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# A NUMERICAL PROCEDURE FOR INVESTIGATION OF IMPACTS OF ROTORS SUPPORTED BY CAVITATED LONG FLUID FILM BEARINGS AGAINST THE STATIONARY PART

**J. Zapoměl**

**Summary** : *The rotor systems consit of two principal components, of a rotor and of a stationary part, and the fluid film bearings are often used as coupling elements. As the clearance between the discs and the casing is usually very narrow, excessive lateral vibration of the rotor can produce impacts between the discs and the housing. Because up to now influence of the fluid film bearings and the impacts on the rotor behaviour has been investigated separately, it is desirable to develope a procedure which would make possible to analyze their mutual interaction. Into the mathematical models both the hydrodynamic bearings and the contacts are implemented by means of nonlinear force couplings. To determine the contact forces a Hertz theory is applied. The hydrodynamic forces depend on the pressure distribution in the oil film. As the bearing gap is very narrow a classical theory of lubrication can be used for this purpose. If pressure at some location of the bearing gap should drop below a critical level, a vapour cavitation occurs. In the noncavitated part of the lubricant layer the pressure distribution is governed by solution of the Reynolds equation and in the cavitated region it remains constant. The developed procedure for determination of the pressure distribution in the oil film satisfies the continuity of flow through the inlet and outlet edges of the cavitated area. Components of the hydraulic force are calculated by integration of the pressure distribution along the bearing lenght and circumference. For solution of the equation of motion a modified Newmark method is adopted. Applicability of the developed approach has been tested by means of computer simulations.*

## 1. Introduction

The rotor systems consit of two principal components, of a rotor and of a stationary part, and the fluid film bearings are often used as coupling elements. To ensure efficient performance of rotating machinery the clearance between the discs and the casing is usually very narrow and therefore excessive lateral vibration of the rotor can arrive at impacts between the discs and the housing. This event has two principal consequencies. Friction in the contact area produces local heat, deformation and wear of both colliding bodies a the impacts can lead to irregular or even chaotic vibration of the rotor system. The influence of the fluid film bearings

Doc. Ing. Jaroslav Zapoměl, DrSc., VŠB-Technical University of Ostrava, Department of Mechanics, 17.listopadu 15, Ostrava-Poruba, 708 33, tel.: 59 732 3267, fax.: 59 732 1287, e-mail : jaroslav.zapomel@vsb.cz

and the impacts have been investigated separately up to now. Therefore it is desirable to develope a procedure that would make possible to analyze their mutual action on the rotor system.

An important tool for such investigations is a computer modelling method. It is usual to implement both the fluid film bearings and the impacts into the mathematical models by means of nonlinear force couplings.

To determine components of the bearing forces through which the layer of lubricant acts on the rotor journal and bearing linen it is necessary to know a pressure distribution in the oil layer. As the bearing gap is very narrow a classical theory of lubrication can be applied for this purpose. If pressure at some location of the oil film drops to a critical level a vapour cavitation appears. It represents a complex of physical phenomena that take place in the bearing gap due to boiling the oil and liberation of gases dissolved in it. The cavitation always arrives at occurance of two-phase medium and at a rupture of the oil film.

The kind and substance of the cavitation in hydrodynamic bearings and squeeze film dampers was experimentally studied by a number of researchers. Zeidan and Vance [13] revealed five cavitation regimes. Their observations showed that pressure of the medium in cavitated areas remained approximately constant. A new approach to determination of the inlet and outlet edges of the cavitated regions in long bearings has been developed by Zapoměl and Liu in [9], [7]. This procedure is based on repeated solution of the Reynolds equation by means of a finite difference method. It assumes that pressure in the cavitated regions remains constant and it satisfies the continuity of flow in the whole oil film.

Impacts of the rotors against the stationary part were observed both numerically and experimentally by many investigators. Azeez and Vakakis [1] showed that even if the excitation was a periodic function of time, the produced vibration was not only periodic but it had also a quasi-periodic or chaotic character. The disc - housing impacts can be implemented into the computational models in several ways. The simpliest approach is based on application of a Newton's theory as described in [4], [11], [12]. Another possibility is to implement the contacts into the mathematical models by imploying a Hertz theory [2], [6].

A computational procedure for investigation of nonlinear behaviour or rotors supported by hydrodynamic bearings and performing impacts of the discs against the stationary part has been developed by Zapoměl et al. [8]. The shaft was modelled by a beam like body and properties of the fliud film bearings were linearized in the neighbourhood of the equilibrium position. Chang-Jian and Chen [3] set up an algorithm for analysis of a Jeffcott rotor supported by fluid film bearings and performing impacts of the disc. The bearings were modelled by force couplings and components of the bearing forces were calculated by approximate analytical relations.

This paper presents a computational procedure for analysis of rotors supported by long fluid film bearings. The shaft is implemented into the mathematical model by a beam-like body. The hydraulic forces are calculated numerically, an arbitrary extent of the cavitated area is possible. Magnitudes of the contact forces are determined by imploying a Hertz theory. In addition friction in the contact area is taken into consideration. Applicabilty of the developed procedure has been tested by means of computer simulations.

## 2. Determination of a pressurte distribution in the oil film of a long journal bearing

The fluid film bearings are usually implemented into the computational models by means of nonlinear force couplings. To determine components of the hydraulic force through which the layer of lubricant acts on the rotor journal and bearing shell it is necessary to know a pressure distribution and the velocity field in the bearing gap. As thickness of the oil film is very small, a classical theory of lubrication can be imployed for this purpose. Its assumptions can be found e.g. in [5]. The pressure distribution is then governed by a Reynolds equation.

In the areas where the pressure should drop below a critical level ( pressure of the oil saturated vapour ) a cavitation occurs. Pressure in such regions remains constant and the lubricant flows there only due to its adherance to the surface of the turning journal. The continuity of flow in this area is satisfied by decreasing the lubricant density.

The length to diameter ratio of many fluid film bearings used in real rotating machinery is often between 0.5 and 1.0. Geometry of such bearings enables to consider them as long and then the Reynolds equation describing the pressure distribution in the oil layer takes the form

$$\frac{\partial}{\partial\varphi}\left(h^3\frac{\partial p}{\partial\varphi}\right)=6\eta R^2\omega\frac{\partial h}{\partial\varphi}+12\eta R^2\frac{\partial h}{\partial t} \qquad (1)$$

$$h=c-e.\cos(\varphi-\gamma) \qquad (2)$$

$p$ - pressure,
$\varphi$ - circumferential coordinate (Fig.1),
$h$ - thickness of the oil film,
$R$, $c$ - journal radius, width of the bearing gap ( at centre position of the journal ),
$\omega$, $t$ - angular velocity of the rotor rotation, time,
$\eta$ - dynamic viscosity of the lubricant,
$e$, $\gamma$ - eccentricity of the journal centre, angular position of the line of centres ( Fig.1 ).

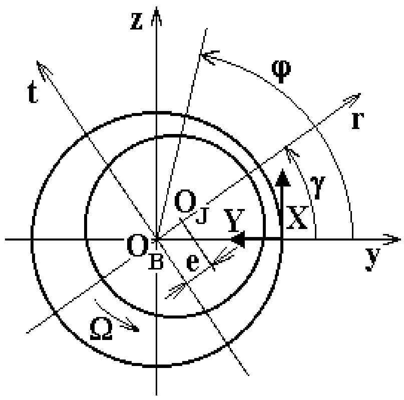


Fig.1 Scheme of a fluid film bearing

To be satisfied the continuity of flow and of the pressure distribution between the cavitated and noncavitated parts of the oil film, solution of the Reynolds equation (1) must satisfy the following boundary conditions [9], [7] :

- magnitude of pressure at the inlet and outlet edges of the cavitated region is equal to the pressure in the cavitated area,

- pressure gradient at the inlet edge of the cavitated region is zero,
- the flow rate at the outlet edge of the cavitated area is equal to the flow rate at the inlet edge,
- pressure at the oil inlets is equal to the pressure under which the oil is supplied into the bearing.

Application of a finite difference method on solving the Reynolds equation (1) requires to divide all portions of the bearing circumference between two adjacent oil inlets by nodes into periods of the same angular length.

First the Reynolds equation is solved using the boundary conditions

$$p = p_{IN1} \qquad \text{for} \qquad \varphi = \varphi_{IN1} \tag{3}$$

$$p = p_{IN2} \qquad \text{for} \qquad \varphi = \varphi_{IN2} \tag{4}$$

$p_{IN1}$, $p_{IN2}$ - pressure at the oil inlets into the bearing gap,
$\varphi_{IN1}$, $\varphi_{IN2}$ - angular coordinates of nodes corresponding to the oil inlets.

Let the pressure minimum corresponds to the node having coordinate $\varphi_{MIN}$. If its magnitude is less then the critical one, a vapour cavitation occurs.

To determine the inlet edge of the cavitated area the border nodes are succesively chosen from the node of the pressure minimum in the direction opposite to the journal rotation and for each such selected border node the Reynolds equation is solved with the boundary conditions

$$p = p_{IN1} \qquad \text{for} \qquad \varphi = \varphi_{IN1} \tag{5}$$

$$\frac{\partial p}{\partial \varphi} = 0 \qquad \text{for} \qquad \varphi = \varphi_{HR1} \tag{6}$$

$\varphi_{HR1}$ - angular coordinate of the chosen border node.

This process is repeated until pressure in the selected border node is equal or greater than is the cavitation pressure $p_{CAV}$. This node is then considered to be the inlet edge of the cavitated region.

To determine the outlet border of the cavitated area the nodes are succesively selected from the node of the pressure minimum in the direction of the journal rotation and for each selected node the Reynolds equation is solved with the boundary conditions

$$p = p_{CAV} \qquad \text{for} \qquad \varphi = \varphi_{HR2} \tag{7}$$

$$p = p_{IN2} \qquad \text{for} \qquad \varphi = \varphi_{IN2} \tag{8}$$

$p_{CAV}$ - pressure in the cavitation area (critical pressure),
$\varphi_{HR2}$ - angular coordinate of the chosen border node.

This manipulation is repeated until the difference between the linean flow rate $\Delta q_{HR2}$ in the node having the coordinate $\varphi_{HR2}$ and in the node corresponding to the inlet edge of the cavitated region is minimum

$$\Delta q_{HR2} = \left| -\frac{1}{12 R \eta} \left[ \frac{\partial p}{\partial \varphi} \right]_{\varphi = \varphi_{HR2}} h_{HR2}^{3} + \frac{1}{2} h_{HR2} R \omega - \frac{1}{2} h_{C1} R \omega \right| \tag{9}$$

$h_{HR2}$ - thickness of the oil film at node having coordinate $\varphi_{HR2}$,
$h_{C1}$ - thickness of the oil film on the inlet side of the cavitation region.

The border node minimizing relation (9) is considered to be the outlet edge of the cavitation region.

This procedure for searching the border nodes of the cavitated area is repeated for all portions of the bearing circumference situated between two adjecant oil inlets. Components of the hydraulic force acting on the rotor journal in y and z directions are given by integration of the pressure distribution around the circumference of the bearing

$$F_{hy} = -RL\int_0^{2\pi} p_d(\varphi)\cos\varphi \, d\varphi \qquad , \qquad F_{hz} = -RL\int_0^{2\pi} p_d(\varphi)\sin\varphi \, d\varphi \tag{10}$$

$p_d$ - pressure distribution in the oil layer,
$F_{hy}$, $F_{hz}$ - components of the hydraulical force acting on the rotor journal,
L - length of the bearing.

Pressure distribution $p_d$ in the full oil film is given by solution of the Reynolds equation. In cavitated areas it remains constant.

## 2. Determination of components of the impact forces

The impacts are implemented into the mathematical models of rotor systems by means of nonlinear force couplings. To calculate components of the impact forces it is assumed that

- the stationary part and the discs are absolutely rigid except a small neighbourhood of the contact point,
- friction in the contact area is of a Coulomb type,
- direction and orientation of the friction force depends only on the sense of the rotor turning,
- the impacts does not influence the angular speed of the rotor rotation.

The contact force consists of two components, of the normal and tangential ( it means frictional ) ones ( Fig.2 ).

The normal component of the contact force is produced by the contact stiffness and damping in material in the neighbourhood of the contact point. Therefore it depends on the contact stiffness and damping and on the displacement and velocity of the disc centre relative to the stationary part in the radial direction

$$\eta_C = y_C \cos\varphi_\zeta + z_C \sin\varphi_\zeta \tag{11}$$

$$\dot{\eta}_C = \dot{y}_C \cos\varphi_\zeta + \dot{z}_C \sin\varphi_\zeta \tag{12}$$

where it holds for the position angle $\varphi_\zeta$

$$\cos\varphi_\zeta = \frac{y_C}{\sqrt{y_C^2 + z_C^2}} \qquad , \qquad \sin\varphi_\zeta = \frac{z_C}{\sqrt{y_C^2 + z_C^2}} \tag{13}$$

$y_C$, $z_C$ - displacement of the disc centre in y, z directions,
$\eta_C$ - displacement of the disc centre in the radial direction,

$\dot{y}_C, \dot{z}_C$ - velocity of the disc centre in y, z directions,
$\dot{\eta}_C$ - velocity of the disc centre in the radial direction

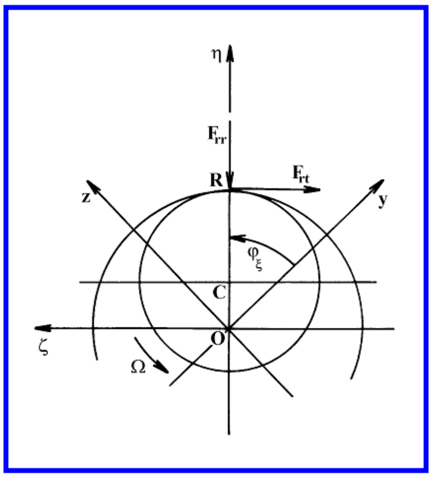


Fig.2 Disc geometry and impact force components

The contact stiffness can be determined by means of a Hertz theory and the coefficient of contact damping is considered to be proportional to the contact stiffness

$$b_{CON} = \beta_{CON} k_{CON} \tag{14}$$

$k_{CON}$ - contact stiffness,
$b_{CON}$ - coefficient of contact damping,
$\beta_{CON}$ - coefficient of proportionality.

The conditions for occuring the contact are geometric and force. The normal component of the impact force can be only compressive and it can act only if the displacement of the disc centre in the radial direction is greater than is the width of the clearance between the rotor and the stationary part.

$$F_{rr} = -k_{CON}\eta_C - b_{CON}\dot{\eta}_C \quad \text{for} \quad -k_{CON}\eta_C - b_{CON}\dot{\eta}_C < 0 \quad \text{and} \quad \eta_C > c_R \tag{15}$$

$$F_{rr} = 0 \quad \text{for} \quad -k_{CON}\eta_C - b_{CON}\dot{\eta}_C \geq 0 \quad \text{or} \quad \eta_C \leq c_R \tag{16}$$

$F_{rr}$, $F_{tt}$ - radial, tangential components of the impact force,
$c_R$ - width of the clearance between the disc and the housing.

According to the Coulomb law the friction force is proportional to the normal component of the impact force

$$F_{rt} = -|F_{rr}|\mu \,\text{sign}(R_D\omega) \tag{17}$$

$R_D$ - disc radius,
$\mu$ - coefficient of friction in the contact area.

## 3. Solving the equation of motion

Lateral vibration of rotors supported by fluid film bearings is described by a nonlinear equation of motion and by the relationship for the boundary conditions

$$\mathbf{M}\ddot{\mathbf{x}} + (\mathbf{B} + \eta_V \mathbf{K}_{SH} + \omega\mathbf{G})\dot{\mathbf{x}} + (\mathbf{K} + \omega\mathbf{K}_C)\mathbf{x} = \mathbf{f}_A + \mathbf{f}_V + \mathbf{f}_R(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}}) \tag{18}$$

$$\mathbf{x}_{BC} = \mathbf{x}_{BC}(t) \tag{19}$$

**M**, **G**, **K** - mass, gyroscopic, stiffness matrices of the rotor system,
$\mathbf{B}$, $\mathbf{K}_C$ - ( external ) damping, circulation matrices of the rotor system,
$\mathbf{K}_{SH}$ - stiffness matrix of the shaft,
$\mathbf{f}_A$, $\mathbf{f}_V$ - vectors of applied, constraint forces acting on the rotor system,
$\mathbf{f}_H$, $\mathbf{f}_R$ - vectors of hydraulical, impact forces acting on the rotor system,
$\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$ - vectors of generalized displacements, velocities, accelerations of the rotor system,
$\mathbf{x}_{BC}$ - vector of boundary conditions,
$\omega$ - angular speed of the rotor rotation,
$\eta_V$ - coefficient of viscous damping ( material of the shaft ).

Solution of the equation of motion (18) utilizing a Newmark method [10] arrives at each integration step at solving a set of algebraic equations that are nonlinear due to the bearing forces ( elements of vector $\mathbf{f}_H$ ). To avoid this operation elements of the vector of hydraulic forces $\mathbf{f}_H$ at time t+Δt are determined by means of their expansion into a Taylor series in the neighbourhood of time t

$$\mathbf{f}_{H,t+\Delta t} = \mathbf{f}_{H,t} + \mathbf{D}_{B,t}.(\dot{\mathbf{x}}_{t+\Delta t} - \dot{\mathbf{x}}_t) + \mathbf{D}_{K,t}.(\mathbf{x}_{t+\Delta t} - \mathbf{x}_t) + \ldots \tag{20}$$

Substitution only of the linear portion of the Taylor series (20) into (18) results into a modified equation of motion related to the instant of time t+Δt

$$\mathbf{M}\ddot{\mathbf{x}}_{t+\Delta t} + (\mathbf{B} + \eta_V \mathbf{K}_{SH} + \omega\mathbf{G})\dot{\mathbf{x}}_{t+\Delta t} + (\mathbf{K} + \omega\mathbf{K}_C)\mathbf{x}_{t+\Delta t} =$$
$$= \mathbf{f}_{A,t+\Delta t} + \mathbf{f}_{R,t+\Delta t} + \mathbf{f}_{V,t+\Delta t} + \mathbf{f}_{H,t} + \mathbf{D}_{B,t}(\dot{\mathbf{x}}_{t+\Delta t} - \dot{\mathbf{x}}_t) + \mathbf{D}_{K,t}(\mathbf{x}_{t+\Delta t} - \mathbf{x}_t) \tag{21}$$

Square matrices of partial derivatives $\mathbf{D}_{K,t}$ and $\mathbf{D}_{B,t}$

$$\mathbf{D}_{K,t} = \left[\frac{\partial \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}})}{\partial \mathbf{x}}\right]_{\mathbf{x}=\mathbf{x}_t, \dot{\mathbf{x}}=\dot{\mathbf{x}}_t} , \qquad \mathbf{D}_{B,t} = \left[\frac{\partial \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}})}{\partial \dot{\mathbf{x}}}\right]_{\mathbf{x}=\mathbf{x}_t, \dot{\mathbf{x}}=\dot{\mathbf{x}}_t} \tag{22}$$

are calculated at time t for which all kinematic parameters of the rotor system have been calculated at the previous integration step.

It is assumed that the time increment Δt is very short. Therefore magnitudes of the components of the impact forces at time t+Δt can be determined utilizing kinematic parameters of the rotor calculated at time t

$$\mathbf{f}_{R,t+\Delta t} = \mathbf{f}_{R,t+\Delta t}(\dot{\mathbf{x}}_t, \mathbf{x}_t) \tag{23}$$

or supposing the vibration can be considered as a uniformly accelerated motion during a short time step

$$\mathbf{f}_{R,t+\Delta t} = \mathbf{f}_{R,t+\Delta t}\left(\dot{\mathbf{x}}_t + \Delta t\,\ddot{\mathbf{x}}_t, \mathbf{x}_t + \Delta t\,\dot{\mathbf{x}}_t + \frac{1}{2}\Delta t^2\,\ddot{\mathbf{x}}_t\right) \tag{24}$$

To be satisfied the boundary conditions at any moment of time the equation of motion (18) referred to time t+Δt is converted to the form

$$\mathbf{A}_{2,t+\Delta t}.\ddot{\mathbf{y}}_{t+\Delta t} + \mathbf{A}_{1,t+\Delta t}.\dot{\mathbf{y}}_{t+\Delta t} + \mathbf{A}_{0,t+\Delta t}.\mathbf{y}_{t+\Delta t} = \mathbf{b}_{t+\Delta t} \tag{25}$$

Matrices $\mathbf{A}_{2,t+\Delta t}$ , $\mathbf{A}_{1,t+\Delta t}$ , $\mathbf{A}_{0,t+\Delta t}$ and vectors $\mathbf{b}_{t+\Delta t}$ and $\mathbf{y}_{t+\Delta t}$ are obtained from (26) - (29) and $\mathbf{x}_{t+\Delta t}$ respectively by omitting the appropriate rows and columns corresponding to the degrees of freedom to which the boundary conditions are assigned

$$\mathbf{A}^*_{2,t+\Delta t} = \mathbf{M} \tag{26}$$

$$\mathbf{A}^*_{1,t+\Delta t} = \mathbf{B} + \eta_V \mathbf{K}_{SH} + \omega \mathbf{G} - \mathbf{D}_{B,t} \tag{27}$$

$$\mathbf{A}^*_{0,t+\Delta t} = \mathbf{K} + \omega \mathbf{K}_C - \mathbf{D}_{K,t} \tag{28}$$

$$\mathbf{f}^*_{t+\Delta t} = \mathbf{f}_{A,t+\Delta t} + \mathbf{f}_{R,t+\Delta t} + \mathbf{f}_{H,t} - \mathbf{D}_{B,t}\,\dot{\mathbf{x}}_t - \mathbf{D}_{K,t}\,\mathbf{x}_t - \mathbf{A}^*_{2,t+\Delta t}\,\ddot{\mathbf{x}}_{BC,t+\Delta t} - \mathbf{A}^*_{1,t+\Delta t}\,\dot{\mathbf{x}}_{BC,t+\Delta t} - \mathbf{A}^*_{0,t+\Delta t}\,\mathbf{x}_{BC,t+\Delta t} \tag{29}$$

In addition the mentioned modification eliminates unknown values of the vector of constraint forces $\mathbf{f}_V$.

## 4. Example

Applicability of the described approach has been tested by means of computer simulations. Rotor of the investigated rotor system ROT6 ( Fig.3 ) consists of a shaft ( SH ) and of two discs ( D1, D2 ). Disc D2 is placed in a cylindrical hole in the stationary part and the width of the clearance between the disc and the housing is small. The shaft is coupled with the stationary part ( FP ) through two hydrodynamical bearings ( journal diameter 80 mm, bearing length 50 mm, oil dynamical viscosity 0.006 Pas ) whose geometry enables to consider them as long. Each bearing is equipped with two deep axial grooves into which the oil is supplied. The rotor rotates at constant angular speed ( 300 rad/s ) and is loaded by its weight. In addition the system is excited by centrifugal forces caused by imbalance of both discs ( 12 gm, 16 gm ).

The task was to analyze the steady state component of the induced vibration.

In the computational model the shaft was represented by a beam-like body that was discretized into finite elements ( Fig.3 ). Both discs were considered as thin and absolutely rigid. The imbalance loading was modelled by two pairs of mutually perpendicular concentrated harmonic forces whose time histories are shifted by the phase leg of $\pi/2$.

Some results of the performed analysis are summarized in the following figures. The equilibrium positions of the rotor journal centres in bearings B1 and B2 and of the disc D2 are drawn in Fig.4. It shows that no contact between the disc and the housing occurs. The steady state trajectory od the disc D2 centre is evident from Fig.5. The orbit is periodic or close to periodic. The detail in Fig.6 shows the indentations of the disc into the stationary part during the impacts. The steady state orbits in bearings B1 and B2 are drawn in Fig.7 and 8. Time history of the normal component of the impact force is evident from Fig.9 and 10. The former gives information about the magnitudes and frequency of the impact force, the letter makes possible to determine durations of the individual impacts. The results show that they last between 400 and 500 μs.

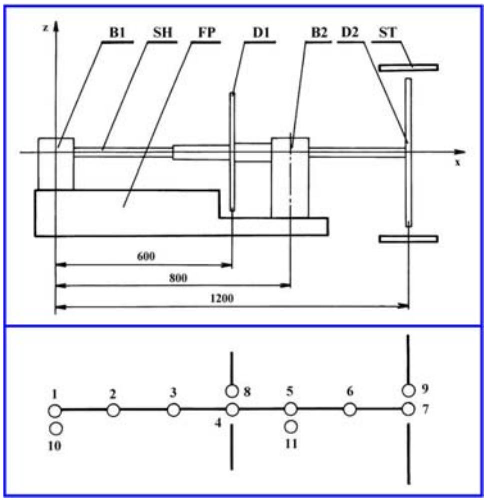


Fig.3 Scheme of the rotor system ROT6

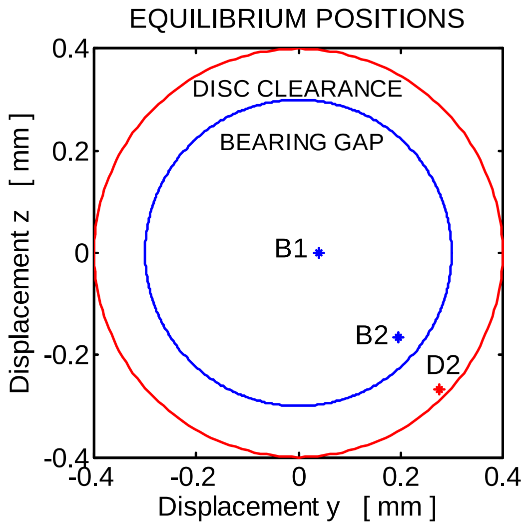


Fig.4 Equilibrium positions ( bearing, disc )

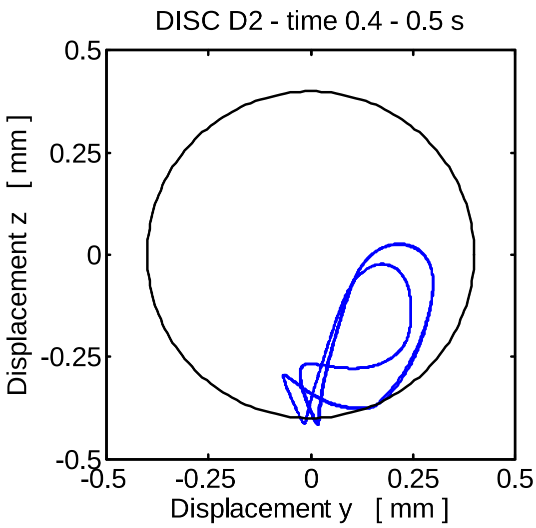


Fig.5 Orbit of the disc D2 centre

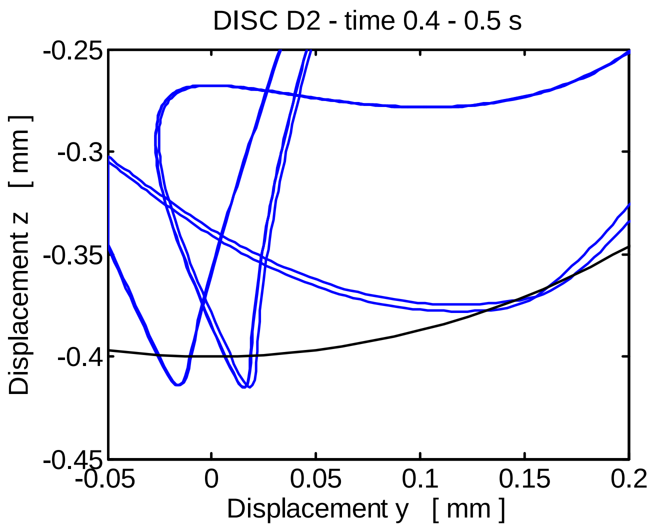


Fig.6 Orbit of the disc D2 centre ( detail )

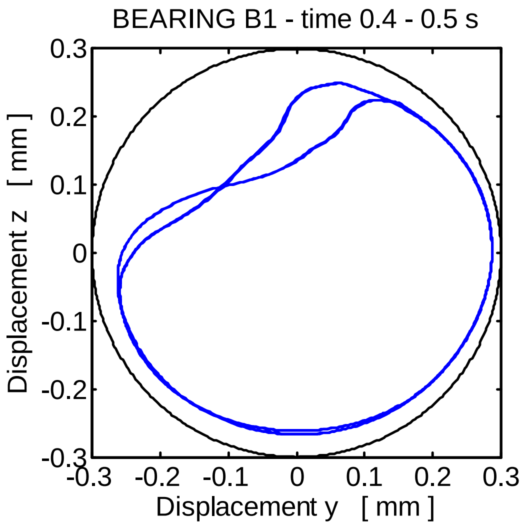


Fig.7 Orbit of the journal centre in B1

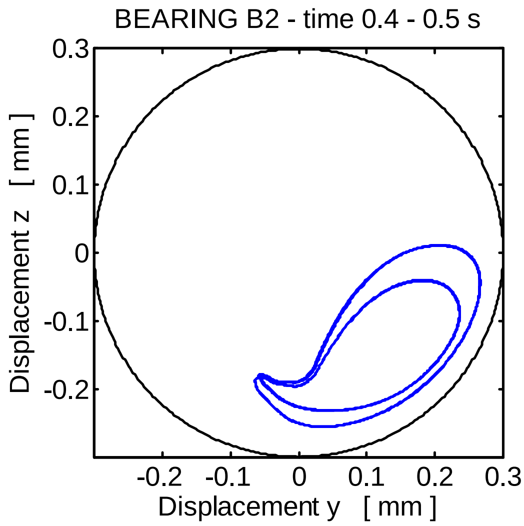


Fig.8 Orbit of the journal centre in B2

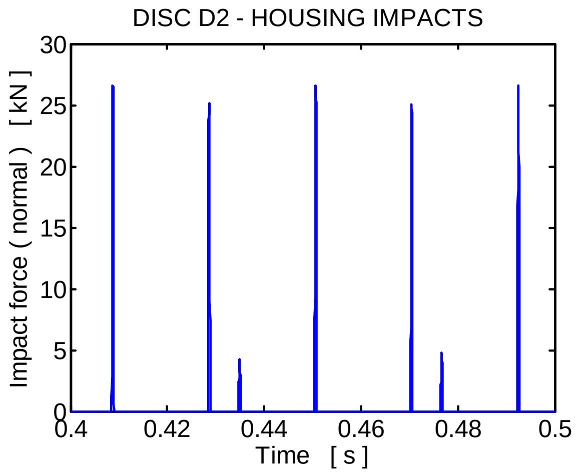


Fig.9 Time history of the impact force

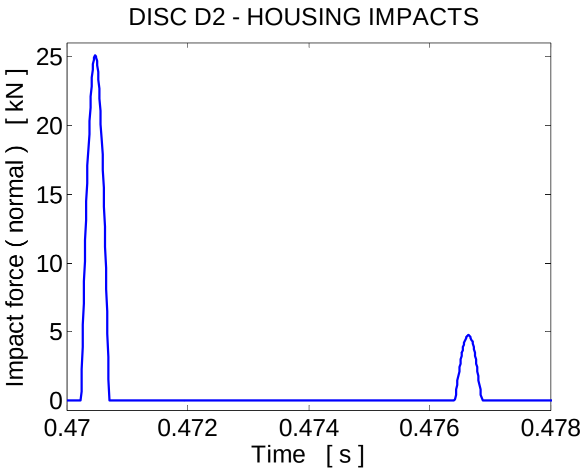


Fig.10 Time history of the impact force

## 3. Conclusions

The described approach represents a numerical method for investigation of a transient response of rotors supported by long hydrodynamic bearings carrying discs that are placed in holes in the stationary part. The developed approach implements both the fluid film bearings and the impacts into the computational models by means of nonlinear force couplings. It takes into consideration a rupture of the oil film caused by a vapour cavitation if pressure at some location of the oil film drops to a critical value. Pressure of the medium in cavitated areas is assumed to be constant and the algorithm for determination of edges of the cavitated regions satisfies the continuity of pressure course and the flow between the cavitated and noncavitated areas. Normal component of the impact force is calculated by application of a Hertz theory and the friction in the contact area is assumed to be of a Coulomb type. To perform solution of the equation of motion a modified Newmark method is applied. The modification consisting in repeated linearization of the vector of hydraulic forces in the neighbourhood of the current position enables to avoid solving a set of nonlinear algebraic equations at each integration step.

The results of computer simulations showed that the influence of both sources of nonlinear vibration of rotor systems ( hydrodynamic bearings, impacts ) could be analyzed together and that the procedure proposed for solving the equation of motion was numerically stable and made possible to apply an acceptable short integration step even in cases when the induced vibration was chaotic. The length of the integration step is determined more by the requirements put on calculation of the impact forces than the forces acting in the hydrodynamic bearings.

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