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IDENTIFICATION OF PARAMETERS OF CONSTITUTIVE MODELS FOR SOFT TISSUES FROM BIAXIAL TENSION TESTS

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Summary: *Soft tissues are pseudoelastic materials, modelled mostly as hyperelastic. In this theory, the non-linear relations between stress and strain components are described by various strain energy density functions, preferably by anisotropic ones. For any hyperelastic materials, isotropic as well as anisotropic, uniaxial extension tests are not sufficient for a reliable determination of parameters of multi-dimensional constitutive models that aim to predict the material behaviour in different types stress-strain states. Therefore it is necessary to use some appropriate additional types of tests for experimental identification of constitutive parameters. Based on experimental data from various types of biaxial tension tests, this paper presents a determination of parameters of anisotropic hyperelastic models.*

Introduction

Biaxial mechanical testing is required to quantify mechanical properties of hyperelastic materials that exhibit anisotropic behaviour. In addition, constitutive models for anisotropic or orthotropic materials that are assumed to be incompressible cannot be derived from uniaxial testing alone. These models are used to predict mechanical behaviour of the material under any biaxial loading condition.

Experimental equipment

The experimental equipment (Fig. 1) consist of bedplate with two servo motors and orthogonal screws, four carriages, equipment for clamping of the specimens, specimen bath with physiological saline solution, support stand with programmable camera and computer with software system for test control. Equipment for clamping of the specimens consist of

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four carriages with four clips per each one and two sensing head with force transducer. The specimen must be clamped by two or four clips on every edge because the loading must be distributed uniformly. The method of clamping should not damage the specimen. The specimen is immersed in physiological saline solution with specified temperature and *pH*. Contacting strain gauges is unacceptable in soft tissue applications therefore contactless strain evaluation is required. A programmable CCD camera is used as one of the noncontacting methods that involves the tracking of a finite number of closely spaced markers that are marked or affixed to the specimen. Measurement of strains in loading directions is based on evaluation of markers position before and during the loading.

The process of testing

The specimen of square or rectangle shape is cut-out, for example, from porcine thoracic aorta (Fig. 2). The reference markers - four black points are marked on specimen [1] or 1 mm diameter steel balls are glued onto the specimen surface [5], [6] (Fig. 3). Then the specimen is loaded by defined forces or displacements. Position of reference points or steel balls are on-line monitored by CCD camera and processed by software for off-line image analysis. The postprocessing data consist of complete deformation gradient tensor (inferred from the black points motions), loading forces and Cauchy true stresses.

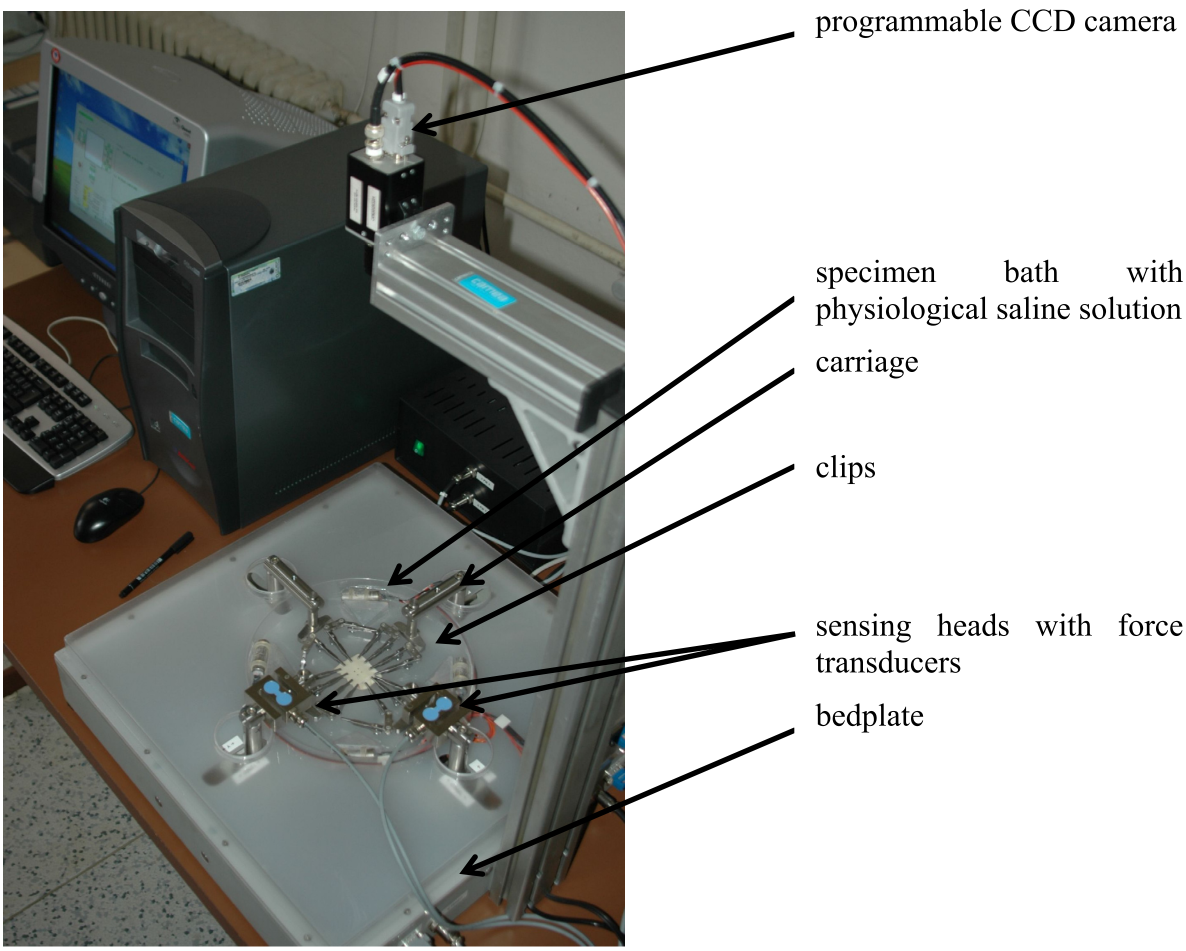

Fig. 1: Experimental equipment

Fig. 2: Porcine thoracic aorta

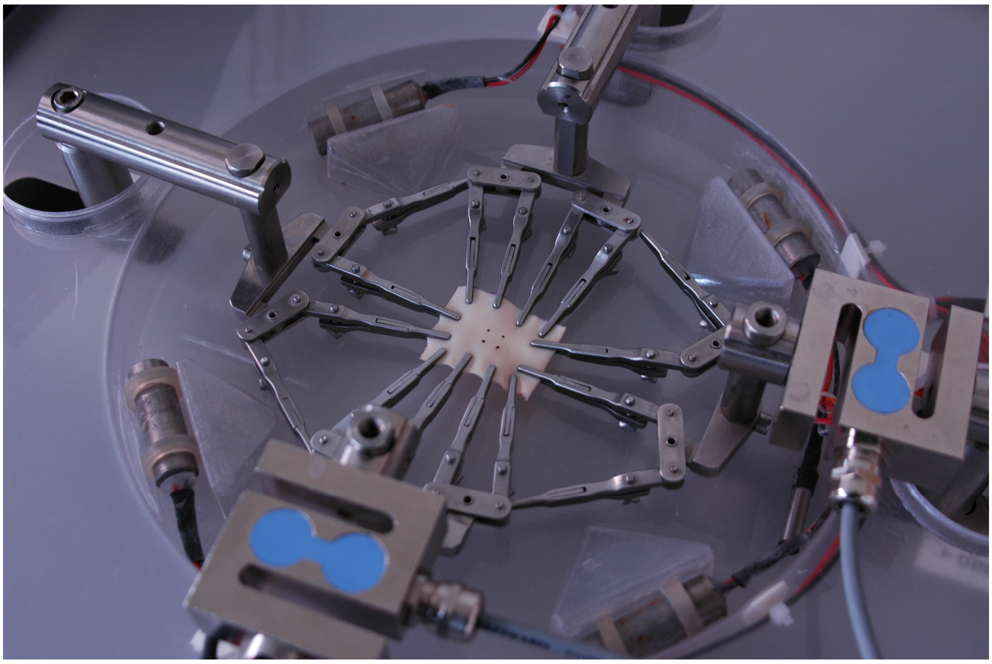

Fig. 3: Gripped of rectangular specimen with the reference points

Types of tests

Mechanical tests of soft tissues are realized „in vitro“ using a square or rectangular specimen. The independent control of displacements in both directions enables us to obtain the stress-strain characteristics for various states of biaxial tension. It is possible to obtain stress-strain characteristics in the following types of test [2], [4] (Fig. 4):

a) equibiaxial tension test - with equal strains in both principal directions (curve 1)

b) biaxial tension tests - uniaxial with constraining of transversal contraction (curves 2, 3)

c) biaxial tension test with proportional strain components (curve 4)

d) biaxial tension test with constant strain in either „1“ or „2“ principal direction (curves 5, 6)

e) uniaxial tension tests in either „1“ or „2“ principal direction (curve 7, 8)

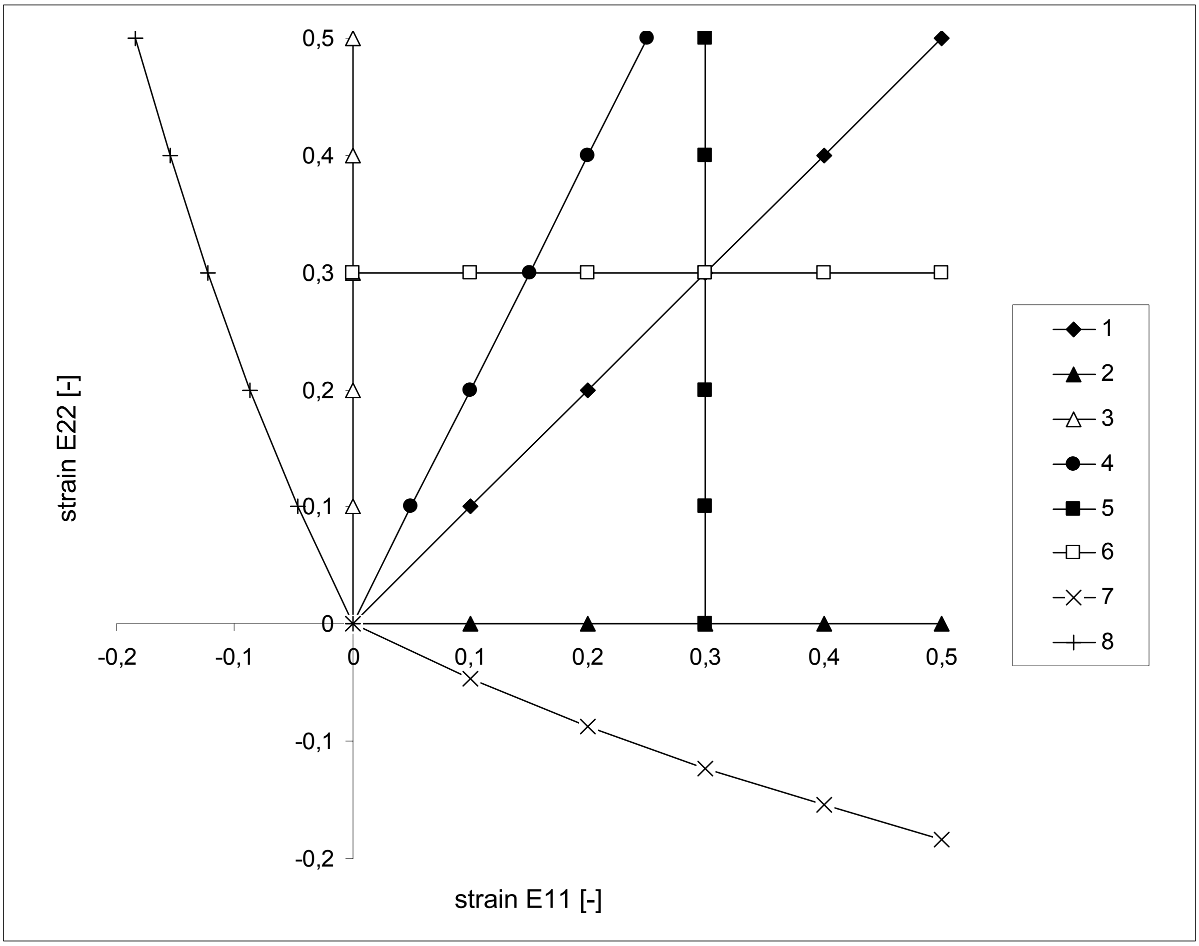

Fig. 4: Strain states in various types of tests

Constitutive relations

Appropriate constitutive relations are needed for computational modelling of different types of stress-strain states in arteries. Isotropic as well as orthotropic hyperelastic constitutive relations represent a mathematical description of relations between stress and strain components that are derived from strain-energy function *W*. If such a strain-energy function exists, the stress components can be obtained as derivatives of *W* with respect to strain components:

$$S_{11} = \frac{\partial W}{\partial E_{11}} \qquad S_{22} = \frac{\partial W}{\partial E_{22}} \tag{1}$$

where S_{ij} is 2. Piola Kirchhoff stress tensor which is conjugated with Green Lagrange strain tensor E_{ij}. Let us consider a rectangular specimen as shown in Fig. 5, then it is possible to define the components of stress tensor by several relations [1] presented in Eq.(2, 3, 4):

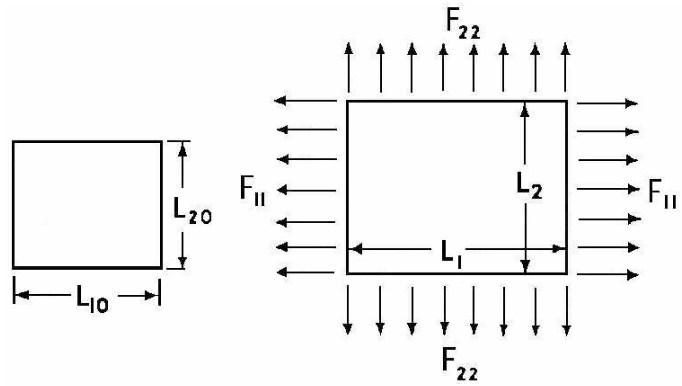

Fig. 5: Geometry and deformation state of the specimen

$$\sigma_{11}=\frac{F_{11}}{L_2\cdot h}\qquad \sigma_{22}=\frac{F_{22}}{L_1\cdot h}\qquad \textit{Cauchy and Euler} \tag{2}$$

$$T_{11}=\frac{F_{11}}{L_{20}\cdot h_0}\qquad T_{22}=\frac{F_{22}}{L_{10}\cdot h_0}\qquad \textit{Piola and Lagrange}\ (1.P.K.) \tag{3}$$

$$\left.\begin{aligned} S_{11}&=\frac{1}{\lambda_1}\cdot T_{11}=\frac{1}{\lambda_1^2}\cdot\sigma_{11}\\ S_{22}&=\frac{1}{\lambda_2}\cdot T_{22}=\frac{1}{\lambda_2^2}\cdot\sigma_{22}\end{aligned}\right\}\qquad \textit{Kirchhoff}\quad (2.P.K.) \tag{4}$$

where F_{11},F_{22} are two pairs of forces acting on the edges of the specimen, L_{10},L_{20},h_0 are the original dimensions of specimen at the zero stress state L_1,L_2,h are the deformed dimensions of specimen, the ratios $\lambda_1=\frac{L_1}{L_{10}},\ \lambda_2=\frac{L_2}{L_{20}}$ are the principal stretch ratios.

Rem. *Incompressible material (such as material of arteries where relation* $\lambda_1\lambda_2\lambda_3=1$ *is valid) is a presumption for equation (4).*

The corresponding components of strain tensor can be define by the following relations [1]:

$$E_1=\frac{1}{2}\cdot\left(\lambda_1^2-1\right)\qquad E_2=\frac{1}{2}\cdot\left(\lambda_2^2-1\right)\qquad \textit{Green and St.Venant}\ (\textit{Lagrange}) \tag{5}$$

$$\mathrm{E}_1=\frac{1}{2}\cdot\left(1-\frac{1}{\lambda_1^2}\right)\qquad \mathrm{E}_2=\frac{1}{2}\cdot\left(1-\frac{1}{\lambda_2^2}\right)\qquad \textit{Almansi and Hamel}\ (\textit{Euler}) \tag{6}$$

$$e_1=\ln\lambda_1\qquad e_2=\ln\lambda_2\qquad \textit{Logarithmic (natural) strain} \tag{7}$$

Relations between differently defined stress and strain tensors are:

Cauchy stress tensor is conjugated with logarithmic strain tensor because both of this tensors are related to the actual dimensions.

Kirchhoff stress tensor (2.P.K) is conjugated with Green (Lagrange) strain tensor.

Data Fitting

The next step in the analysis is fitting of the nonlinear material model to experimental data. For soft tissues applications the most frequent two-dimensional models of the strain-energy function describing orthotropic hyperelastic material behavior [1], [7] are:

Polynomial model: Patel and Vaishnav (1972):

$$W = AE_{11}^2 + BE_{11}E_{22} + CE_{22}^2 + DE_{11}^3 + EE_{11}^2E_{22} + FE_{11}E_{22}^2 + GE_{22}^3 \qquad (8)$$

A, B, C, D, E, F, G are material constants

Exponential model:

a) Fung (1973); Fung, Fronek, Patitucci (1979):

$$W = \frac{C}{2}\exp\left[c_1E_{11}^2 + c_2E_{22}^2 + 2c_3E_{11}E_{22}\right] \qquad (9)$$

C, c_1, c_2, c_3 are material constants

b) Maltzahn (1984):

$$W = \frac{C}{2}\left[\left(\exp Q\right) - 1\right] \quad where \quad Q = c_1E_{11}^2 + c_2E_{22}^2 + 2c_3E_{11}E_{22} \qquad (10)$$

C, c_1, c_2, c_3 are material constants

Logarithmic model: Takamizawa and Hayashi (1987) [3]:

$$W = -C\ln\left(1 - Q\right) \quad kde \quad Q = \frac{1}{2}c_1E_{11}^2 + \frac{1}{2}c_2E_{22}^2 + c_3E_{11}E_{22} \qquad (11)$$

C, c_1, c_2, c_3 are material constants

Rem. *this logarithmic model is available for Q in the range 0<Q<1. For Q=1 ln(1-Q) tends to infinity.*

Fitting of the material model to experimental data is achieved by optimizing (minimizing) the stress-based nonlinear function [8]:

$$f_s = \sum_{i=1}^{n}\left[w_1\left(\frac{\partial W}{\partial E_{11}}\Big|_i - S_{11}^i\right)^2 + w_2\left(\frac{\partial W}{\partial E_{22}}\Big|_i - S_{22}^i\right)^2\right] \quad n = number\ of\ experimental\ data\ records \qquad (12)$$

where w_1 and w_2 are weighting factors, and $\frac{\partial W}{\partial E_{11}}\Big|_i$ and $\frac{\partial W}{\partial E_{22}}\Big|_i$ are 2. Piola Kirchhoff stresses predicted by the constitutive model for *i*-th data record. The experimental 2. Piola Kirchhoff stresses S_{11}^i and S_{22}^i are calculated directly from the original data according Eq. 2, 4.

Alternatively to the stress-based approach expressed by (12), an energy-based nonlinear function f_w may also be chosen. Thus:

$$f_w = \sum_{i=1}^{n}\left(\psi_i - W_i\right)^2 \quad n = number\ of\ experimental\ data\, records \qquad (13)$$

where ψ_i is the strain energy for i-th data record predicted by the constitutive model and

$$W_i = \int_0^{E_{11}^i} S_{11}^i dE_{11}^i + \int_0^{E_{22}^i} S_{22}^i dE_{22}^i \qquad (14)$$

is the strain energy computed from experimental data. From a mathematical point of view both approaches are equivalent.

Transformation of experimental data to mathematical model is shown in the example described bellow:

In this example the energy-based approach and numerical integration for Eq.(14) are used. Thus:

$$W = \sum_{i=1}^{n} \sum_{j=11}^{22} \frac{S_j^i + S_j^{i-1}}{2} \left(E_j^i - E_j^{i-1} \right), \quad n = number\ of\ experimental\ data \qquad (15)$$

Strain-Energy function W is computed from experimental data of three types of tests:

- two biaxial tension tests with constrained transversal deformation: (Fig. 4) - (curves 2, 3)

- equibiaxial tension test: (Fig. 4) - (curve 1)

Using an off-line image analysis software Tibixus (produced by P. Skácel), Cauchy stress and principal stretch ratios in two orthogonal directions are obtained. These data are then expressed as Kirchoff stresses (2.P.K.) and Green (Lagrange) strain (Eq. 4, 5).The parameters C, c_1, c_2, c_3 are then obtained by means of the standard nonlinear Levenberg-Marquardt algorithm for multivariate nonlinear regression by minimizing the strain-energy function. In physiological range of loading, the best-fit parameters are obtained by using logarithmic model for the strain-energy function (Fig. 6):

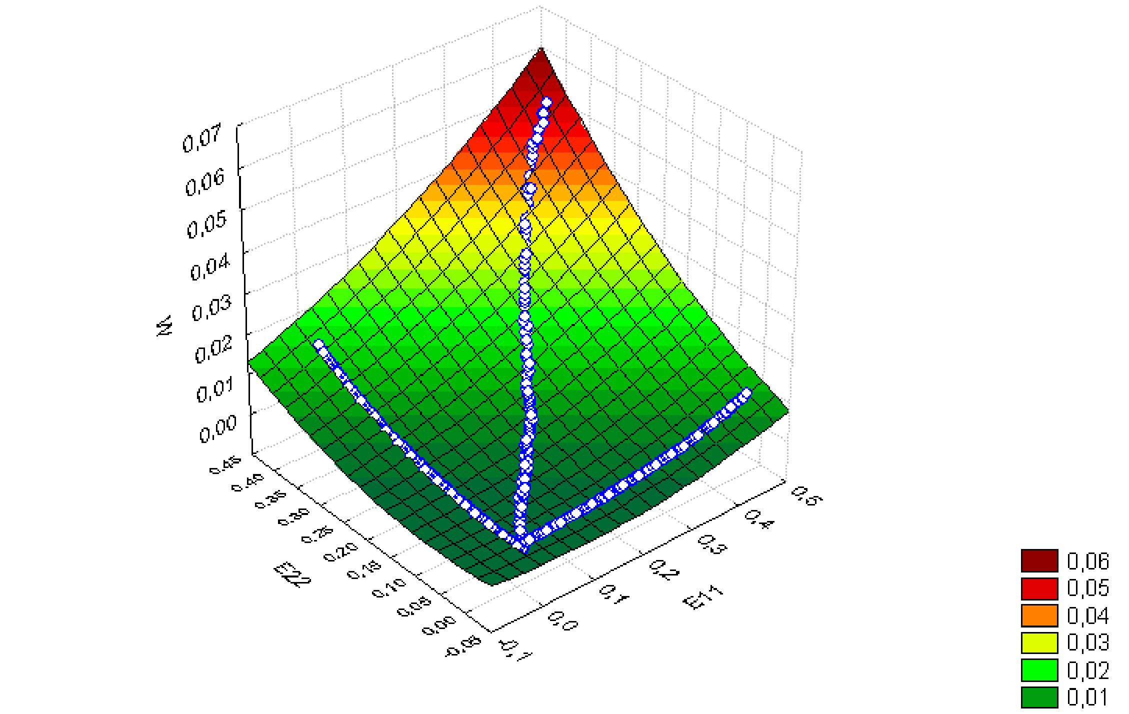

Fig. 6: The regression surface of strain-energy function

When the strain-energy function with the known constants exists, the stress components can be obtained as derivatives of W (Eq. 1) with respect to strain components:

$$S_{11}=\frac{\partial W}{\partial E_{11}}=\frac{c\left(c_1E_{11}+c_3E_{22}\right)}{1-\left(\frac{1}{2}c_1E_{11}^2+c_3E_{11}E_{22}+\frac{1}{2}c_2E_{22}^2\right)} \tag{16}$$

$$S_{22}=\frac{\partial W}{\partial E_{22}}=\frac{c\left(c_3E_{11}+c_2E_{22}\right)}{1-\left(\frac{1}{2}c_1E_{11}^2+c_3E_{11}E_{22}+\frac{1}{2}c_2E_{22}^2\right)} \tag{17}$$

Relations between Cauchy σ_{11},σ_{22} and Kirchoff stresses (2.P.K.) stress S_{11},S_{22} are:

$$\sigma_{11}=S_{11}\cdot\lambda_1^2 \tag{18}$$

$$\sigma_{22}=S_{22}\cdot\lambda_2^2 \tag{19}$$

Comparison between experimental data and data calculated by Eq. 16, 17, 18, and 19 are on Fig. 7, Fig. 8, Fig. 9 (stress-strain curves):

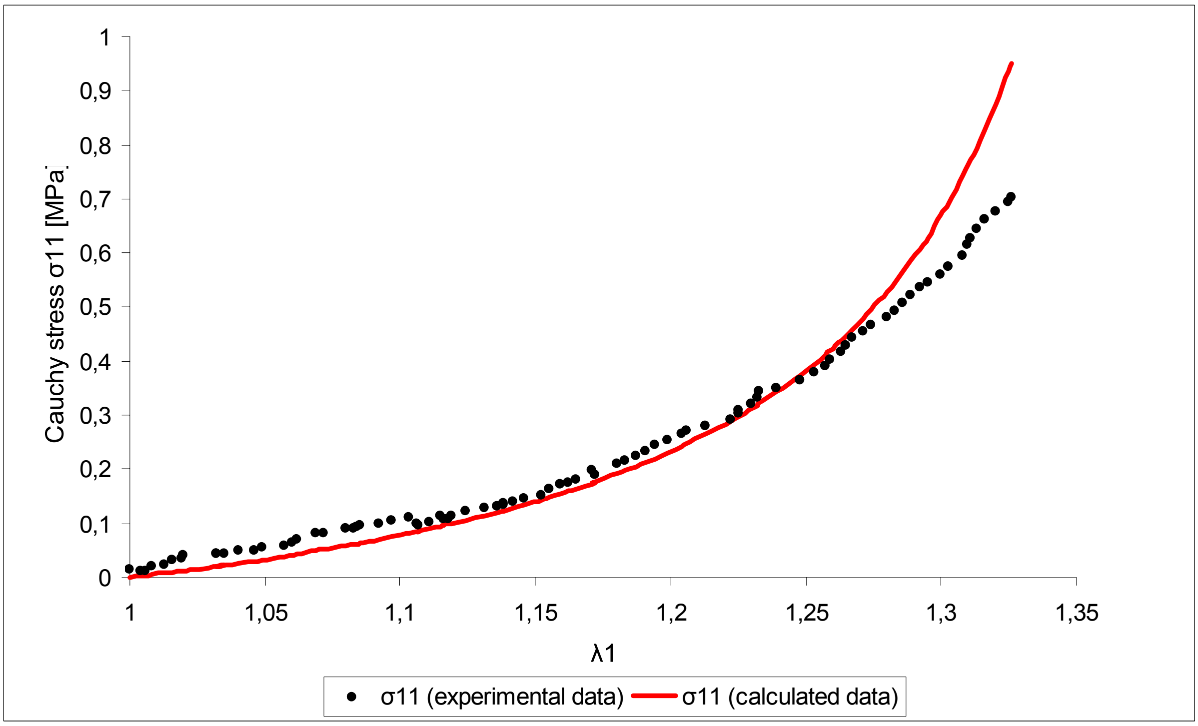

Fig. 7: Biaxial tension test – uniaxial in „1“ direction with constraining of transversal contraction in „2“ direction

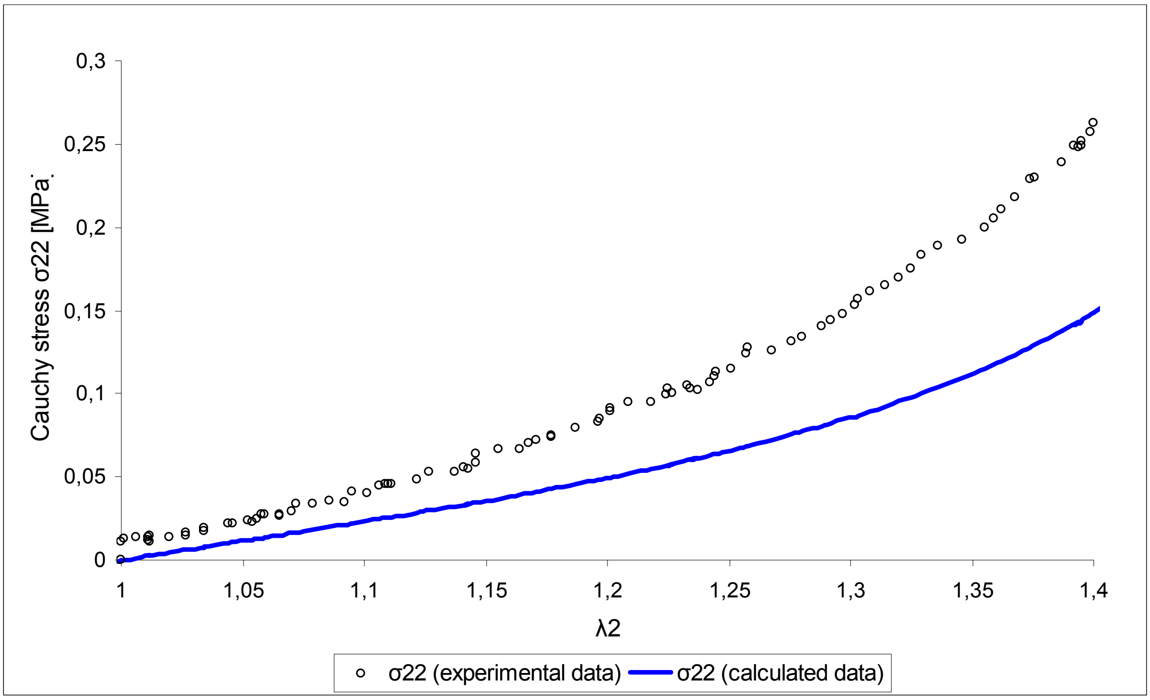

Fig. 8: Biaxial tension test – uniaxial in „2“ direction with constraining of transversal contraction in „1“ direction

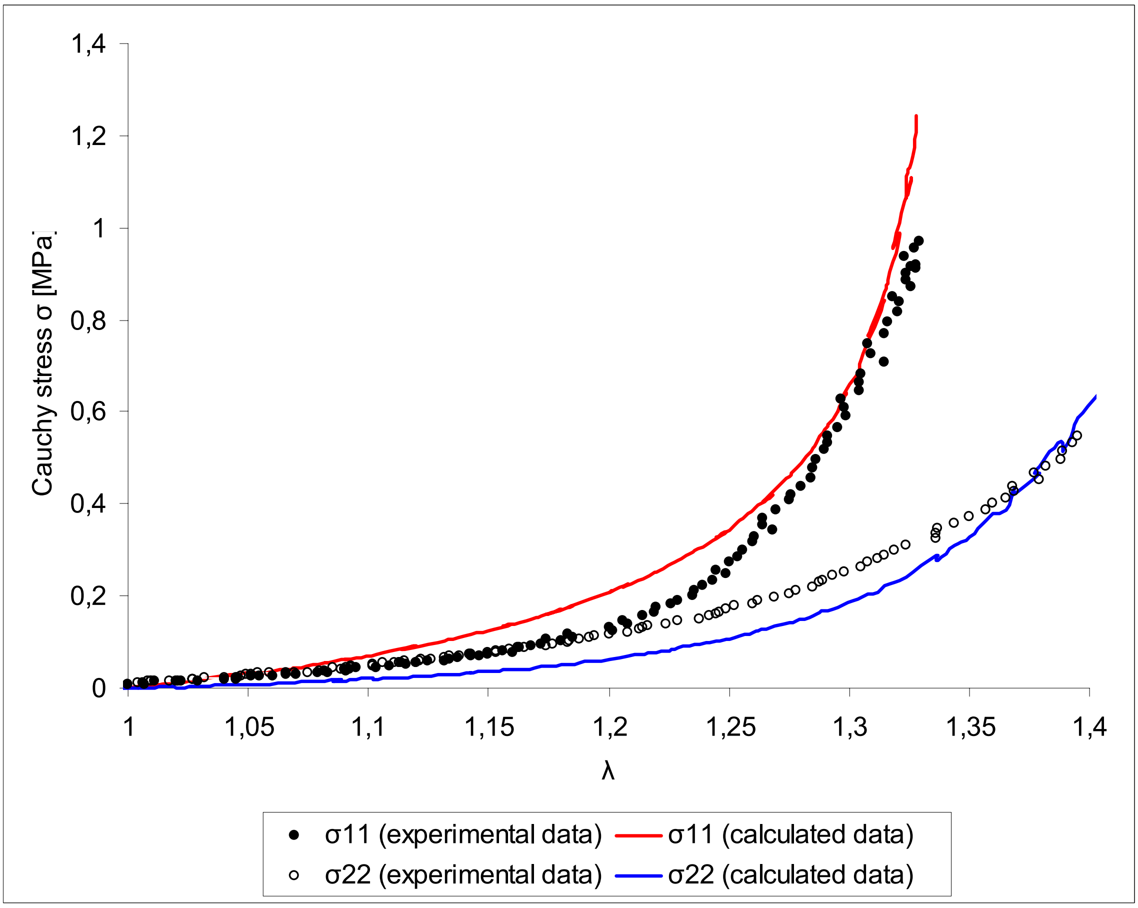

Fig. 9: Equibiaxial tension test

Conclusion

This paper presents a structural design of the equipment for biaxial testing of soft tissues, an analysis of the test types necessary for a credible identification of constitutive relations and their parameters and a method allowing the identification of parameters for constitutive relations. At soft tissues, relationships between stress and strain components are nonlinear anisotropic therefore it is necessary to use several appropriate types of tests for experimental determination of constitutive parameters. A cylindrical specimen of the whole artery fixed on both ends and loaded by internal pressure is often used for biaxial tension testing; the advantages of the presented type of testing are the possibility to change the strains in both directions independently and the possibility of testing of particular arterial layers (media, adventitia).

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