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# FATIGUE DAMAGE FUNCTION AND ITS EXPERIMENTAL VERIFICATION

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**Summary:** *The authors proposed a method for assessing the increase of deflections of structural elements exposed to cyclic loading – the fatigue damage function. The fatigue damage function was verified experimentally with very good results.*

## 1. Introduction

Fatigue is a process of permanent progressive changes in the structure of a material exposed to cyclic loading. Research related to fatigue of metals started in 1840's with the construction of railways. Fatigue of concrete and concrete structures was first described at the beginning of the 20th century and became a significant topic in 1920's with the development of highways.

Nowadays, the use of high strength materials results into design of more slender structures with high live load proportion of the total load. High stress ranges in structures like bridges or crane-ways can result into accelerated crack propagation, higher deflections, structural stiffness reduction and consequently into fatigue failure.

## 2. Strain development in concrete under cyclic loading

### 2.1 Strain development under cyclic loading

Cyclic load acts on a material with a slightly changed structure in every new load cycle. The cracks do not close during the unloading phases. Stress concentrations at crack tips cause damage in every load cycle. The cyclic load causes further development of existing cracks. Cracks propagate, unite and finally they develop in the whole specimen section. In the end they cause the element to fail, though the stress it was subjected to was lower than the static strength of the material.

The cyclic creep curve (Figure 1) plots the change of strain or deformation with the number of load cycles the specimen is being exposed to.

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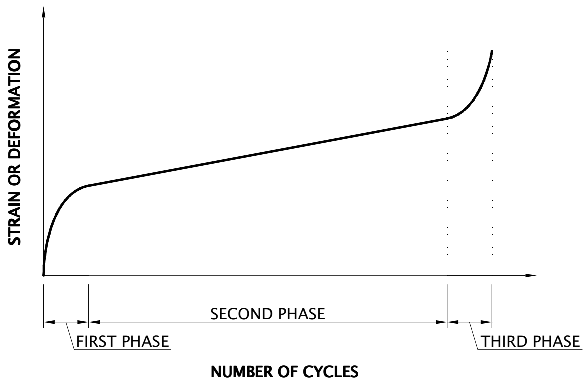


Figure 1 The Cyclic creep curve

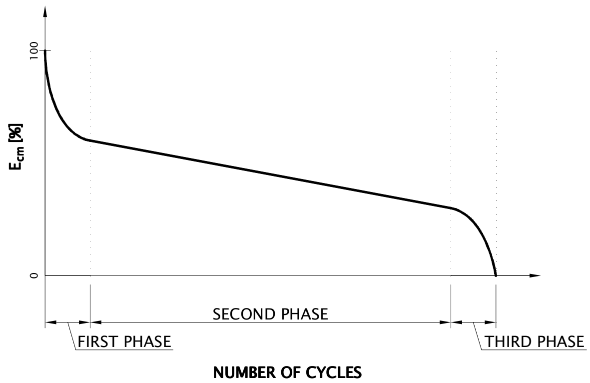


Figure 2 Development of secant modulus of elasticity of concrete under cyclic loading

The cyclic creep curve consists of three phases which are similar to the fatigue phases of ferrous materials:

- Phase 1
  The first phase is the initiation phase. Within this one, microcracks develop in the weaker parts of the cement paste and the strain increases rapidly. Yet the rate of strain increase is decreasing progressively with the number of load cycles. The first phase is completed after approximately 5-10% of the total load cycles.
- Phase 2
  In the second phase, the cracks propagate stably and the strain increases approximately linearly with the number of load cycles. The microcracks which developed in the initiation phase propagate slowly further until they reach their critical length. Some authors name this phase as microcracking – the growth of microcracks. The second phase usually takes about 80% of the total load cycles.
- Phase 3
  The third phase represents the instable crack growth which leads to the fatigue failure of the specimen. It starts when there are already enough instable cracks present, i.e. cracks which have reached their critical length. These microcracks than unite into one macro crack which can cause the fatigue failure. In the third stage, the bearing capacity of the structural element is already severely weakened and the strain increases under progressively increasing rate until failure. The third stage takes the remaining 10-15% of the total load cycles.

## 2.2 Development of secant modulus of elasticity of concrete under cyclic loading

The development of secant modulus of elasticity under cyclic loading reflects the development of strain or deformation, et versa. The development of secant modulus of elasticity under cyclic loading was described by J.O. Holmen in 1979, see Figure 2.

In his research, Holmen used a loading frequency of 5Hz and minimum stress equal to $0.05*f_{cm}$. The maximum stress varied from $0.95*f_{cm}$ to $0.67*f_{cm}$. Higher values of the maximum stress gave smaller proportion of the first phase from the total number of load cycles (less than 10%) and a steeper slope in the second phase. Lower values of maximum stress had a bigger proportion of the first phase from the total number of load cycles (approximately 10-15%) and a flatter slope in the second phase of secant modulus of elasticity development. The first phase of development of secant modulus of elasticity finished at 75-95% of $E_{cm}$, the second phase at 65-75% of $E_{cm}$ depending on the maximum stress applied.

## 3. Mathematical description of strain development under cyclic loading

### 3.1 Parametric description of the development of secant modulus of elasticity of concrete under cyclic loading

Based on the experiments of J.O. Holmen, parametric description of the development of secant modulus of elasticity of concrete under cyclic loading is proposed.

Constants *a* and *b* are introduced, *a* for the decrease of the secant modulus of elasticity in the first phase of its development under cyclic loading, *b* for the remaining proportion of the original secant modulus of elasticity at the beginning of the third phase of its development. The graphical meaning of the constants is explained in Figure 3.

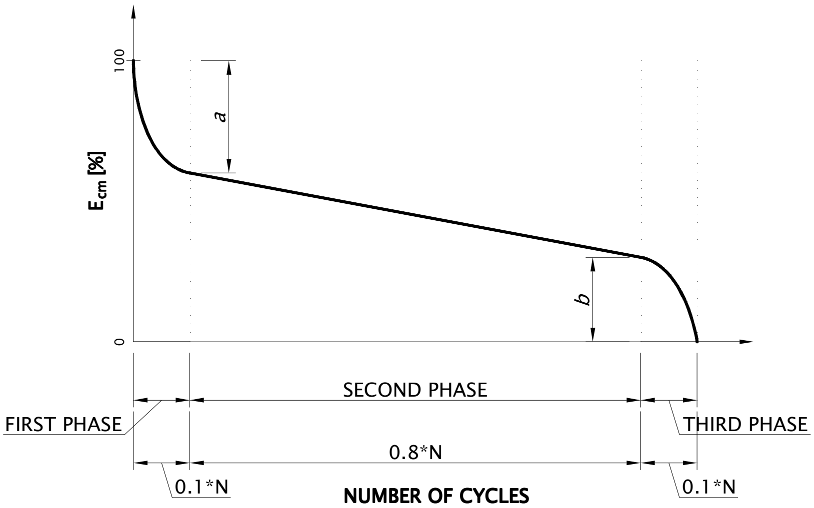


Figure 3 Graphical explanation of the meaning of constants *a* and *b*

The decrease of the secant modulus of elasticity in the first phase of its development under cyclic loading, the constant *a*, is greater at lower stress levels. On the other hand, remaining proportion of the original secant modulus of elasticity at the beginning of the third phase of its development, the constant *b*, is smaller at lower stress levels. This means, expressed in a different way, that a structural element under lower stress level loses higher proportion of the secant modulus of elasticity during the first phase of its development than under higher stress level, while it is still operational at the beginning the third phase of the strain development with a smaller proportion of the original secant modulus of elasticity which would cause fatigue failure at higher stress levels.

The formulas for the constants *a* and *b* were obtained with the use of linear regression:

$$a = 0{,}47 - 0{,}4 \cdot S_{max} \tag{1}$$

$$b = 0{,}57 + 0{,}17 \cdot S_{max} \tag{2}$$

Due to the method used for their assessment and input data based on higher stress levels, formulas for constants *a* and *b* are valid only for stress levels $S_{max}$ > 0,174. New formula, which would incorporate lower stress levels, has to be developed on an experimental basis.

### 3.2 Fatigue damage function

A mathematical function for describing the strain development in a concrete specimen under cyclic loading is sought for. This function should be able to give a value of modulus of elasticity in every particular moment of cyclic loading, thus respecting the three phases in strain development under cyclic loading. The reduced value of modulus of elasticity can be then used for calculating deflections increased by damage accumulation caused by cyclic loading.
This function should be used for assessing deflections of structures subjected to cyclic loading either in "in-hand" calculations or inserted into FEM software to produce a complete useful life analysis of a particular structure.

The instant value of modulus of elasticity after $n_i$ load cycles is:

$$E_{n_i} = \omega_{F_i} \cdot E_{n_o} \tag{3}$$

where $E_{n_i}$ is the value of modulus of elasticity after $n_i$ load cycles,

$\omega_{F_i}$ is the value of fatigue damage function after $n_i$ load cycles,

$E_{n_o}$ is the value of modulus of elasticity at the start of the cyclic loading.

The fatigue damage function decreases modulus of elasticity of concrete at the start of cyclic loading to quantify the deteriorative effect of $n_i$ load cycles of cyclic loading.

It comprises the sum of a power function and an exponential function. While the exponential part is dependent on $S_{max}$ and represents rapid decrease of modulus of elasticity in the first phase of cyclic loading, together with the stable progressive decrease of modulus of elasticity in the second phase of cyclic loading, the exponential part is independent of $S_{max}$ and represents rapid decrease of modulus of elasticity in the third phase of cyclic loading:

$$\omega_{F_i} = 1 - \left\{ \left[ a^{c_4 \cdot \frac{1}{S_{max}^{c3}}} \cdot \frac{n_i}{c_1 \cdot N} \right]^{\frac{1}{c_4 \cdot \frac{1}{S_{max}^{c3}}}} + b \cdot \exp\left[ \left( \frac{n_i}{N} - 1 \right) \cdot c_2 \right] \right\} \tag{4}$$

where $\omega_{F\,i}$ is the value of fatigue damage function after $n_i$ load cycles,

$n_i$ is the number of load cycles the structural element has already resisted,

$N$ is the total number of load cycles the structural element is able to resist. This value can be calculated by formulas given in Eurocode 2 or more conservatively in Model Code 1990,

$a$, $b$ are constants dependent on load level,

$c_1$ = 0.1,

$c_2$ = 70,

$c_3$ = 2.72,

$$c_4 = \begin{cases} = 1.0 \text{ for } S_{\max} \in (0;0.377\rangle \\ = 2.1436 * S_{max} + 0.19037 \text{ for } S_{\max} \in (0.377;0.736) \\ = 1.771 \text{ for } S_{\max} \in \langle 0.736;1) \end{cases}$$

Figure 4 gives example of the fatigue damage function for various load levels.

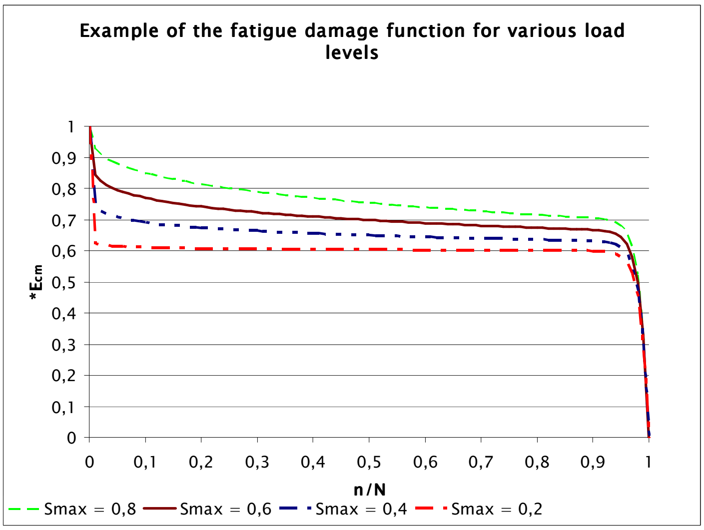


Figure 4 Example of the fatigue damage function for various load levels

## 4. Experimental verification of the fatigue damage function

### 4.1 Layout of the experiments

For the purpose of the experiments of Dr. Plachý, the company SMP CONSTRUCTION, a.s. made three prestressed concrete slabs. The slabs were prestressed from one side by 11 15,7mm prestressing tendons. Strength class of the concrete of the prestressed slabs was C 45/55. The prestressed slabs were subjected to four-point bending. Layout and static scheme of the experiments can be seen in Figures 5 and 6.

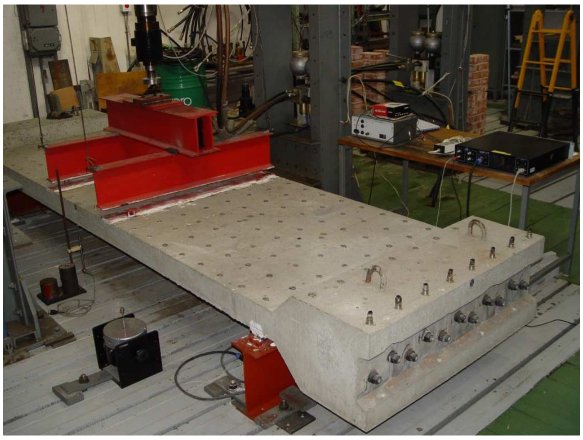

Figure 5 Layout of the experiments

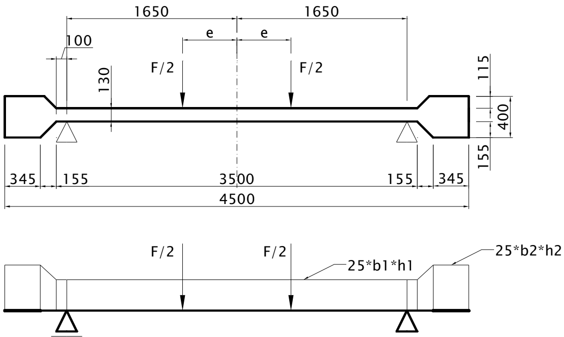


Figure 6 Schematic layout and static scheme of the experiments

Loading frequencies, maximum / minimum force, its eccentricity and number of cycles at specific load level can be seen in Table 1.

Table 1 Loading frequencies, maximum / minimum force and its eccentricity, number of cycles at specific load level for Slab No. 2

| Slab No. | Loading frequency [Hz] | Cyclic force F [kN] | | Force couple eccentricity e [m] | Load cycles applied[1] |
|---|---|---|---|---|---|
| | | Min. | Max. | | |
| 2 | 2,5 | 5 | 133 | 0,375 | 2 268 570 |

Deflections of the prestressed slab No.2 were measured by the inductive track recorder HBM W50. The arrangement of the deflection measurement can be seen in Figure 7.

Figure 7 Detail of the deflection measurement

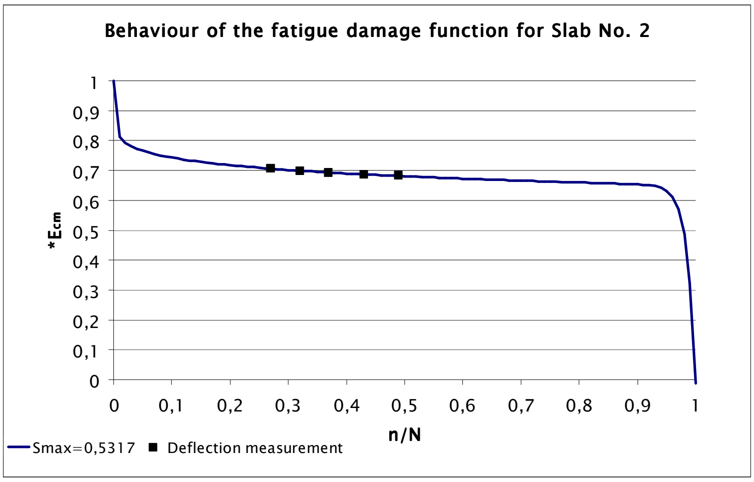


Figure 8 Behaviour of the fatigue damage function for the Slab No. 2

[1] Number of load cycles applied to Slab No. 2 to 22nd November 2007.

## 4.2 Comparison of measured and calculated values of deflection

Five deflection measurements were made in total. Date of measurements, measured deflections and the number of load cycles they were measured at can be seen in Table 2.

Table 2 Deflection measurements on Slab No. 2

| Measurement No. | Date of measurement | Days between measurements [days] | No. of load cycles at measurement [-] | Measured deflections [mm] |
|---|---|---|---|---|
| 1 | 17.7.2007 | - | 1 280 040 | 14,437 |
| 2 | 1.8.2007 | 14 | 1 504 000 | 17,937 |
| 3 | 26.9.2007 | 55 | 1 740 360 | 18,303 |
| 4 | 23.10.2007 | 27 | 2 019 370 | 17,688 |
| 5 | 22.11.2007 | 29 | 2 268 570 | 17,991 |

Cracks propagated on the soffit of the slab after approximately 1 350 000 load cycles. These cracks resulted into irreversible increase of deformations between measurement 1 and 2 (see Table 2).

The maximum stress level $S_{max}$ = 0,5317 was obtained from the detailed fatigue analysis of Slab No. 2. The behaviour of the fatigue damage function for this maximum stress level was calculated and can be seen in Figure 8 with emphasized position of deflection measurements from the point of view of the fatigue endurance of the Slab No. 2.

Table 2 Calculated values of the fatigue damage function for all the measurements

| Measurement No. | n/N [-] | Value of the fatigue damage function [-] | $E_{n_i}$ [MPa] | Calculated deflections [mm] |
|---|---|---|---|---|
| 1 | 0,27415 | 0,70525 | 29 973 | 14,586 |
| 2 | 0,32212 | 0,69878 | 29 698 | 19,071 |
| 3 | 0,37274 | 0,69279 | 29 444 | 19,236 |
| 4 | 0,43249 | 0,68659 | 29 180 | 19,410 |
| 5 | 0,48587 | 0,68216 | 28 992 | 19,536 |

The measured and calculated values of deflection are summarized in Table 4.

Table 3 Comparison of the measured and with the use of fatigue damage function calculated values of of deflection of Slab No. 2

| Measurement No. | No. of load cycles at measurement [-] | Measured deflections [mm] | Calculated deflections [mm] | Measured / calculated deflections [mm] |
|---|---|---|---|---|
| 1 | 1 280 040 | 14,437 | 14,586 | 98,98 % |
| 2 | 1 504 000 | 17,937 | 19,071 | 94,05 % |
| 3 | 1 740 360 | 18,303 | 19,236 | 95,15 % |
| 4 | 2 019 370 | 17,688 | 19,410 | 91,13 % |
| 5 | 2 268 570 | 17,991 | 19,536 | 92,09 % |

The measured and the calculated values show a very good agreement, the difference varies from 1 % to 8,9 %. Figure 9 compares the measured and with the use of fatigue damage function predicted values of deflection of Slab No. 2 with the static deflection.

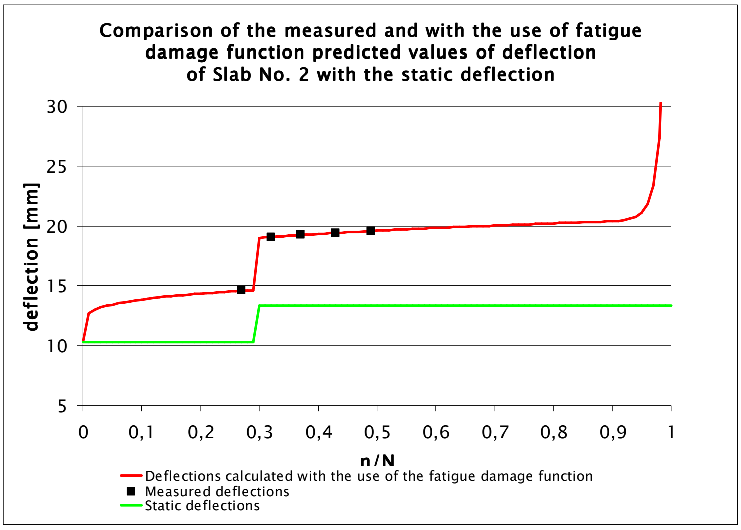


Figure 9 Comparison of the measured and with the use of fatigue damage function calculated values of deflection of Slab No. 2 with the static deflection

The predicted behaviour of deflection of the Slab No. 2 under cyclic loading shows that the values of the static deflection can increase up to 1,6 times of the value of the static deflection before the slab collapses (i.e. enters the third phase of the strain development under cyclic loading).

## 5. Conclusions

The authors proposed a method for assessing the increase of deflections of structural elements exposed to cyclic loading – the fatigue damage function – a mathematical function which describes the strain development in concrete under cyclic loading. It produces a decreasing multiplier of the original modulus of elasticity at the start of the cyclic loading which represents the deteriorative effect of cyclic loading on a concrete structural element. With the help of fatigue damage function the increase of deformations of cyclic loaded structural elements can be assessed and the total fatigue endurance predicted.

This text presented an experimental verification of the fatigue damage function on prestressed concrete slab. The measured and with the use of fatigue damage function calculated values of deflection showed a very good agreement. The detailed analysis showed that the deflection of a cyclic loaded concrete structural element can reach up to 160% of the static deflection before the structural element fails due to fatigue of concrete.

In conclusion, it has to be emphasized that the fatigue damage function can be easily used in „in-hand" calculations or inserted into FEM based software and used in common praxis for assessing the increase of deformations of concrete structural elements caused by cyclic loading.

## Acknowledgement

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