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DETERMINATION OF AIRCRAFT STRUCTURE WHIRL FLUTTER STABILITY BOUNDARIES BY MEANS OF OPTIMIZATION BASED SOLUTION

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Summary: The submitted paper deals with the aircraft structure whirl flutter analysis. It gives a summary of the regulations requirements, the theoretical background and the aircraft certification relating issues. The main part deals with determination of critical engine suspension stiffness parameters considering the structure whirl flutter stability at the certification speed. The optimization-based analytical procedure is demonstrated on the two examples. Finally, the future exploitation of the method is outlined.

1. Introduction

Aircraft structures are required to have a reliability certificate including the flutter stability. The turboprop aircraft are required to be certified also considering the whirl flutter. Rotating parts like a propeller or a turbine increase the number of degrees of freedom and cause additional forces and moments. Moreover rotating propeller causes a complicated flow field and interference effects between wing, nacelle and propeller. Whirl flutter may cause the propeller mounting unstable vibrations, even a failure of the engine, nacelle or whole wing. It has been the cause of a number of accidents (two Lockheed Electra II airliners in 1959 and 1960 and a Beech 1900C commuter in 1991).

At the VZLU, the first tasks regarding the whirl flutter were dealt with in connection with the certification of the Ae-270 small single-engine turboprop aircraft for 8 passengers. Calculations were performed by means of the NASTRAN program system supported by the other specific software, as described in Čečrdle, 2001, 2002. Calculations were performed using the model of the cantilevered engine bed with flexibly suspended engine – propeller system. The analysis included just engine vertical and lateral vibration modes, it didn't take under consideration the dynamic characteristics of the residual structure. For the nose-mounted single-engine aircraft configuration, such simplification was acceptable. However the analysis of the classical twin wing-mounted turboprop structure demands to include the rest structure, particularly wing dynamic characteristics and specific aerodynamic propeller – nacelle – wing interference effects (down-wash). Therefore the analysis procedure was enlarged and adapted also for such configuration. It was demonstrated on the example of commuter aircraft for 40 passengers dynamic model wing – engine component (see Čečrdle, 2006, 2007).

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2. Motivation

Development of a new aircraft can be divided into following phases: conceptual design, virtual prototype, manufacturing of physical prototype and physical prototype testing.

During the conceptual design phase, the main geometric, aerodynamic, inertia or performance characteristics are estimated. Also the engine – propeller system is chosen, therefore the inertia characteristics of these parts should be known. During the virtual prototype phase, the maximum feasible amount of analyses should be performed, since the virtual structure can be easily refined considering various analyses results. During the next two phases, analyses are kept on. Prototypes are tested; analytical models are improved and updated according the test results etc. Structure can be also adjusted considering tests or refined analyses results, however, it is obvious, that there are much more limitations or boundary conditions contrary to the previous phase.

Aeroelastic analyses during the first and second phase have mostly overall character to determine possibly critical areas. Number of parameters isn't set with the sufficient level of probability. The final aeroelastic analyses are usually performed after the ground vibration tests of the prototype, when it is possible to update and verify computational models with a satisfactory level of assurance. Obviously, in this phase, analyses have more or less a checking character. Also relatively short period among ground vibration tests and flight flutter tests should be taken under consideration. Therefore, there are efforts to move aeroelastic analyses to the earlier phases. It would allow the structure corrections to prevent possible aeroelastic problems and also timesaving in the later development phases is important. After the ground vibration tests, just checks of the selected critical areas would be performed. Precondition of successful aeroelastic analyses at the virtual prototype phase is usage of simple and fast tools, good estimation of parameters, automated parametric studies etc. Also the determination of parameter critical values is the acceptable approach.

The most critical parameters influencing the whirl flutter stability are natural frequencies of the flexibly attached engine – propeller system vibrations (vertical and lateral). Airworthiness regulations directly require inclusion of the changes in the stiffness and damping of the propeller – engine – nacelle – structure system (\$23.629(e)(1)(2)). The reliable values of mentioned frequencies aren't at disposal until the ground vibration tests. Assuming the inertia characteristics of the engine – propeller system as reliably determined, the critical parameters are stiffness of the engine system attachment (engine bed, engine mount isolators).

Determination of the critical values of mentioned stiffness parameters, it means the values when the whirl flutter speed is equal to the certification speed would allow replacing large parametric studies varying the stiffness. It would considerably decrease number of necessary analyses, particularly for the twin wing-mounted engine aircraft, when the number of secondary parameters, like wing inertia and stiffness must be also included. Obviously, it would allow moving the whirl flutter analysis to the early phase of the aircraft development. After the ground vibration tests, just rate of a reserve towards the critical values would be evaluated.

The main aim of the presented paper is a development of such analytical procedure. Parameters are vertical and lateral vibration frequencies of a flexibly attached aircraft engine – propeller system. The optimization – based solution is employed.

3. Theoretical Background



Fig.1: Gyroscopic system with propeller

Engine flexible mounting is represented by two rotational springs (stiffness K_{Ψ} , K_{Θ}) as illustrated in fig.1. Propeller is considered as rigid, rotating with angular velocity Ω . System is in the airflow of velocity V_{∞} .

Neglecting propeller rotation and the aerodynamic forces, the two independent mode shapes (yaw – around vertical axis, pitch – around lateral axis) will emerge with angular frequencies ω_{Ψ} and ω_{Θ} . Considering the propeller rotation, the primary system motion changes to the characteristic gyroscopic motion. The gyroscopic effect makes two independent mode shapes merge

to rotational motion. The propeller axis makes an elliptical movement. The orientation of the propeller axis movement is

backward relatively to the propeller rotation for the mode with lower frequency (backward whirl mode) and relatively forward to the propeller rotation for the mode frequency with higher (forward whirl mode). The mode shapes of gyroscopic modes are complex, since independent yaw and pitch modes have a phase shift 90°.



Fig.2: Stable and unstable state of gyroscopic vibrations for backward flutter mode

Condition of real mode shapes corresponds to the state of the undamped system.



Fig.3: Kinematical scheme of the gyroscopic system

The described gyroscopic mode shapes make harmonic changes of propeller blades angles of attack. They give rise non-stationary to aerodynamic forces. which may under the specific conditions induce a flutter. Possible states of gyroscopic the system from the flutter stability point of view for backward mode are explained in fig.2. Provided that the air velocity is lower than

critical value ($V_{\infty} < V_{FL}$), the system is stable and the motion is damped. If the airspeed exceeds the critical value ($V_{\infty} > V_{FL}$), the system becomes unstable and motion is diverging. The limit state ($V_{\infty} = V_{FL}$) with no total damping is called critical flutter state and V_{FL} is called critical flutter speed.

The basic problem of the analytical solution grounds on determination of the aerodynamic forces caused by the gyroscopic motion for the specific propeller blades. The equations of motion were set up for system described in fig.1. The kinematical scheme including gyroscopic effects is shown in fig.3. The independent generalized coordinates are three angles (ϕ, Θ, Ψ) . We assume the propeller angular velocity constant ($\phi = \Omega t$), mass distribution symmetric around X-axis and mass moments of inertia $J_Z \neq J_Y$.

Considering the small angles simplification, the equations of motion become:

$$J_{Y}\ddot{\Theta} + \frac{K_{\Theta}\gamma_{\Theta}}{\omega}\dot{\Theta} + J_{X}\Omega\dot{\Psi} + K_{\Theta}\Theta = M_{Y,P} - a.P_{Z}$$

$$J_{Y}\ddot{\Psi} + \frac{K_{\Psi}\gamma_{\Psi}}{\omega}\dot{\Psi} - J_{X}\Omega\dot{\Theta} + K_{\Psi}\Psi = M_{Z,P} + a.P_{y}$$
(1)

We formulate the propeller aerodynamic forces by means of the aerodynamic derivatives (Ribner, 1945) and make the simplification for the harmonic motion, then the final whirl flutter matrix equation will become:

$$\left(-\omega^{2}[M]+j\omega\left([D]+[G]+q_{\omega}F_{P}\frac{D_{P}^{2}}{V_{\omega}}[D^{A}]\right)+\left([K]+q_{\omega}F_{P}D_{P}[K^{A}]\right)\right)\times\left[\frac{\overline{\Theta}}{\overline{\Psi}}\right]=\left\{0\right\}$$
(2)

The limit state emerges for the specific combination of parameters V_{∞} and Ω , when the angular velocity ω is real. The whirl flutter characteristics are explained in fig.4, which



Fig.4: Influence of the propeller relative velocity $(V_{\infty}/(\Omega R))$ to the stability of undamped gyroscopic system

describes influence of the propeller relative velocity (V_{∞} / (ΩR)) to the stability of undamped gyroscopic system. Increasing the propeller relative velocity makes increasing of the necessary stiffnesses K_{Θ} , K_{Ψ} . Also influences of the structural damping (stabilizing) and a distance propeller – mode shape node (destabilizing) are significant.

The whirl flutter appears at the gyroscopic rotational vibrations, the flutter frequency is the same as the frequency of the backward gyroscopic mode. The critical state may be reached either due to increasing the air velocity or the propeller revolutions. Structural damping is a significant stabilization factor. On the contrary, the propeller pull force influence is barely noticeable. The most critical state is $K_{\Theta} = K_{\Psi}$, it means $\omega_{\Theta} = \omega_{\Psi}$ when the interaction of both independent motions is maximal. A special case of the eq.(2) for $\omega=0$ is the gyroscopic static divergence.

4. Ordinary Analysis Procedure

The whirl flutter solution by means of the NASTRAN program system grounds on the Strip Aerodynamic Theory for the propeller at the windmilling mode. A propeller is assumed rigid. For the rest structure is used a Wing – Body Interference Aerodynamic Theory (Giesing, Kalman, Rodden, 1972). For a flutter stability solution there is used a PK method. The NASTRAN whirl flutter DMAP (Direct Matrix Abstraction Program) procedure is supplemented by the external preprocessor (program propf.for) for calculation of the propeller aerodynamic matrices (formally damping and stiffness matrices) and optionally for calculation of the down / side wash effects.

The FE model can be prepared similarly as for the ordinary flutter analysis; the model must include the grid at the propeller center of gravity with propeller mass characteristics. The aerodynamic model must be prepared for a Wing – Body Interference Theory. Data for calculation of downwash and sidewash angles may be specified by means of the partitioning matrices. The first NASTRAN run calculates the down / side wash angles only. These data and other data concerning the engine and the propeller are inputs to the external preprocessor (program propf.for) which calculates the propeller aerodynamic matrices and possibly down / side wash effects. These data are added to the NASTRAN input, formally as a direct input to the stiffness and damping matrices. Partitioning matrices must be removed. The second NASTRAN run is the final one and makes a flutter stability calculation.

The propeller aerodynamic forces and moments are calculated by eq.(3):

$$P_{Y} = q_{\infty}F_{p}\left(c_{y\Psi}\Psi^{*} + c_{y\Theta}\Theta^{*} + c_{yq}\frac{\Theta^{*}R}{V_{\infty}}\right)$$

$$P_{Z} = q_{\infty}F_{p}\left(c_{z\Theta}\Theta^{*} + c_{z\Psi}\Psi^{*} + c_{zr}\frac{\dot{\Psi}^{*}R}{V_{\infty}}\right)$$

$$M_{Y,P} = q_{\infty}F_{P}D_{P}\left(c_{m\Psi}\Psi^{*} + c_{mq}\frac{\dot{\Theta}^{*}R}{V_{\infty}}\right)$$

$$M_{Z,P} = q_{\infty}F_{P}D_{P}\left(c_{n\Theta}\Theta^{*} + c_{nr}\frac{\dot{\Psi}^{*}R}{V_{\infty}}\right)$$
(3)

Aerodynamic derivatives are given from propeller blade integrals (Rodden, Rose, 1989), effective angles are shown in the fig.5.

An option to include the downwash and sidewash effects may be important for configuration with engines mounted to the wing. Downwash and sidewash angles behind the propeller describe interference between propeller and nacelle. Induced downwash and sidewash angles are added to the effective static angles (fig.5) by the eq.(4):



Fig.5: Effective static angles

$$\Theta^* = \Theta + \frac{\dot{z}}{V_{\infty}} - \frac{W_1}{V_{\infty}}$$

$$\Psi^* = \Psi - \frac{\dot{y}}{V_{\infty}} + \frac{W_2}{V_{\infty}}$$
(4)

Above mentioned induced down / side wash angles dependent on the reduced frequency can be obtained from the lift solution by partitioning the interference coefficients. The downwash effect influences only the aerodynamic stiffness matrix; influence to the aerodynamic damping matrix is neglected. Only interference between propeller and nacelle is included, interference between propeller and a wing is neglected.

5. Optimization – Based Solution

5.1. General Introduction

The optimization is a process of the structure changes aimed to make an improvement considering determined conditions and parameters. Basically the optimization is a mathematical algorithm of seeking the optimal parameters combination to minimize the objective function respecting the determined limitations and conditions. Following text describes main terms used in the optimization.

Design Variable is a quantity, which is changeable. It is possible to specify both boundary values and change rate for an optimization cycle. It is not required to correspond to the FE model property; it may become a combination of several properties. Design variables can be interdependent.

Design Property is a FE model property relating to the design variable. In the case of shape optimization may become a grid position. Also boundary values may be specified. It may become either a linear combination of design variables $(p_j = C_0 + \sum_i C_i x_i)$ or a general,

also non-linear function $(p_i = f({x}, {C}))$, and also a set of discrete values is allowed.

Design Response may become either the objective function or a design constraint. It may be either linear or non-linear combination of design variables, other design responses, sets of discrete values and constants etc. Character of design responses determines the optimization algorithm (linear, non-linear). The **objective function** is a scalar quantity, which is minimized during optimization (maximization is mathematically realized as minimization of reciprocal value). The **design Constraint** is a quantity or function defined as condition, which must be held. It may be an inequality (for example $h_i(\bar{x}) \le 0$), an equality (for example $h_i(\bar{x}) = 0$), side constraints (for example $x_i^I \le x_i \le x_i^U$) etc.

Optimization procedures belong to the family of methods called "gradient-based", since they determine the gradients of the objective function and constraints to determine a direction of searching for the optimum in the design space. Then it proceeds in that direction as far as they can go. After that it investigates if we are at the optimum point, if not the process is repeated until can make no more improvement of the objective without violating some constrain. The optimization may cover various types of analyses. For the static aeroelasticity, apart from the ordinary design responses like displacement, strain, stress, force, etc. the two specific types are applicable (trim variables and stability derivatives). For the flutter solution, the design response is represented by the total damping of the structure for a selected mode, air density (ρ), Mach number and velocity (V). Obviously synthetic responses defined as a function of other design responses, design variables, constants etc. are allowed. However, usage of such advanced synthetic applications assumes a knowledge regarding the flutter behavior of the structure.

For the optimization purposes, only the PK method is applicable. The basic flutter equation in modal coordinates is:

$$\left[M_{hh} p^{2} + \left(B_{hh} - \frac{1}{4} \frac{\rho c V Q_{hh}^{Im}}{k} \right) p + \left(K_{hh} - \frac{1}{2} \rho V^{2} Q_{hh}^{Re} \right) \right] \left\{ u_{h} \right\} = 0$$
(5)

 M_{hh} , B_{hh} and K_{hh} are modal mass, damping and stiffness matrices respectively. Aerodynamic loads are incorporated into damping and stiffness matrices. Aerodynamic matrices are dependent on the reduced frequency (k) at a gentle rate. All matrices are real; Q_{hh}^{Re} and Q_{hh}^{Im} are real and imaginary part of a complex aerodynamic matrix Q_{hh} . The decay rate coefficient is defined in connection with the complex eigenvalue:

$$\mathbf{p} = \omega(\gamma \pm \mathbf{j}) = \mathbf{p}^{\mathrm{Re}} + \mathbf{j}\mathbf{p}^{\mathrm{Im}}$$
(6)

Flutter sensitivities are computed as rate of change of the transient decay coefficient γ with respect to changes in design variables $(\partial \gamma / \partial x_i)$.

5.2. Solution Strategy

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As mentioned in the previous chapters, the most important parameters influencing the whirl flutter are the natural frequencies of the engine vertical and lateral vibration modes; and also a ratio of both ones. Let's assume the inertia characteristics of the engine – propeller system and the residual structure reliably determined. That's why the engine suspension stiffness parameters would be used as optimization parameters. We formulate two design variables: the rotational stiffness of the engine attachment around the vertical and lateral axes. Design properties and the relations to the design variables are to be defined in accordance with the stiffness model type (two springs, system of springs, beams, shells, combined model, etc.).

Design responses are specified particularly as a design constrains. They represent the demand to hold the selected frequency ratio and the whirl flutter stability below the selected certification speed as well. A frequency ratio is specified by the lower and upper bounds, the recommended interval bounds are $\pm(1 - 2)$ %. Demand of the flutter stability is realized through the damping value for the specific mode and speed. Considering the possible problems of the numeric solution, the flutter state damping of g = 0 is moved to a non-zero value, as described later. In the case of more than two degrees of freedom included in the solution, there is necessary to ensure also no other flutter instability below the certification speed. That's why the latter constrain is extended also to the other mode shapes. The objective function is defined as minimization of the both frequencies sum.

During the problem solving, there were several different optimization approaches and options tested. The described solution has appeared as the best one, in particular, it is applicable also for the gyroscopic divergence.

Firstly, the target frequency ratio is set. One of the design variables is fixed at the nominal value; the other one is updated to reach the mentioned target ratio. For this purpose, usage of the optimization solution for normal modes is appropriate. The objective function is defined as:

$$min\left(\frac{frequency(2)}{frequency(1)} - target \ ratio\right) \tag{7}$$

This preparatory analysis provides the initial values of both design variables and the corresponding frequencies of both vertical and lateral engine vibration modes. The frequency ratio agrees with the target ratio as well. The next analysis is the main one. It is a composite solution of both normal modes and flutter optimization solution specified in the two separated subcases. The normal modes solution includes the design constrain of keeping the target ratio, defined as:

$$\frac{frequency(2)}{frequency(1)} = target \ ratio(\pm(1-2)\%)$$
(8)

Besides it includes the objective function, defined as:

$$min(frequency(1) + frequency(2))$$
(9)

The flutter solution includes the design constrain of the flutter stability (g<0) below the certification speed. It is applied for all modes included in the analysis to prevent other type of the flutter instability apart from the whirl flutter below the certification speed. It is defined as:

$$\frac{g\left(V = V_{certif.}\right) - 0.03}{0.1} \in \left\langle -\infty; -0.3 \right\rangle \tag{10}$$

The flutter solution is performed just for the certification speed; input file must satisfy the specific whirl flutter analysis demands and include all the specific data, in particular data from the external preprocessor as described in the chapter 4. The NASTRAN whirl flutter DMAP procedure must have been adapted for the NASTRAN optimization solver. The adapted procedure is applicable for the NASTRAN v2005.0. The results from this main optimization analysis are the final design variables, and corresponding design properties values. The whirl flutter speed is equal to the certification speed; the frequency ratio of the vertical and lateral engine vibration frequencies is equal to the target ratio as well. The described procedure is also applicable for a case of gyroscopic divergence, which may occur for the high target ratio values.

As a final phase, it is recommended to perform standard whirl flutter solution for standard number of velocities to check the flutter behavior of the updated structure. Described procedure is required to be repeated for the range of target frequency ratios, the order of the vertical and lateral modes (lower, higher) must be taken in account as well. It should be noted that in the most cases it is impossible to reach the values of frequency ratio extremely closed to the unity value due to the character of the stiffness model.

6. Application Examples

6.1. Aircraft Engine Component System

The first testing example represents a model of the flexibly attached isolated engine – propeller system of a small (10 seat) single nose-mounted turboprop aircraft (wingspan 13,8 m; length 12,2 m; max. take-of weight 3300 kg – see fig.6).



Fig. 7: Engine vibrations mode shapes (vertical - $f_0 = 10,43$ Hz; lateral - $f_0 = 14,84$ Hz)

moments of inertia. The flexibility of the mount isolators is modeled by means of two rotational springs placed at the node points of vertical and lateral engine vibrations mode shapes obtained from the ground vibration tests. The nominal stiffness of each spring was tuned to reach natural the frequencies from ground vibration tests as well. The described system was attached to the beam model of the cantilevered engine bed.

There were two modes included in the flutter analysis, despite that the system has more degrees of freedom due to the beam engine bed model. Modes are listed in the fig.7. Flutter analyses were performed for the aircraft





The FE model includes engine, propeller and engine bed inertia characteristics and the engine bed and engine mount isolators' stiffness characteristics as well. The engine – propeller inertia is modeled by of concentrated means relevant with masses



Fig. 8: V-g - f diagram (nominal state)



Fig. 9: Whirl flutter stability boundaries (critical values of natural frequencies)

frequency ratios, the flutter transforms to divergence.

Whirl flutter stability boundaries are presented in the fig.9. Axes represent both vertical and lateral frequencies; the ratio value of unity was extrapolated. Whirl flutter frequencies are presented in the fig.10. V-gf diagrams for the selected frequency ratios are presented in the fig.11-12. evaluation For the of diagrams, it must be taken into account, that there was a subsonic aerodynamic theory used, therefore the results for speed in the transonic or supersonic ranges doesn't aeroelastic certification altitude of H = 2500 mcertification speed for optimization was $1,2V_D = 150 \text{ m.s}^{-1} \text{ TAS}$. At whirl flutter first, the calculation for the nominal state was performed. There was no flutter state found as shown in the fig.8. After that. the optimization analyses for selected frequency ratios were performed as described in the previous chapter. Analyses included both options of the modes order. The critical flutter frequency is maximal for the frequency ratio close to the unity value, for the high



Fig. 10: Whirl flutter frequencies

represent the real state, but just artificial values.



6.2. Aircraft Engine - Wing Component System

The second testing example represents a model of a cantilevered wing with a flexibly attached engine – propeller system of a twin wing-mounted engine 42 seat turboprop commuter (wingspan 25,6 m; length 21,4 m; max. take-of weight 14500 kg – see fig.13).

The FE model doesn't represent a full-scale aircraft, but the aeroelastic model with the length



Fig. 14: Aeroelastic model model is attached to the beam model of the cantilevered model is prepared in a number of configurations concerning the



Fig. 13: Twin-engine commuter

scale of 1/5 and the velocity scale of 1/6 (fig.14). The FE model includes engine, propeller and wing inertia characteristics and the engine mount isolators, engine bed and the wing stiffness characteristics as well. Character of the FE model corresponds to the hardware aeroelastic model. The engine – propeller and the wing inertias are modeled by means of concentrated masses with relevant moments of inertia. Flexibility of the engine attachment is modeled by springs. The two rotational springs simulate the engine bed flexibility in the vertical and lateral directions. Two further translational springs simulate the mount-isolators flexibility in pitch. The engine – propeller model is attached to the beam model of the cantilevered

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Fig. 15: FE model (structural, aerodynamic)

fuel filling, stiffness of the aileron drive and the stiffness of the engine suspension. For the purpose of presented task, there was one specific configuration selected. Aerodynamic model

consist of the wing – aileron and spliter Doublet - Lattice elements, the nacelle was modeled as a Slender and Interference Body. Interference effects were taken into account. An interconnection between structural (fig.15a) and aerodynamic (fig.15b) part was realized via beam splines, the spliter was grounded by a surface spline.

Since on the full-scale aircraft, there is the engine attached via mount isolators in two planes (engine bed ring and rear attachment). contrary to the previous example, the engine vibrations modes represent the vibrations of the engine bed. The modal characteristics of the selected configuration are listed in the tab.1. Listed frequencies compared with the were experimental values obtained from the ground vibration of the hardware test aeroelastic model. An agreement between analytical and experimental results was



Fig. 16: V-g-f diagram (nominal state)

#	Title	f ₀ [Hz]
1	1 st wing bending	3,094
2	Engine vertical vibration	3,791
3	Engine lateral vibration	4,628
4	1 st wing horizontal bending	8,353
5	2 nd wing bending	8,522
6	Aileron flapping	13,567
7	1 st wing torsion	16,591
8	2 nd wing bending	19,674
9	2 nd wing horizontal bending	22,407
10	2 nd wing bending	24,886

Tab.1: Modal characteristics summary

assessed as satisfactory. As the former example, firstly the whirl flutter calculation for a nominal state was performed. There was no whirl flutter instability found, just a wing bending – torsion – aileron flutter occurred ($V_{FL} = 68,5 \text{ m.s}^{-1}$; $f_{FL} = 12,2 \text{ Hz}$), as shown in the V-g-f diagram in the fig.16.

Optimization based calculations were performed in the same way as have been already described in the previous subchapter. The results look similar to those ones as well. A certification speed scaled down to the aeroelastic model scale was

1.2*V_D = 32 m.s⁻¹. Whirl flutter stability calculations were performed for subset of two modes (engine vertical and lateral vibrations) and for set of 10 modes listed in the tab.1 as well. Selected V–g–f diagrams for the two modes calculations are presented in the fig.17-18. There can be found just a critical state at the velocity of 32 m.s⁻¹ of the whirl flutter or



divergence respectively.

Selected V–g–f diagrams for the 10 modes calculations are presented in the fig.19-20. There can be found a critical state at the velocity of 32 m.s^{-1} of the whirl flutter or divergence respectively. Furthermore, the aileron flutter instability remains; it wasn't affected by the gyroscopic effect. Diagrams roughly correspond to those ones presented for the two modes calculation.



Whirl flutter stability boundaries are presented in the fig.21. Axes represent both vertical and lateral frequencies; the value for the ratio of unity was extrapolated. Stability boundaries for both two modes and 10 modes option are compared. The former case is more stable, it

means, that remaining modes representing the flexibility of the wing have a slightly destabilizing effect. It gives for demands reasons to include the rest structure, particularly the wing elasticity to the whirl flutter analysis. This fact causes а considerable increasing of configurations for the whirl flutter analysis (wing fuel filling etc.). The presented procedure can be employed to assess the influences of such secondary parameters. Whirl flutter frequencies for the 10 modes solution are presented in the fig.22 as well.



Fig. 21: Whirl flutter stability boundaries (critical values of natural frequencies)



Fig. 22: Whirl flutter frequencies

7. Conclusion

Submitted paper presents a procedure for determination of the critical values of parameters regarding the whirl flutter stability. Parameters are stiffness of the engine suspension or vibration engine modes natural frequencies respectively. An optimization-based solution is employed. The procedure was tested on the two examples: the engine – propeller system of a single turboprop small transport aircraft and the engine propeller - wing system of a twin turboprop commuter.

The procedure replaces large parametric analytical studies varying the stiffness parameters by assessment of the critical values of them. It allows assessing easily influences of secondary parameters (residual structure of the aircraft). Furthermore it requires no experimental data from ground vibration tests, that's why it allows to move the whirl flutter analysis to the earlier phase of the aircraft development.

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