 3 – cubic.

3. The exemplification of analysis of nonlinear damping influence on properties of resonance characteristics and bifurcation in gear mesh

By reason that in gear mesh of kinematic pairs are still unknown *in general* both theoretical and experimental patterns about vibration damping, were placed in the equation systems (1) the expression (7) for vibration damping in gear mesh and for the qualitative and quantitative analysis then combination of damping according to the Tab.1. By the substitutive mathematical – physical model of system, s.Fig.1 and by means of its motion equations we are able to describe all motions, which can during the mesh occur, i.e. the normal mesh, the impact effects in gear mesh with contact bounces of cog profiles and the inverse mesh, when the inverse cog faces come in mesh. The damping in the all combination of meshes is described by the coefficients k, k_m, s. Tab.1. Besides so-called conservative system in mesh with $k, k_m = 0$, here are given the damping combination in gear meshes both linear, quadratic, cubic and their variation.

This study continue for example the works [4], [5], [6], where are solved some dynamic problems connected mainly with the linear damping in gear mesh. In relation to the limiting extent of the contribution we present in next only some dynamic properties of the damping of relative motion *y(t)* in gear mesh of kinematic pairs of given parametric system.

All the resonance bifurcation characteristics $\{\nu; y(t)\}$ have been solved for $\varepsilon = 1{,}569; \kappa = 0{,}5879;\ C_{max} = 4.10^5[\mathrm{Nmm}^{-1}]; m_{red} = 3{,}123.10^{-3}[\mathrm{kg}]$. The values of material damping $k \equiv k_{2,3}$ both in the area of normal mesh and inverse one, as well as the values of viscous damping in the tooth backlash $k_m \equiv k_{2m,3m}$ (for reason of simplicity we do not take into consideration the temperature influence) are considered in all next given examples of solution identical, i.e. $k_1 = k_{1m} = 3{,}95$, which corresponds to proportional damping $\beta = \beta_m = 0.062$.

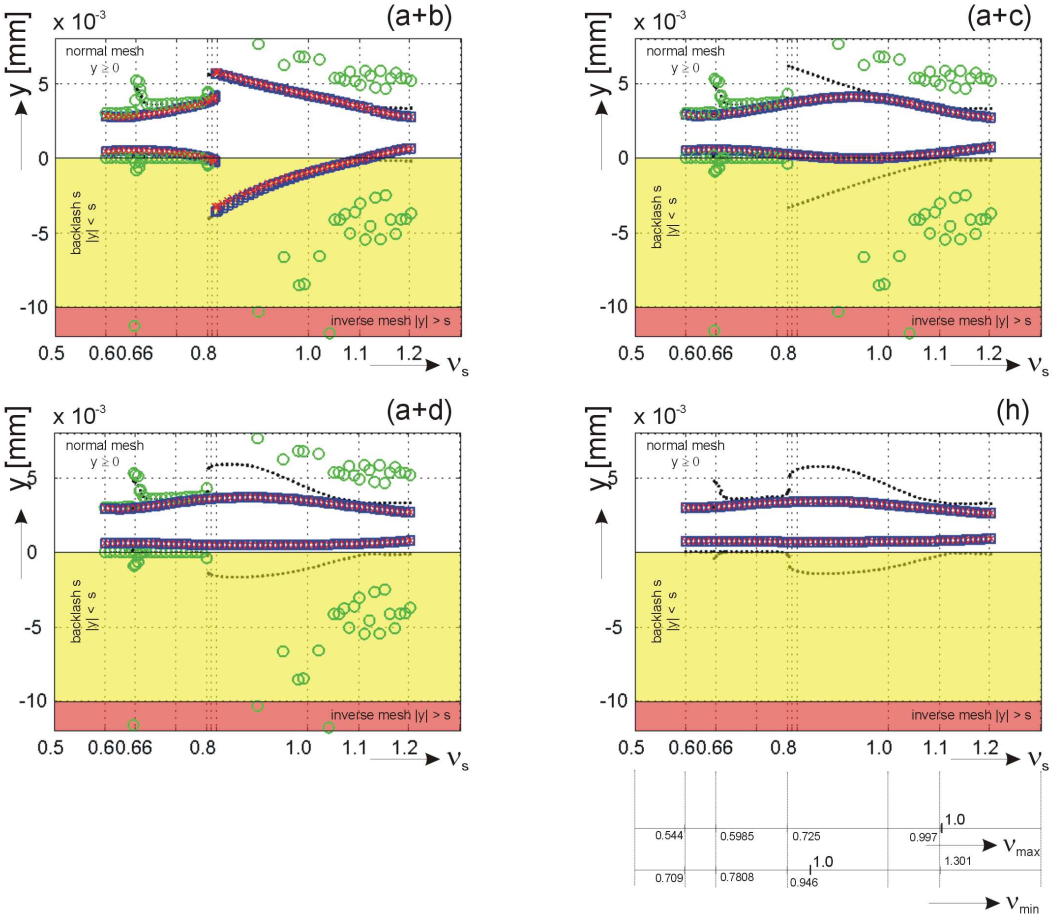

Fig.2 – Resonance characteristics of relative motion $y(t)$ of time heteronomous system excited by the parametric stiffness function $C(t)$ for the combination of damping in the gear mesh according to Tab.1:

- (a+b) … conservative (○) and linearly damped gear mesh (□,×,•)
- (a+c) … conservative (○) and quadratic damped gear mesh (□,×,•)
- (a+d) … conservative (○) and cubic damped gear mesh (□,×,•)
- (h) …quadratic and cubic damped gear mesh (□,×,•)

and for the fifth quasisteady revolution of gear wheels from the mathematical – physical model in accordance to Tab.1.

In the Fig.2 are colour coded the area of normal gear mesh for $y(t) \geq 0$, the area of the phase of teeth profile contact loss ($|y(t)| < s(t), s(t) \ldots$ the tooth backlash) and the area of inverse mesh ($|y(t)| > s(t)$) with the ordering of colours white, yellow and red.

The first picture with the damping variant (a+b) and similarly the courses (a) - (○) in the Fig.2 forms just comparative courses of resonance characteristics both of linear damping in gear mesh and conservative system with the nonlinear courses, i.e. with nonlinear damping in the gear mesh, they will be discussed furthermore.

The courses with nonlinear variants of damping in the gear meshes differ from linear ones, in detail discussed in [4], [7], … , not only qualitatively by the singular incontinuity locality – jumps, but also quantitatively in the courses of resonance characteristics. The give courses with nonlinear damping in the gear mesh differ above all in the variants (□,×) and (•). Whereas the variants (□,×) marks out with continuous courses in whole given resonance range $\nu_s \in \langle 0.6;1.2 \rangle$ and quantitatively in the normal mesh, i.e. for $y(t) \geq 0$, differ just slightly (the influence of the second and the third power of $y'(t)$ in (7)), the variants with the marker (•) differ by the existence of singular i.e. jump location in the vicinity $\nu_s \approx 0.66$ and $\nu_s \approx 0.8$ but with different quantitative dissimilarity likewise in linear cases. Simultaneously these courses de facto do not differ here in the area $\nu_s \approx \langle 0.66;0.8 \rangle$, in the area $\nu_s \in \langle 0.8;1.2 \rangle$ differ from the singular location $\nu_s \approx 0.8$ next both qualitatively and quantitatively. The courses (•) round off from the jump location $\nu_s \approx 0.8$ for the cubic damping (d) and for the composite damping (h) in gear mesh and the peak $y_{max}(t)$ shifts from $\nu_s \approx 0.8$ continuously to higher values of ν_s.

In Fig.3 are colour-coded marked the areas of gear mesh, i.e. with white colour the area of normal mesh ($y(t) \geq 0$) and with yellow colour the area of jump effects ($|y(t)| < s(t)$, where $s(t) \ldots$ teeth backlash).

In the first column of this picture are plotted the bifurcation resonance characteristics of *quadratic* variant damping (c), in the first row for the combination (•), in the second one then for the combination (□).By analogy forms the second column the bifurcation resonance characteristics of cubic variant damping (d), withal in the first row by analogy with the previous case for the combination (•), in the second for the combination (□). Similarly is it in the third case of column, where are plotted the resonance bifurcation characteristics for the combinational variant damping (h), where in the first row are bifurcation characteristics for the combination (•), in the second then for the combination (□).

From the comparison of the first row of bifurcation diagrams for the combination (•) is seen, that the resonance course $y(t)$ is in the given frequency range $\nu_s \in \langle 0.6;1.2 \rangle$ both in the area of normal mesh ($y(t) \geq 0$) and in the area of the jump effects ($|y(t)| < s(t)$). In the range $\nu_s \in \langle 0.6;0.8 \rangle$ forms this course with the exception of $\nu_s \approx 0.66$, when the amplitudes $y(t)$ hard grow, the well-ordered bifurcation characteristics. The stiffness function $C(t)$ by the normal mesh $y(t) \geq 0$ converts in the vicinity $\nu_s \approx 0.66$ and $\nu_s \approx 0.8$ into the modify stiffness function *C(t)(H1+H2)* (*H1,H2* … Heaviside´s function) by the contact bounce of cog faces in gear mesh $y(t)<0$. These jump effects of the bifurcation amplitudes here are caused by the resonance tuning at the corresponding stiffness level of the function *C(t)*, alternatively *C(t)(H1+H2)*, in the given case at the minimal stiffness level C_{min}. In the vicinity of the jump $\nu_s \approx 0.66$ i.e. $\nu_{min} = 0.7806$ against $\nu_{max} = 0.5985$, next then in the interval $\nu_s \in (0.8;0.9)$ is the solved system in the vicinity $\nu_s \approx 0.8$ at the stiffness level C_{min} directly in the resonance $\nu_{min} = 1$. The size of the time course interval of the amplitude size of the relative motion *y(t)* is impacted by the *phase shift* of relative motion $y(t)$ towards stiffness function *C(t)*, alternatively towards its modify form *C(t)(H1+H2)*, caused by non-linear damping effects both the material in gear mesh.

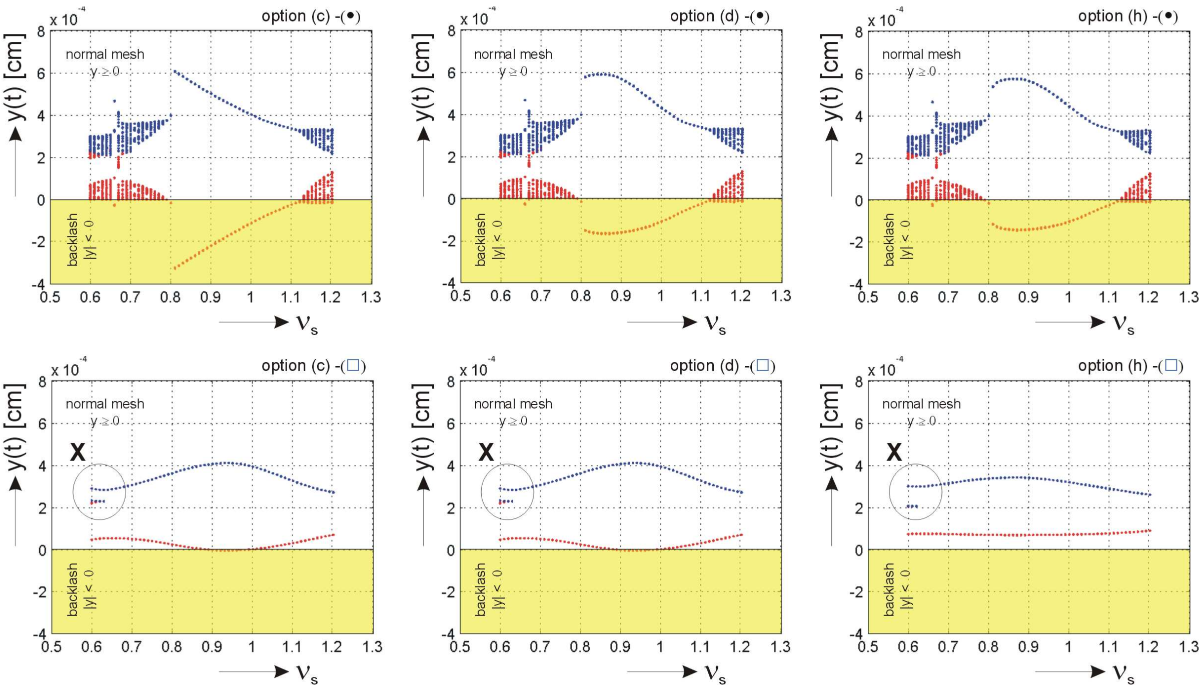

Fig.3 – Bifurcation resonance characteristics of the system with the nonlinear quadratic variant damping (c) – (□) $k_2 > 0, k_{2m} = 0$; (•) $k_2 = 0, k_{2m} > 0$, variant cubic damping (d) – (□) $k_3 > 0, k_{3m} = 0$; (•) $k_3 = 0, k_{3m} > 0$ and combinational variant of damping (h) – (□), $k_2 > 0, k_3 > 0$`, $k_{2m} = 0, k_{3m} = 0$ and (•) $k_2 = 0, k_3 = 0, k_{2m} > 0, k_{3m} > 0$, in the area of frequency tuning $\nu_s \in \langle 0.6;1.2 \rangle$ in the phases of normal gear mesh and impact effects in the gear mesh.

With growing ν_s then next the resonance bifurcation course decreases tidily – continuously. From $\nu_s \approx 1.1$ the orderly course becomes into the bifurcation characteristics of the maximal $y_{max}(t)$ and minimal $y_{min}(t)$ resonance courses. On the reasons of the growing of the unsteady motions can we conclude from the time courses of $C(t)$, modify stiffnesses $C(t)(H1+H2)$, courses $y(t)$ and velocity $y'(t)$ inclusive of the phases planes $\{y'(t); y(t)\}$ and frequency tuning $\nu_s = 0.60; \nu_s = 1.05$ and $\nu_s = 1.20$, s. Fig.4.

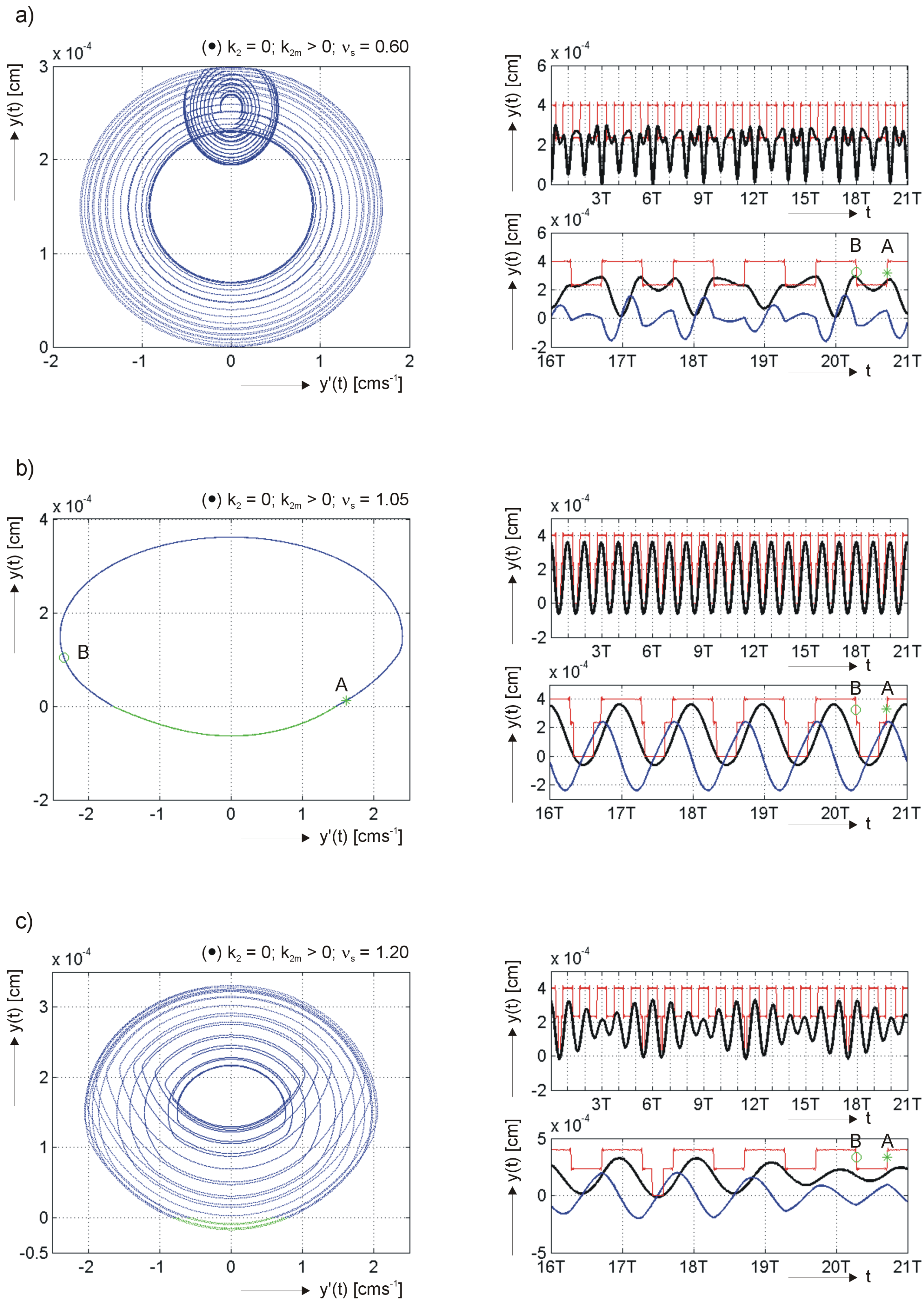

Fig.4 - Phase planes $\{\, y'(t); y(t)\}$ and time course *y(t), y´(t), C(t)* and *C(t)(H1+H2)* of nonlinear quadratic damped system for the variant damping (c) – $(\bullet)\, k_2 = 0; k_{2m} > 0$ in accordance with Tab.1 and the frequency tuning $\nu_s = 0.60; \nu_s = 1.05$ a $\nu_s = 1.20$.

From the fullness of phase planes, the time mutual position of $C(t)$ alternatively $C(t)(H1+H2)$ towards $y(t)$, $y'(t)$ result then the conclusions for the steady or the heterodyne time course of relative motion.

In the Fig.4 are given the time courses both $y(t)$, $y'(t)$ and $C(t)$ alternatively $C(t)(H1+H2)$ for 21 and 5 last periods from the revolution cycle of the cog wheels with the number of teeth $Z_{3,2} = 21$ and with the tuning $\nu_s = 0.60; \nu_s = 1.05$ and $\nu_s = 1.20$. When we compare the applicative characteristics from the Fig.3 with the phase planes and time courses from Fig.4 for the nonlinear quadratic damping (c) – (•), we see that the bifurcations are the result of the phenomena of the heterodyne courses of the relative motion $y(t)$. They are however also the function of frequency tuning ν_{min}, ν_{max} of the relevant resulting stiffness functions in the gear mesh, pertinently $\nu = 0$ by the contact bounces – impact phenomena in the gear mesh and are likewise the functions of the damping both material one and viscous mediums in the gear mesh. The steady motion mark out in the phase planes with the thin lines of function courses, s. e.g. Fig.4b.

We pay attention to the influence of the phase shift of $y(t)$ towards $C(t)$ alternatively towards $C(t)(H1+H2)$ and the time duration of the component stiffness levels in these functions onto the possible growth or the descent of the function amplitude $y(t)$.

The points of discreteness A,B in the courses – the jump location in the stiffness function $C(t)$ alternatively $C(t)(H1+H2)$ mark the discreteness locations in the phase planes. The exceptionality in the time courses of relative motion $y(t)$ in the time heteronomous – parametric systems is the periodicity of the courses [7].

In Fig.5 is given the exemplification of phase planes $\{y'(t); y(t); \nu_s\}$ for the nonlinear quadraticaly damped system of gearing with six degrees of freedom in the frequency range $\nu_s \in \langle 0.6;1.2 \rangle$ for the variant damping (in accordance to Tab.1) (c) – (□) $k_2 > 0, k_{2m} = 0$. The function courses - loops in the phase planes disappear with the increasing frequency tuning ν_s and only melt into the discreteness location A,B. They are in accordance to Fig.4 the locations, where the stiffness function $C(t)$ alternatively $C(t)(H1+H2)$ skips from the stiffness level C_{min} on C_{max} and conversely. The extinction of loops is also the function of damping in the gear mesh [5].

The influence of the damping size is given e.g. in the Fig.6 in the phase portrait $\{y'(t); y(t); \nu_s\}$ for the frequency range $\nu_s \in \langle 0.6;0.8 \rangle$. The blue courses of phase planes present the courses with nonlinear quadratic variant damping (c) - (□), $k_2 > 0; k_{2m} = 0$ for $k_2 = 3.9537$ (i.e. the proportional damping $D_2 = 0.062$), the red courses then for the damping 5x greater, i.e. $k_2' = 19.7685$ (the proportional damping $D_2' = 0.31$).

From the phase portrait in Fig.6a is evident, that the red courses of more damped system with the amplitude $y(t)$ exceed in the range $\nu_s \approx 0.61$ the blue courses, i.e. courses of the less damped system. It is the consequence of the phase shift of the amplitude $y(t)$ in the given frequency tuning ν_s towards the stiffness function $C(t)$. This fact is then also evident from Fig.6b, where is plotted the phase plane from view in the direction ν_s. In the red courses

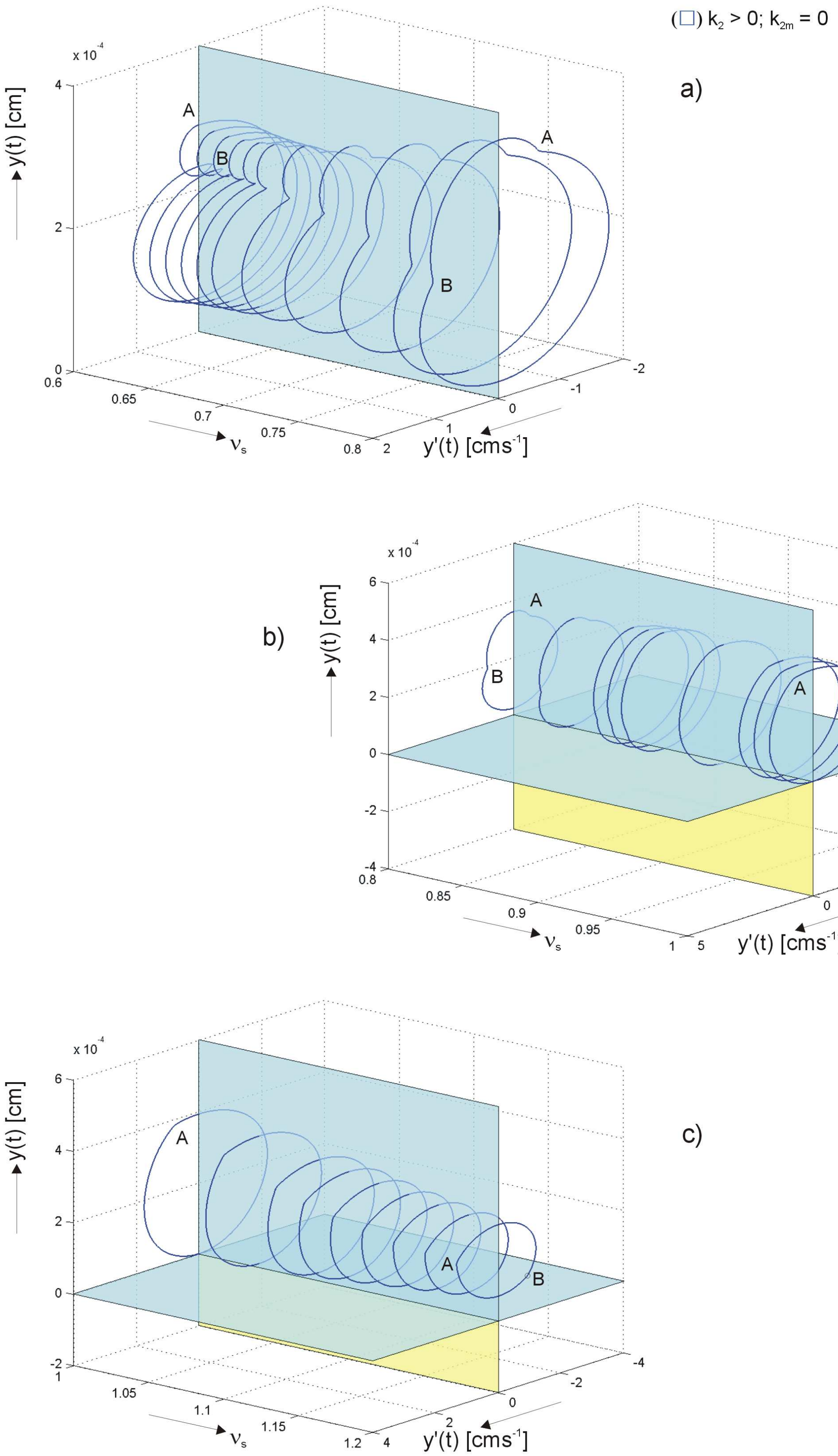

Fig.5 – Resonance phase portrait $\{ y'(t); y(t); \nu_s \}$ of nonlinear quadratic variant damping (c) - (□), $k_2 > 0; k_{2m} = 0$ (s.Tab.1) in the frequency range $\nu_s \in \langle 0.6;1.2 \rangle$ for the nonconservative system of cog wheels with six DOF (in accordance to Fig.1).

occur for given frequency tuning and the material damping only the discreteness location A‘,B‘.

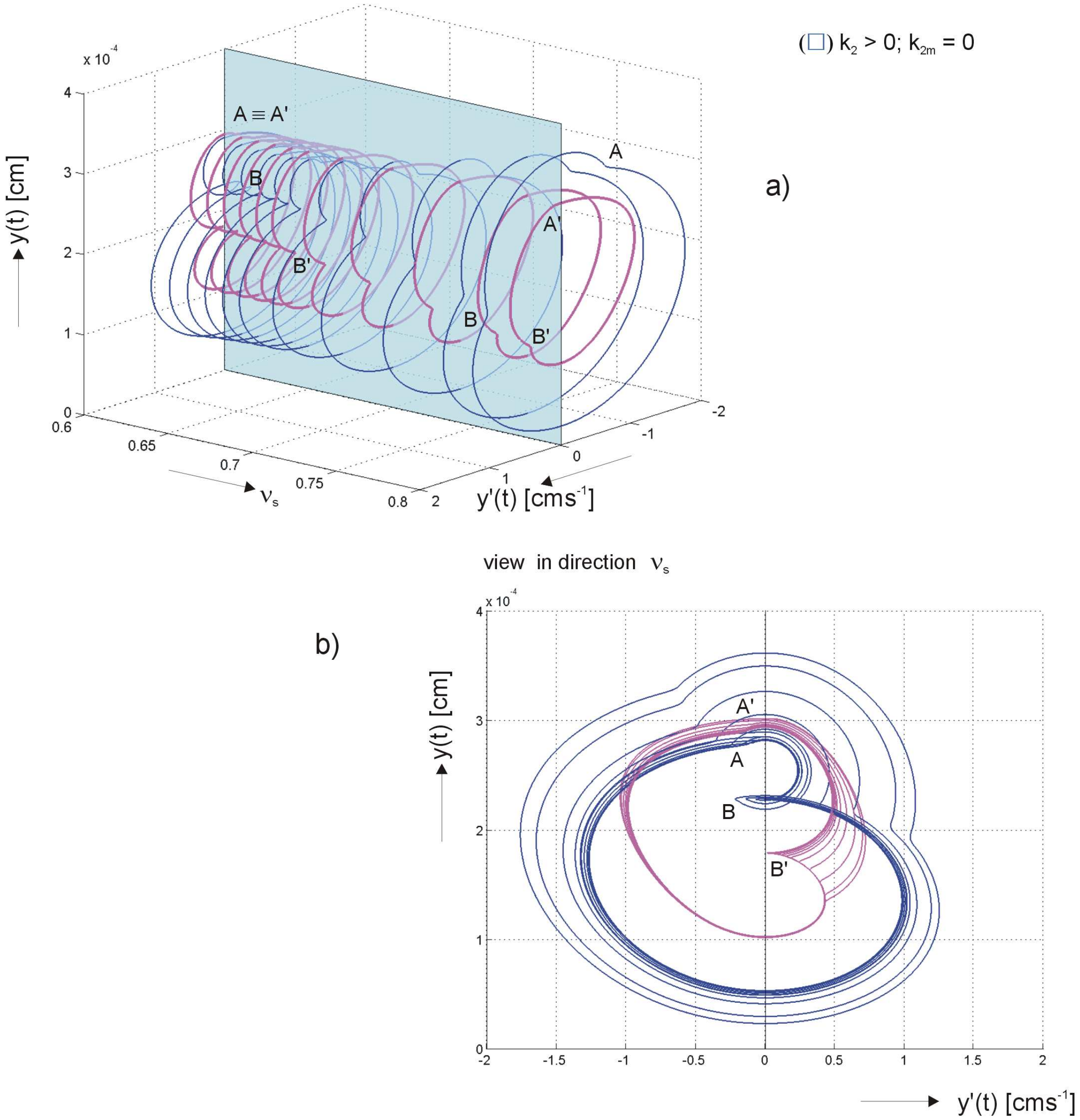

Fig.6 – Resonance phase portrait $\{ y'(t); y(t); \nu_s \}$ of nonlinear gearing system with 6DOF (Fig.1) with nonlinear quadratic damping in gear mesh for the variant damping (Tab.1) (c) - (□), $k_2 > 0; k_{2m} = 0$ and the material damping $k_2 = 3.9537$ and $k_2' = 19.7685$.

The same phenomenon is with the detailed analysis evident from Fig.7, where are plotted besides the courses of two phase planes (blue and red) also the time courses $y(t), y'(t), y''(t)$, stiffness function $C(t)$, dynamic $F_{dyn}(t)$ and frictional $F_T(t)$ forces in the mesh of gearing with spur gears for two periods 2T.

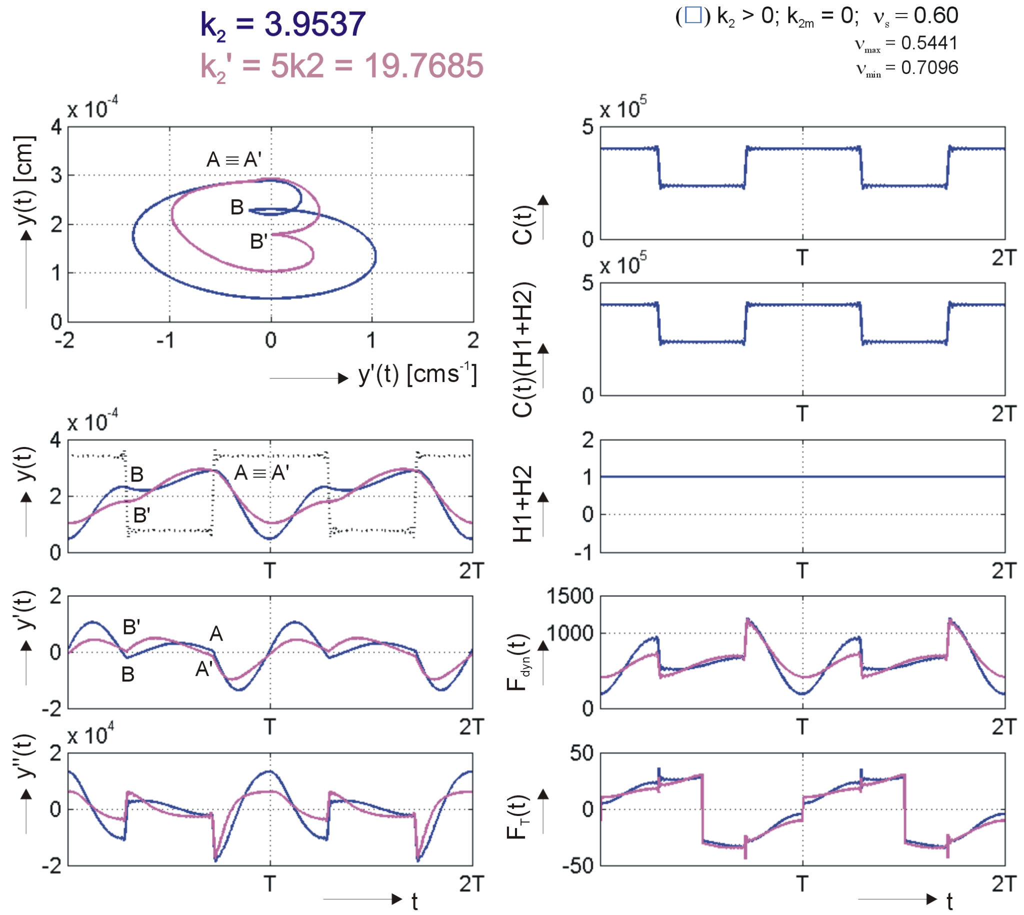

Fig.7 – Resonance phase planes $\{y'(t);y(t)\}$ of system with kinematic couplings (Fig.1) with 6DOF for the nonlinear quadratic variant damping (Tab.1) (c) - (□), $k_2>0;k_{2m}=0$ (c) - (□), $k_2>0;k_{2m}=0$ and for the material damping $k_2=3.9537$ (the proportional damping $D_2=0.062$) and $k_2'=19.7685$ (the proportional damping $D_2'=0.31$).

The Fig.6 and Fig.7 explain also the phenomenon from DETAIL X (Fig.3), i.e. the issue of the next resonance bifurcation courses. The loops in the phase courses (Fig.6, Fig.7) forms in the frequency range $\nu_s\in\langle 0.6;0.64\rangle$ for the given nonlinear quadratic variant damping (c) - (□) in the resonance characteristics (Fig.3) the next bifurcation course. For the given material damping $k_2=3.9537$ (the proportional damping $D_2=0.062$) becomes the loop by the tuning $\nu_s\geq 0.64$ into the discreteness location B in the phase course, as is seen from blue courses in the Fig.6b. The red courses form the solution of given system for the greater damping $k_2'=19.7685$ (the proportional damping $D_2'=0.31$) with the discreteness location B‘.
The existence of loops in the courses of the phase planes is the function of damping in gear mesh and the frequency tuning ν_s of the resulting stiffness function $C(t)$ alternatively of its modify form $C(t)(H1+H2)$.

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