National Conference with International Participation

ENGINEERING MECHANICS 2008

Svratka, Czech Republic, May 12 – 15, 2008

KINEMATICS OF 16 DOF ROBOTIZED VEHICLE CHASSIS

M. Denk*, M. Šír**

Summary: *This paper deals with elaboration of a mathematical model of a robotized chassis with sixteen degrees of freedom. This model will be used for setting of gear parameters, it will be a part of control system, and it will be a base for creation of a simulator in the future. The chassis is equipped with four arms ended with wheels. Each arm has four degrees of freedom. The kinematics is solved within the range of location and speed with maximal observance of rolling conditions between the wheels and the surface. Some animations of the vehicle basic motions which can be seen on internet pages are conclusion of the paper.*

1. Introduction

The paper was written in the frame of the research project "*Optimalizace vlastností strojů v interakci s pracovními procesy a člověkem*" and it deals with a robotized chassis for social/medical applications with the aim to develop a device enabling handicapped people and laying patients to move on a rough surface.

This article describes an elaboration of a mathematical model of a robotized chassis with sixteen degrees of freedom. The model will be used (i) for gear parameters setting, (ii) as a part of the vehicle control system, and (iii) as a base for a vehicle simulator in the future. The chassis is equipped with four axles made as legs ended with wheels. Each axle has four degrees of freedom. The kinematics is solved for the vehicle position and velocity with the observance of the rolling conditions between the wheels and the surface.

2. Concept

In order to enable a user to move freely without any help of another person both in urbanized environment and outdoor, the concept should enable the robotized chassis to perform at least the following maneuvers keeping the seat in a horizontal position:

- motion both straightforward and in varying direction on a flat and complicated terrain as wall,
- the change of the chassis clearance height,
- motion on stairs of various parameters,

* Ing. Miroslav Denk: Department of Engineering Mechanics, Technical University of Liberec; Komenského 2; 460 01 Liberec; tel.:+420 485 354 159; e-mail: miroslav.denk@tul.cz

** doc. Ing. Miroslav Šír, CSc.: Department of Engineering Mechanics, Technical University of Liberec; Komenského 2; 460 01 Liberec; tel.:+420 485 354 136; e-mail: miroslav.sir@tul.cz

- crossing an obstacle by stepping over,
- going through a narrow passage without a stability loss.

For these reasons, a configuration with four arms ended with a wheel has been chosen. The wheel is ball-shaped, because such wheel has a good contact with a supporting surface even if the rotation axis is greatly diverted from the tangential plane. Each arm has four degrees of freedom which are directly or indirectly controlled by separate electro motors. Motions corresponding to these DOFs are marked in the Fig. 1.

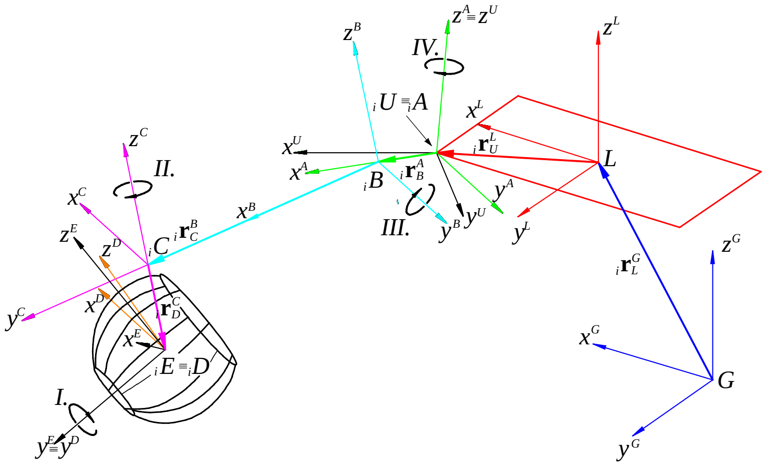

Fig. 1: Degrees of Freedom

3. Coordinate systems

To describe possible motions of all the chassis parts, there were used twenty six different coordinate systems - five of them were used for each axle, four of them represent the position of the joints between the axles and the chassis, one represents the vehicle chassis and the last one belongs to the global reference frame.

In the Fig. 1, there is a demonstration of a coordinate systems group which belongs to the i - th axle, the vehicle chassis and the global reference frame. There is drawn each degree of freedom and there are also position vectors determining the locations of the local coordinate systems origins.

To describe the relations between all the chassis parts, extended transformation 4 x 4 matrices were used. For instance, transformation matrix transforming vectors described in the i - th coordinate system ${}_i\left(D, x^D, y^D, z^D\right)$ into global coordinate system $\left(G, x^G, y^G, z^G\right)$ is

$${}_i\overline{\mathbf{T}}^{GD} = \overline{\mathbf{T}}^{GL}\,{}_i\overline{\mathbf{T}}^{LU}\,{}_i\overline{\mathbf{T}}^{UA}\,{}_i\overline{\mathbf{T}}^{AB}\,{}_i\overline{\mathbf{T}}^{BC}\,{}_i\overline{\mathbf{T}}^{CD}\text{, or }{}_i\overline{\mathbf{T}}^{GD} = {}_i\overline{\mathbf{T}}^{GC}\,{}_i\overline{\mathbf{T}}^{CD}\,. \qquad (1), (2)$$

4. Location

First, we put together a set of transformation equations determining the location of the wheel centre, that means the location of the point ${}_iD$ in the $\left(G, x^G, y^G, z^G\right)$ using the

$\left(L, x^L, y^L, z^L\right)$, ${}_i\left(U, x^U, y^U, z^U\right)$, ${}_i\left(A, x^A, y^A, z^A\right)$, ${}_i\left(B, x^B, y^B, z^B\right)$, ${}_i\left(C, x^C, y^C, z^C\right)$, ${}_i\left(D, x^D, y^D, z^D\right)$ coordinate systems

$$ {}_i\overline{\mathbf{r}}_D^G = {}_i\overline{\mathbf{T}}^{GD}\, {}_i\overline{\mathbf{r}}_D^D, \quad i = 1, 2, 3, 4\,. \tag{3}$$

Secondly, we determine the position of the same ${}_iD$ point in $\left(G, x^G, y^G, z^G\right)$ using the supporting surface unit normal vector ${}_i\mathbf{n}^{0G}$ and wheel radius r. The normal vector is erected in the point ${}_iT$ (or more precisely ${}_i\mathbf{r}_T^G$), which is the contact point between the i - th wheel and the supporting surface, see Fig. 2.

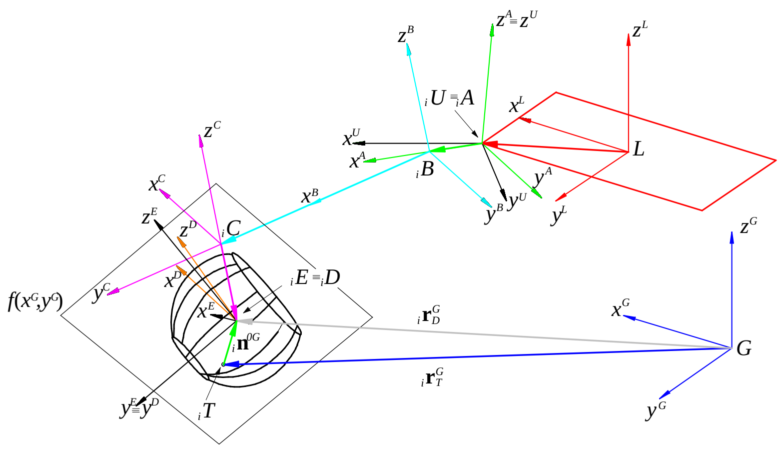

Fig. 2: Determination of the location centre of the wheel

$$ {}_i\overline{\mathbf{r}}_D^G = {}_i\overline{\mathbf{r}}_T^G + r\, {}_i\overline{\mathbf{n}}^{0G}, \; i = 1, 2, 3, 4\,. \tag{4}$$

If the supporting surface is described by a function $f\left(x^G, y^G\right)$, the unit normal vector ${}_i\mathbf{n}^{0G}$ is

$$ {}_i\mathbf{n}^{0G} = \frac{{}_i\mathbf{n}^G}{\left|{}_i\mathbf{n}^G\right|}, \quad {}_i\mathbf{n}^G = \begin{bmatrix} -\dfrac{\partial f\left(x^G, y^G\right)}{\partial x^G} \\ -\dfrac{\partial f\left(x^G, y^G\right)}{\partial y^G} \\ 1 \end{bmatrix}, \quad i = 1, 2, 3, 4\,. \tag{5}$$

The equations (3) and (4) express position of the same point and therefore we can proceed to the equality of the right - hand sides. This way we get a set of 12 constraint equations for all the wheels

$$ {}_i\overline{\mathbf{T}}^{GD}\, {}_i\overline{\mathbf{r}}_D^D = {}_i\overline{\mathbf{r}}_T^G + r\, {}_i\overline{\mathbf{n}}^{0G}, \quad i = 1, 2, 3, 4\,. \tag{6}$$

5. Velocity

In the set of equations (6), there are 30 unknowns. Let's assume 12 of them being controlled by a vehicle manual directional control. Additionally, the two spherical angles of the $\left(L, x^L, y^L, z^L\right)$ frame are kept at zero values and the chassis clearance height is kept at given value by a chassis horizontal position automatic control system. This reduces the number of unknowns to 15 in the equations set (6), so there are 3 missing equations. These equations are provided by the rolling conditions.

The rolling condition are written for the i - th wheel point of contact with the supporting surface $f\left(x^G, y^G\right)$. In the case of a perfect rolling of a ball - shaped object, the velocity of the contact point equals zero.

Consider the point ${}_iT$ with global extended position vector ${}_i\overline{\mathbf{r}}_T^G$ to be a particle of the wheel with local coordinate system ${}_i\left(E, x^E, y^E, z^E\right)$. Its local extended position vector is

$$ {}_i\overline{\mathbf{r}}_T^E = {}_i\overline{\mathbf{T}}^{EG}\, {}_i\overline{\mathbf{r}}_T^G, \;\; i = 1, 2, 3, 4 , \tag{7}$$

and therefore its velocity expressed in global coordinates is

$$ {}_i\overline{\mathbf{v}}_T^G = {}_i\dot{\overline{\mathbf{T}}}^{GE}\, {}_i\overline{\mathbf{T}}^{EG}\, {}_i\overline{\mathbf{r}}_T^G, \;\; i = 1, 2, 3, 4 \;\; . \tag{8}$$

Because the component of ${}_i\overline{\mathbf{v}}_T^G$ in ${}_i\mathbf{n}^{0G}$ direction is identically equal to zero, the rolling conditions

$$ {}_i\overline{\mathbf{v}}_T^G = \overline{\mathbf{0}}, \;\; i = 1, 2, 3, 4 \tag{9}$$

give additional 4 × 2 = 8 equations. But in the system (6) there are only 3 equations missing. So it is impossible in the kinematical model to fulfill concurrently rolling conditions at all the wheels.

For this reason, we are using the rolling conditions applied to two wheels only. For the first wheel we expect a perfect rolling, which gives two additional constraint equations. The second wheel can satisfy the rolling condition in one direction only, in the other direction (the one of the wheel axis rotation) there is allowed to slip. This gives the last constraint equation.

The third and the fourth wheel are considered not to completely fulfill the rolling conditions. In such a case they don't influence the vehicle instantaneous position and corresponding equations

$$ {}_i\overline{\mathbf{T}}^{GD}\, {}_i\overline{\mathbf{r}}_D^D = {}_i\overline{\mathbf{r}}_T^G + r_i\, {}_i\overline{\mathbf{n}}^{0G}, \; i = 3, 4 \tag{10}$$

can be split from the system (6) and solved separately after the vehicle position has been determined.

Finally, the vehicle position and velocity result from the equations system made of 3 of 4 scalar equations

$$ {}_i\dot{\overline{\mathbf{T}}}^{GE}\, {}_i\overline{\mathbf{T}}^{EG}\, {}_i\overline{\mathbf{r}}_T^G = \overline{\mathbf{0}}, \;\; i = 1, 2 . \tag{11}$$

and 6 scalar equations

$$ {}_i\bar{\mathbf{T}}^{GD}\,{}_i\bar{\mathbf{r}}_D^D = {}_i\bar{\mathbf{r}}_T^G + r_i\bar{\mathbf{n}}^{0G},\ i = 1, 2. \tag{12} $$

Because of the terms ${}_i\dot{\bar{\mathbf{T}}}^{GE}$, the rolling conditions (11) are first order differential equations and complete system of constraint equations is algebraic-differential. In order to enable solution using a standard integrating method, the equations (12) must be replaced by these equations differentiated by time as follows

$$ {}_i\dot{\bar{\mathbf{T}}}^{GD}\,{}_i\bar{\mathbf{r}}_D^D = {}_i\dot{\bar{\mathbf{r}}}_T^G + r_i\dot{\bar{\mathbf{n}}}^{0G},\ i = 1, 2. \tag{13} $$

6. Solution

Under these conditions it is necessary to solve a set of nine first order nonlinear differential equations. This set of differential equations has been solved using the numerical method of Runge - Kutta.

7. Results and discussion

Using the mathematical model, there were modeled numerous animations of vehicle basic maneuvers on a flat or an undulated surface. These animations, illustrating the vehicle ability to manage every complicated terrain, can be seen on the internet page http://www.kmp.vslib.cz/lide/index.php?clovek=denk&lang=cs.

8. Conclusion

The created mathematical model of the vehicle chassis kinematics will be extended with an acceleration part in the future. This will enable us to compute all the forces and torgues in the system that will be used to design engines and to dimension each part of the chassis. Such a complete model will be also used to design a structure and parameters of the vehicle control system.

Acknowledgment

The research has been supported by project MSM 4674788501 "*Optimalizace vlastností strojů v interakci s pracovními procesy a člověkem*".

References

M., Denk (2007) *Mathematical Model of Robotized Chassis Kinematics with Sixteen Degrees of Freedom*, Diploma Thesis Work, TUL Liberec