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APPLICATION OF EXPLICIT FORMULATION OF DAMPING FORCE OF POLYURETHANE FOAM IN CASE OF HARMONIC KINEMATIC EXCITATION

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Summary: *Article deals with approximation of damping force of PU foam specimen being compressed. Two different methods of determination of approximating function parameters are used and outcomes compared. First method is based on keeping position of damping force extreme, and the other on keeping value of damping force extreme.*

1. Introduction

In technical handbooks as Juliš & Brepta (1987) or Brebta & Půst & Turek (1994) it is possible to find out that damping force of elastic elements, at which damping depends on displacement *x* and is realized by energy dissipation inside the material, can be defined by equation (1). This formulation appears to be applicable in case of description of damping force of polyurethane foam being compressed.

$$F_{\mathrm{d}}(x,\dot{x}) = b_{\alpha}\left|x^{\alpha}\right|\dot{x}\,. \tag{1}$$

Because the specimen was loaded by only positively oriented deformations (with respect to suitable coordinate system) it is possible to leave out the absolute value in equation (1) and rewrite it in form (2). The task then lies in finding the coefficient b_{α} and exponent α.

$$F_{\mathrm{d}}(x,\dot{x}) = b_{\alpha}x^{\alpha}\dot{x}\,. \tag{2}$$

2. Experimental part

Experimental data of damping force was acquired during uniaxial compressing of a specimen of PU foam of cuboidal shape with base dimensions (100 × 100) mm and hight 50 mm by harmonical course of displacement *x*. Measurements were carried out for many combinations of mean value $A_0 \in \{20, 25, 30\}$ mm, amplitude $A \in \{1,3,5,7,\mathrm{resp.}10\}$ mm and frequency $f \in \{0.1, 0.5, 0.7, 1, 1.5, 2, 3, 4, 5, 6, 8, 10\}$ Hz of harmonical exciting signal.

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3. Method

In case of dynamical compression of the PU foam specimen by harmonical displacement $x(t)$ given by eqn. (8) for one period we get the hysteresis course of the measured total force F as shown in Fig. 1. Let us assume that the damping force F_d is distributed around the skeleton curve of hysteresis loop symmetrically. Let us also assume that this skeleton curve represents just restoring force F_R. In Fig. 1 at the bottom there is course of dampig force in dependence on displacement x. This dependency shows a typical pear-like character with significant extrem value F_{de} at position x_e.

Work of damping force (dissipated energy) is given as an curve integral of damping force F_d with respect to x:

$$W_d = \oint F_d \, dx \, . \tag{3}$$

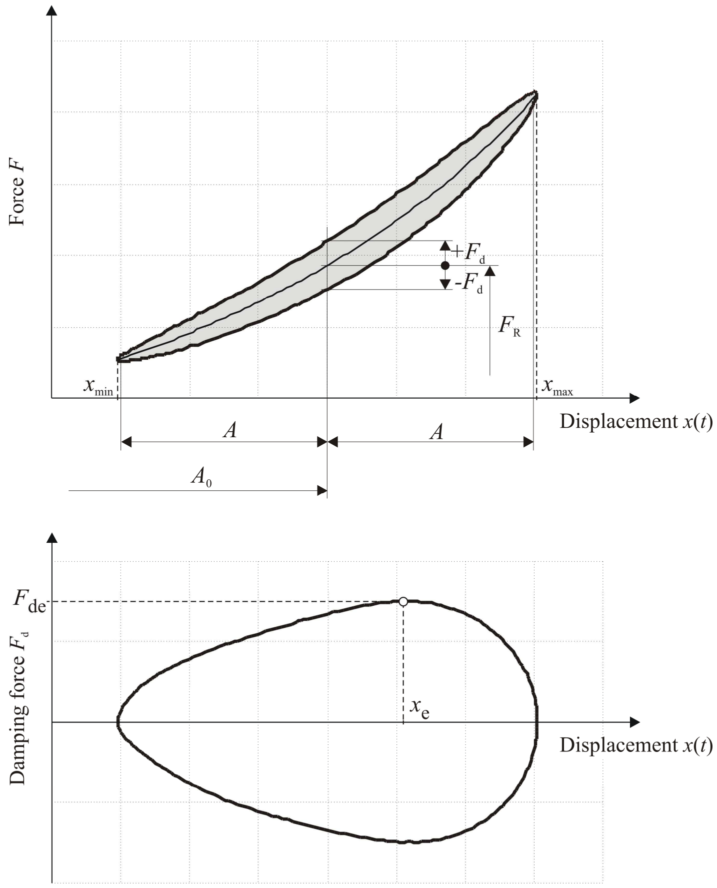


Fig. 1: Measured force responce to kinematic excitation by harmonical course of displacement and its decomposition

In Fig. 2 there is dependency of W_d on parameters of harmonical exciting function of displacement x and its approximating function $\hat{W}_d$ reached by least square method. For a good approximation it is enough to consider its linear dependency on frequency f. The same is considerable in case of dependency of value of damping force extreme F_{de} and its approximation $\hat{F}_{de}$ which is expressed by eqn. (5) and plotted in Fig. 3. Last needed function is dependency of position of damping force extreme on parameters of exciting function. Experience shows that we can assume the extreme position x_e independent on frequency f for given mean value and amplitude of harmonical excitation. This measured dependency and its approximation $\hat{x}_e$ are plotted in Fig. 4.

$$\begin{aligned} \hat{W}_d(A_0, A, f) = & -63.55292557\, A + 6.304231862\, A A_0 - 0.1365379833\, A A_0^2 + \\ & + 59.05597091 A^2 - 5.120196969 A^2 A_0 + 0.1176524597\, A^2 A_0^2 + \\ & -16.48336754\, f\, A + 1.428604046 F\, A A_0 - 0.3098332460 \cdot 10^{-1}\, f\, A A_0^2 + \\ & + 7.224179462\, f\, A^2 - 0.6317837830\, f\, A^2 A_0 + 0.1436008971 \cdot 10^{-1}\, f\, A^2 A_0^2 \end{aligned} \tag{4}$$

$$\begin{aligned}\hat{F}_{\mathrm{de}}(A_0,A,f) = &-2.110178391A + 0.2897513956AA_0 - 0.4064362851\cdot10^{-2}AA_0^2 + \\ &+5.060438039A^2 - 0.4586896672A^2A_0 + 0.1015867499\cdot10^{-1}A^2A_0^2 + \\ &-0.4526828028fA + 0.3357626873\cdot10^{-1}fAA_0 - 0.5011891803\cdot10^{-3}fAA_0^2 + \\ &+0.3554509419fA^2 - 0.3115516380\cdot10^{-1}fA^2A_0 + 0.7011392733\cdot10^{-3}fA^2A_0^2.\end{aligned} \tag{5}$$

$$\begin{aligned}\hat{x}_{\mathrm{e}}(A_0,A) = &\ 0.4533461420 - 0.5219584498\,A + 0.0256748155\,A^2 + \\ &+0.9763572537\,A_0 + 0.0308546553\,A_0\,A + 2.3507212384\cdot10^{-4}A_0\,A^2\end{aligned} \tag{6}$$

3.1 Determination of α exponent

3.1.1 Calculation of α from position of damping force extreme

For finding the exponent of function (2) extreme we put its derivative with respect to x equal to zero. If we have unequivocaly assigned x and $\dot{x}$ – e.g. by choosing of certain time course – it is possible to assume the eqn. (2) as a function of one variable. Position of extreme we denote as x_e.

$$\frac{\mathrm{d}F_{\mathrm{d}}}{\mathrm{d}x} = b_\alpha\left[\alpha x_{\mathrm{e}}^{(\alpha-1)} + x_{\mathrm{e}}^{\alpha}\frac{\mathrm{d}\dot{x}}{\mathrm{d}x}\right] = 0,$$

where

$$\frac{d\dot{x}}{dx} = \frac{\ddot{x}}{\dot{x}}.$$

Thus we get the equation

$$\frac{\alpha}{x_{\mathrm{e}}} + \frac{\ddot{x}}{\dot{x}^2} = 0. \tag{7}$$

Specially for harmonical function of displacement x we get system of equations:

$$x = A_0 + A\sin(\omega t), \tag{8}$$

$$\dot{x} = A\omega\cos(\omega t), \tag{9}$$

$$\ddot{x} = -A\omega^2\sin(\omega t). \tag{10}$$

From this we can now express the velocity $\dot{x}$ and the acceleration $\ddot{x}$ as functions (11) and (12) depending only on displacement x.

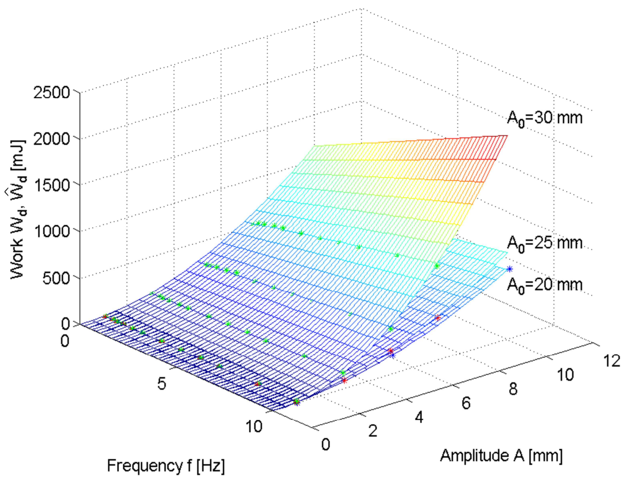


Fig. 2: Dependence of work of damping force W_d on frequency f, amplitude A and mean value A_0 of harmonic excitation signal (W_d – measured values (*), $\hat{W}_d$ - approximation (surface))

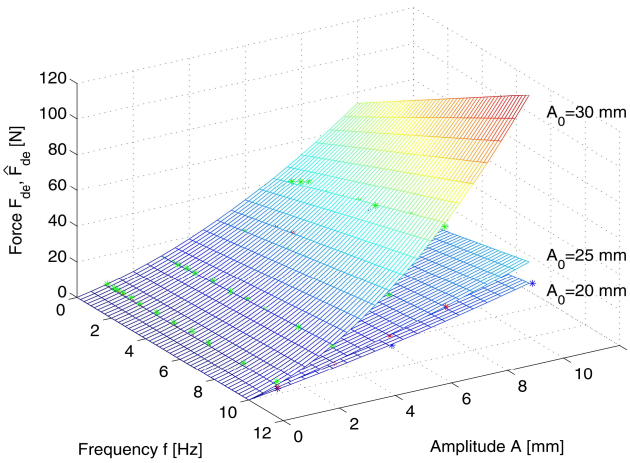


Fig. 3: Dependence of value of damping force extreme F_{de} on frequency f, amplitude A and mean value A_0 of harmonic excitation signal (F_{de} – measured values (*), $\hat{F}_{de}$ - approximation (surface))

$$\dot{x}(x) = \pm\omega\sqrt{A^2 - (x - A_0)^2}\,, \tag{11}$$

$$\ddot{x}(x) = -\omega^2 (x - A_0)\,. \tag{12}$$

We substitute these results into eqn. (7) where $x = x_e$ and we get

$$\frac{\alpha}{x_e} + \frac{\ddot{x}(x_e)}{\dot{x}^2(x_e)} =$$

$$= \frac{\alpha}{x_e} + \frac{-\omega^2 (x_e - A_0)}{\omega^2 \left[A^2 - (x_e - A_0)^2\right]} = 0$$

After abbreviation of square of angular velocity ω we obtain an implicit function (13) which is possible to use for calculation of damping force extreme position. In this equation the angular velocity ω, respectively exciting frequency f, is not present.

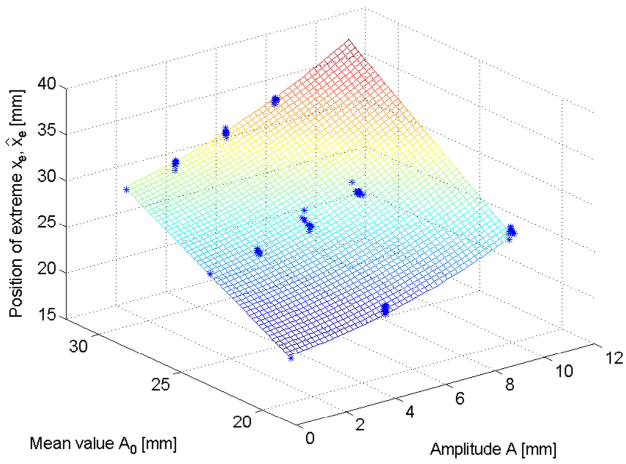


Fig. 4: Dependence of damping force extreme position on amplitude and mean value of harmonic excitation signal (x_e – measured values (*), $\hat{x}_e$ - approximation (surface))

$$\frac{\alpha}{x_e} - \frac{(x_e - A_0)}{A^2 - (x_e - A_0)^2} = 0\,. \tag{13}$$

By this it has been proved that damping force formulation (2) does not appear the dependency of extreme position on exciting frequency, what is in accordance with observations.

Because the position of extreme x_e is defined by aproximating function (5) as $\hat{x}_e$ it is possible to use the eqn. (13) for exponent α calculation in form (14).

$$\alpha = \frac{(\hat{x}_e - A_0)}{A^2 - (\hat{x}_e - A_0)^2}\,\hat{x}_e\,. \tag{14}$$

As experience shows this way of exponent α determination has a tendency to overestimate its values for positions of extreme more displaced from mean values. Because of this there is also a reduced exponent α_{red} calculated by multiplication of α by constant k, where k was set to 0.75 in entire range of calculations.

$$\alpha_{red} = \alpha \cdot k\,. \tag{15}$$

3.1.2 Calculation of α from value of damping force extreme

Another way of α exponent determination is an numerical iterational method where equation (16) is solved until the difference between its two terms is less then set treshold. In this way calculated exponent is further called α_F.

$$\hat{F}_{de} - \max\left(b_\alpha x_e^\alpha \dot{x}\right) = 0\,. \tag{16}$$

3.2 Determination of b_α coefficient

As has already been written above the work of damping force W_d for one loading period is given by line integral of damping force F_d with respect to displacement x. In eqn. (3) we substitute eqn. (2) for F_d and we get:

$$W_d = \oint F_d \mathrm{d}x = b_\alpha \oint x^\alpha \dot{x} \mathrm{d}x = b_\alpha \int_0^{2\pi/\omega} x^\alpha(t)\, \dot{x}^2(t) \mathrm{d}t \,. \tag{17}$$

Using time courses (8) and (9) where differential $\mathrm{d}x = A\omega\cos(\omega t)\,\mathrm{d}t$, and substituting $\omega t = \varphi$ we transform this integration to angular displacement domain φ. Under consideration of constant angular velocity ω during one loading period we can write:

$$W_d = b_\alpha \omega \int_0^{2\pi} [A_0 + A\sin(\varphi)]^\alpha [A\cos(\varphi)]^2 \mathrm{d}\varphi \,.$$

Using expression $\omega = 2\pi f$ we rewrite this equation into form

$$W_d = b_\alpha 2\pi\, f\, I_\alpha \,, \tag{18}$$

where

$$I_\alpha = I_\alpha(A_0, A, \alpha) = \int_0^{2\pi} [A_0 + A\sin(\varphi)]^\alpha [A\cos(\varphi)]^2 \mathrm{d}\varphi \tag{19}$$

is integral independent on frequency of harmonical exciting signal. Otherwise the work of damping force W_d depends on frequency linearly which is in accordance with experimental observation. Value of integral (19) is for given mean value, amplitude and α exponent computed numerically. From eqn. (18) we express the coefficient b_α where we substitute work W_d for our purpose by approximation $\hat{W}_d$:

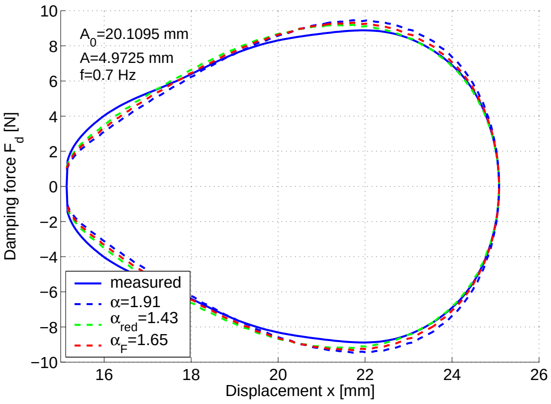
$$b_\alpha = \frac{\hat{W}_d}{2\pi f I_\alpha} \,. \tag{20}$$

4. Achieved results

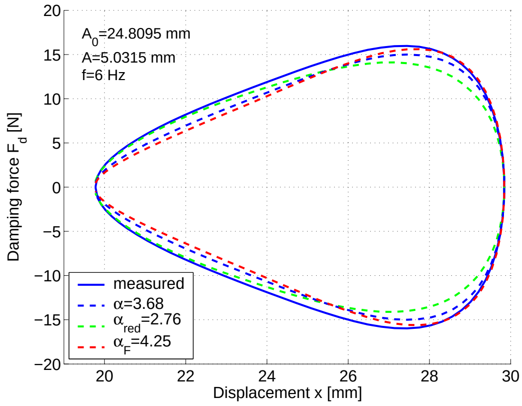
Eqn. (2) has been calculated for many combinations of mean values, amplitudes and frequencies of exciting harmonical signal. There were used and compared three alternatives of exponent denoted α, α_{red} and α_F. For selected combinations of exciting signal parameters they are pictured in Fig. 5 (a), (b), (c). Simulated damping force was calculated using real (measured) kinematical quantities x and $\dot{x}$, not under consideration of their theoretical harmonical course. From this reason there is noted in each graph real mean value and amplitude for which the data were acquired and which are different from desired theoretical values. This difference is given by control deviation of experimental device. Little deviations of mean values have been neglected for the purpose of picturing of measured data denoted (*) in Fig. 2, and Fig. 3.

For quantification of simulation error there has been defined quantity R_2 given by eqn. (21), where S_e is summation of deviations squared and S is summation of measured values squared. F_{dmi} here is denoted measured value of damping force and F_{dsi} is corresponding sample of simulated value. The quantity R_2 has been evaluated in interval $x \in \langle x_{\min}; x_{\max} \rangle$; n is number of measured samples in this interval. Values of R_2 are in Fig. 6, 7, 8 for entire range of exciting frequency from 0.1 Hz to 10 Hz.

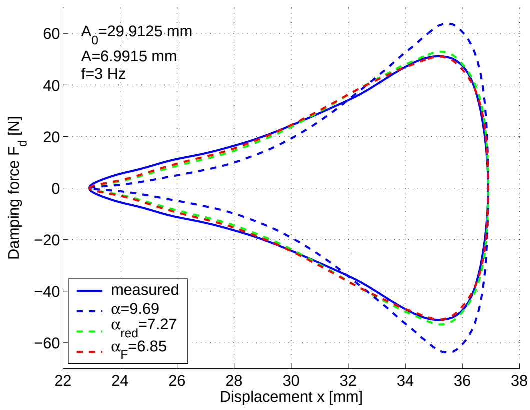
$$R_2 = \sqrt{\frac{S_e}{S}}, \qquad S_e = \sum_{i=1}^{n} \left(F_{dmi} - F_{dsi}\right)^2, \qquad S = \sum_{i=1}^{n} F_{dmi}^2 \tag{21, 22, 23}$$



(a) A_0=20 mm, A=5 mm



(b) A_0=25 mm, A=5 mm



(c) A_0=30 mm, A=7 mm

Fig. 5 Comparison between damping force F_d measured and calculated for different alternatives of α exponent determination and for selected combinations of parameters of exciting harmonical signal (in captions there are noted desired theoretical parameters A_0 and A and inside the picture there are presented real measured ones)

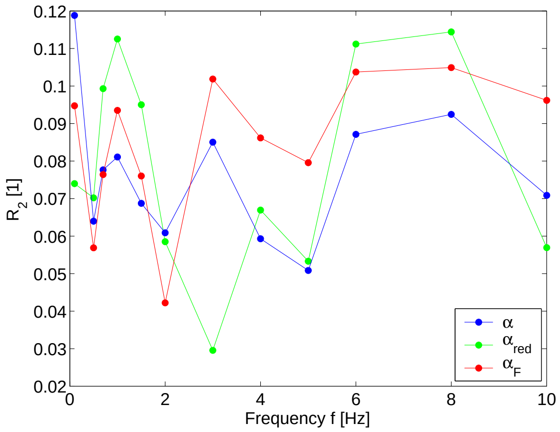


Fig. 6 Error of simulation; A_0=20 mm, A=5mm, $f \in \langle 0.1 \div 10 \rangle$ Hz

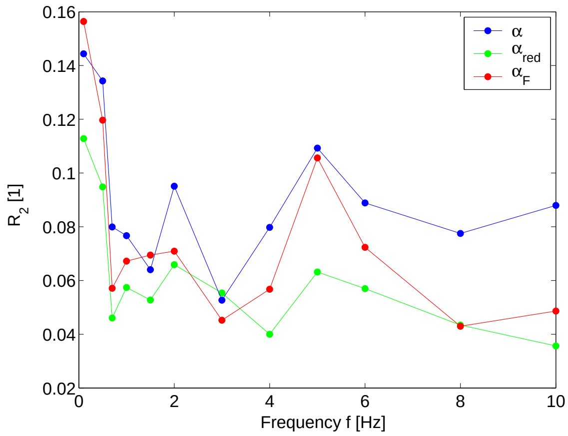


Fig. 7 Error of simulation; A_0=25 mm, A=5mm, $f \in \langle 0.1 \div 10 \rangle$ Hz

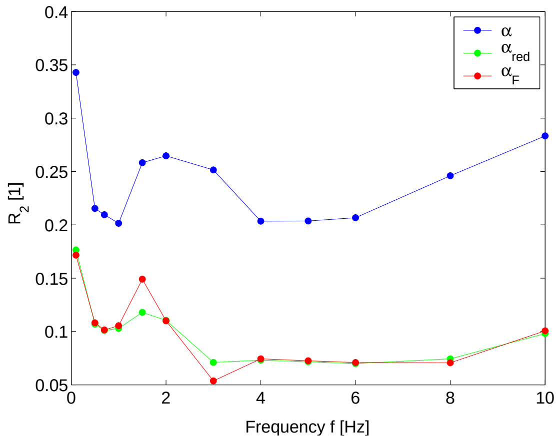


Fig. 8 Error of simulation; A_0=30 mm, A=7mm, $f \in \langle 0.1 \div 10 \rangle$ Hz

5. Conclusion

Eqn. (2) has been used for simulation of damping force of specimen of PU foam being compressed. Three alternatives of coefficient α determination are presented and outcomes compared. Numerical simulations show that for smaller α there are not significant differencies between them. For greater α the first way of its determination has a tendency to give the overestimate values which is well seen in Fig. 5 (c) and Fig 8. Simulation with reduced exponent α_{red} and α_F gives qualitatively similar outcomes.

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