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WHEEL VERTICAL FORCES DISTRIBUTION AS THE SOURCE OF VEHICLE STABILIZATION

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Summary: *The paper presents specific approach in terms of finding the dependency between corrective torque, which is acting to a vehicle in yaw direction, and the tire vertical forces redistribution among four corners in a vehicle. The formula for wheel vertical forces evaluation was derived from the system of equations. There is described the relationship between corrective torque and the tire vertical force variations. Derived body control system algorithm offers the ability to control the lateral dynamics of the vehicle. The limitation of the system has been defined. Generally, the limits are given by the static load of the wheels. The efficiency of the control system was observed and expressed numerically by the vehicle movement close to the adhesion limit.*

1. Introduction

There are some ways how to directly or indirectly affect behaviour of the vehicle in yaw direction. All these methods have the same principle of finding the optimal tire forces distribution among four corners in the vehicle. The system of dynamic body control modifies the vertical forces distribution in order to change the adhesion threshold of individual tire up and down. The question arises, whether there is any potential to stabilize the vehicle in the extreme driving maneuvres using the wheel vertical forces variation.

2. Vehicle model

For simulations there was used 3DOF vehicle model with mass *m* and velocity *v*, where the lateral dynamics was defined by equations (1).

$$\dot{\beta} = \frac{-m\dot{v}\sin\beta + \cos\delta_f\left(S_{FL} + S_{FR}\right) + S_{RL} + S_{RR}}{mv\cos\beta} - \dot{\psi}$$
$$\ddot{\psi} = \frac{S_{FL}\left(l_f\cos\delta_f + \frac{T}{2}\sin\delta_f\right) + S_{FR}\left(l_f\cos\delta_f - \frac{T}{2}\sin\delta_f\right) - S_{RL}\cdot l_r - S_{RR}\cdot l_r + M_{Corr}}{J} \qquad (1)$$

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Parameter δ_f means the average value of the front wheels steering angles δ_{FL}, δ_{FR}. Variables S_{FL}, S_{FR}, S_{RL}, S_{RR} are the tire lateral forces, l_f and l_r are the CG (centre of gravity) distances from front and rear axle. J means the moment of inertia around the z-axis of the vehicle. As for the parameter T, the simplification which is based on the average value of the front and rear track T_F, T_R was used.

As a product of the equations (1), there are evaluated two parameters: the sideslip angle of the vehicle β and the yaw rate $\dot{\psi}$. If M_{corr} is used as an input variable, then it has the ability to affect turning of the vehicle in positive or negative way directly. Positive M_{corr} makes the vehicle more oversteered, negative M_{corr} makes the vehicle more understeered. This subject of corrective torque magnitude, that represents the corrective torque demand for vehicle stabilization as an idealized problem, was comprehensively described in Drobný & Valášek (2007). The direct control of the vehicle using the M_{corr} input variable can be solved from the mathematical point of view only. As for the real vehicle behavior modeling, there rises the certain demand on finding the energy source for vehicle dynamics control in relation to M_{corr}. This paper extends previous results and is mainly focused on the mentioned source of M_{corr} generation using the vertical forces variation and as well as on the relation between vertical forces variation and gained M_{corr}.

2.1. Vertical forces model

The 4-wheel mathematical model of the vehicle was extended by comprising of the vertical forces. When a vehicle turns, the weight is transferred from the inside wheels to the outside wheels. The magnitude of this weight transfer is the function of mass, speed, yaw rate, and location of the CG. This weight transfer must be balanced by the tire vertical forces.

$$\begin{gathered} N_{FL} + N_{FR} + N_{RL} + N_{RR} = mg \\ -h_{CG} m a_y \cos\beta + \frac{T}{2} mg - T\left(N_{FL} + N_{RL}\right) = 0 \end{gathered} \tag{2}$$

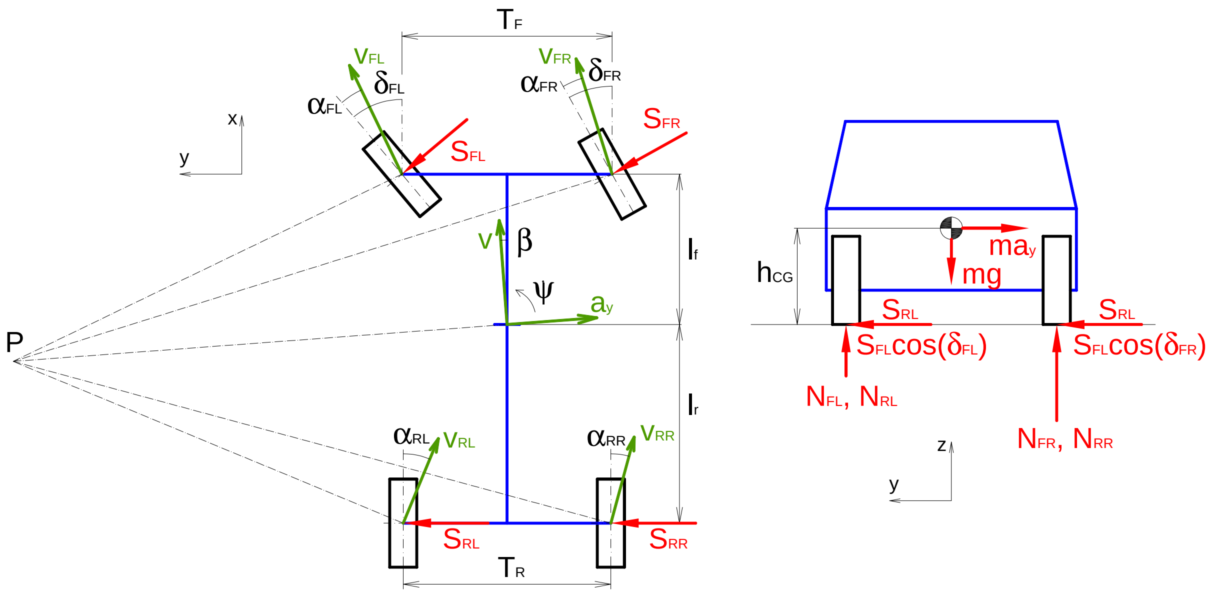

Fig. 1: Scheme of the lateral and vertical mathematical model of the vehicle

The parameters which were used in the extended mathematical model of the vehicle are shown on the Fig. 1. In the addition to equations (1), there are used equations (2) for roll response definition. They describe the sum of vertical forces and torque equilibriums. Forces N_{FL}, N_{FR}, N_{RL}, N_{RR} represent the tire vertical forces and distance h_{CG} represents the CG height above the road surface.

2.2. Vertical forces variation

The main strategy of the stability control system is based on the controlling of the lateral tire forces transfer among the four corners of the vehicle. The tire vertical load affects longitudinal and lateral forces, which are acting on the tire. By increasing the vertical tire force, the lateral force increases nonlinearly with apparent degradation at high vertical forces. The lateral force dependency generated by using Pacejka Magic Formula is represented on Fig. 2.

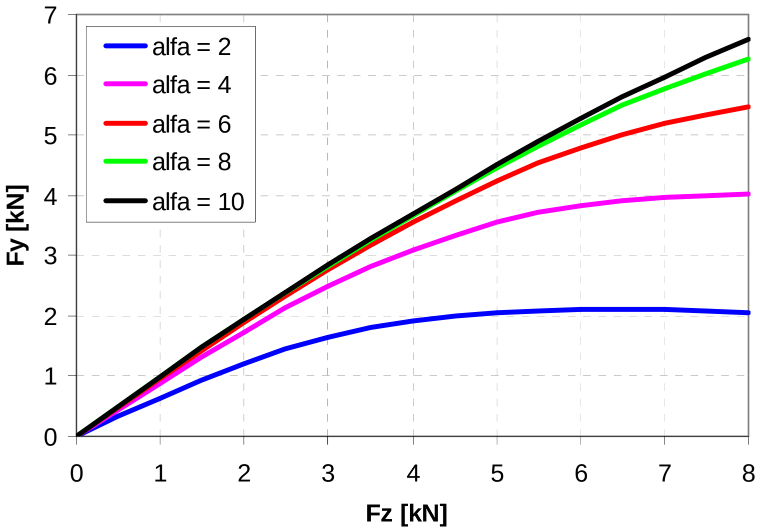

Fig. 2: Lateral tire force *Fy* over wheel load *Fz* at different slip angles

The lateral stability of the vehicle keeps effective as far as the potential to transfer the lateral forces is sufficient. With the usage of the lateral force degradation on the appropriate tire, we can generate the yaw stabilization movement of the vehicle. Described situation occurs at the vertical force lowering. On the other side, by increasing the tire vertical force, the response leads to enlarging the tire lateral force. The resulted dynamic effect of the 4-wheel vertical force variations is sufficient to be described by parameter M_{corr}. It characterizes the stabilization demand and among others it allows to make the comparison to the passive vehicle.

By the formulation of the solved problematic there was expected, that the weight distribution among the 4 wheels did not cause any movement in vehicle roll and pitch. In the ideal case the driver wouldn't be able to recognize any change of the vehicle's behavior. Development of the expressed problematic led to the system of four equations (3). The variable dN represents one input parameter. Solving the equation for specific dN results to the actual values of the vertical forces N_{FL}, N_{FR}, N_{RL}, N_{RR}. Defined formula was integrated to the vehicle control system as the actuator and was connected as a part of the feedback control loop, which compares the movement of the vehicle with the movement of the reference model.

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} N_{FL} \\ N_{FR} \\ N_{RL} \\ N_{RL} \end{bmatrix} = \begin{bmatrix} -\frac{h_{CG} m a_y \cos\beta}{T} + \frac{mg}{2} \\ \frac{mgl_r}{l_f + l_r} \\ mg \\ \left(\frac{h_{CG} m a_y \cos\beta}{T} + \frac{mg}{2}\right) \cdot \left(1 - \frac{l_r}{l_f + l_r}\right) + dN \end{bmatrix} \tag{3}$$

2.2.1. Stabilization process

Simulation outputs of the controlled vehicle model were analyzed. The correlation between dN and M_{corr} was determined. The corrective torque is generated in positive direction, if the vertical load at inner front and outer rear wheel increases and at the same time the outer front and inner rear tire vertical force decreases. The vertical force shift dN has the same absolute value for all wheels. The difference consists in the force direction. The final conclusions for positive and negative corrective torque generation are shown on Fig. 3. Plus sign on the appropriate wheel means the increase of the vertical tire force and the minus sign means the decrease.

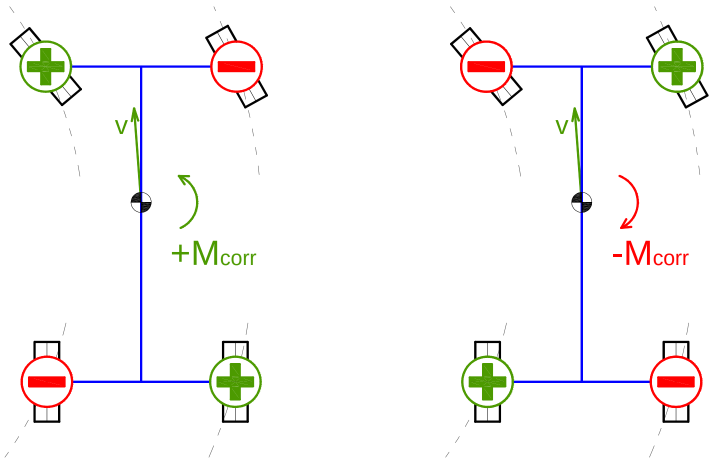

Fig. 3: Vertical tire force shift (dN) and corrective torque (M_{corr}) correlation

2.3. Equivalent torque

The corrective torque M_{corr}, which is the product of the vertical forces’ change, could not be simply measured. There was used a special method how to obtain the value of corrective yaw torque. As for the initial state of the vehicle, the constant velocity cornering near the adhesion limit was chosen. This steady state cornering was disturbed by application of the negative torque $-M_{corr}$ into the mathematical model of the vehicle (1). Subsequently the deviated trajectory of the vehicle caused changes in observed parameters β and $\dot{\psi}$ compared to the original vehicle model. In the next step there was used the correction of the vehicle movement by using the control algorithm, namely the vertical forces distribution specified in chapter 2.2. The purpose was to figure out the dependency between vertical force shift dN and generated corrective torque M_{corr}. The magnitude of M_{corr} gives us the information about the efficiency of the proposed control method when compared to the system with directly applied M_{corr} in the mathematical equations (published in Drobný & Valášek (2007)). For several constant M_{corr} increment steps there were found appropriate *dN* values. Satisfying the condition where: using adequate force variation *dN* the vehicle moves along the original trajectory - the same trajectory as the passive vehicle would have moved without any disturbances. The solutions led to the minimizing of β and $\dot{\psi}$ deviations between the original and disturbed vehicle. The multiple parameter optimization method was used. The results are shown on Tab. 1.

Tab. 1: Torque M_{corr} and force absolute value *dN* correlation

M_{corr}	*dN*
-500 Nm	603 N
-1000 Nm	1213 N
-1500 Nm	2362 N
-2000 Nm	2321 N

The optimization process for $M_{corr} = -2000\ Nm$ induced loosing the contact of one tire. The intention was to increase *dN* much more, but the potential for stabilizing the vehicle has been completely used. Such a high corrective torque could not be fully compensated. The efficiency of the system falls rapidly down in case of loosing the contact of any tire.

3. Conclusions

The clear dependency was found between the corrective torque of the vehicle and the tire vertical forces redistribution among four corners of the vehicle. The results give to the dynamic body control system the opportunity to control the lateral dynamics of the vehicle. The limitation of the system is caused by the contact of the tire. The maximal force *dN* should be less then tire static load force.

The efficiency of the mentioned control system could be higher than presented in this paper, if the longitudinal tire forces will be implicated into the mathematical model of the vehicle. Driving and braking forces cause destabilization of the vehicle lateral movement. Compared to the presented model it will occur at lower vehicle speeds where the potential for stabilizing is higher. It is an object for the future work.

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