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INTERACTION PROBLEM FOR A BLOOD VESSEL

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Summary: *Blood – aortic wall interaction was investigated in a current work using the uncoupled method. The method is based on alternating, relatively independent solutions of two problems – the problem of hydromechanics for a blood flow in the vessel and the problem of elastostatics for a blood vessel wall, with consideration of appropriate conditions, predescribed on their common bound. In order to solve single problems requested for a complex problem solution the Finite Element Method was used. Blood flow is modeled as isothermal laminar steady flow of incompressible Newtonian fluid. Blood vessel is simplified to a thick-walled linear elastic tube, composed of two fiber-reinforced composite layers. The results of the solution of the interaction problem are used later in order to state the failure index for the blood vessel by means of the Tsai-Hill failure theory obtained for composite structures.*

1. Introduction

Fluid – structure interaction occurs when a fluid interacts with a solid structure, exerting pressure that may cause deformation in the structure and, thus, alter the flow of the fluid itself. Fluid – structure interaction is required for many industry applications, such as biomedicine – elastic artery or urethra modelling, aerospace and civil engineering – for a particular case – wind loading of structures.

The solution of the interaction problem, were investigated in the current work on the example blood – aorta interaction. The solution was further used in order to state the blood vessel wall damage prediction, using one of the composite material failure theories, see work of Decolon, (2002). The mechanical properties of the blood vessels could have probably later found application in surgery or tissue engineering.

2. The Aorta. Blood Flow and Blood Vessel

The aorta is the largest artery in the human body, originating from the left ventricle of the heart and bringing oxygenated blood to all parts of the body in the systematic circulation.

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The wall of the blood vessel is composed of few layers, which function-ability is important to the proper functioning of the whole circular system, for details see Holzapfel et al., (2000). The layers of the aorta are composed of similar components such as fibers of elastin, collagen and smooth muscle, however, in various volume fractions, that gives an explanation to different mechanical properties of single aortic layers. Blood flow in the healthy aorta is laminar isothermal and steady. In case of different pathologies like a decrease of the blood viscosity or narrowing of a part of the vessel, the velocity of blood flow increases, which leads to rise of turbulence. Blood circulation differs from the non-alive model with capability of vessel to change its volume in passive and active states. This capability is conditioned by the presence of different fibers mentioned above in the blood vessel wall. The fibers of elastin and collagen represent elastic component in the blood vessel wall. The main function of these fibers is to generate elastic stresses of the blood vessel and to function against distention of the blood vessel. The aorta has significantly more elastic fibers as any other arteries, this is why it is able to balance pulsatile blood flow.

Blood ejected from the left ventricle widens the wall of the aorta. This way a part of kinetic energy turns to potential elastic energy of the wall. Later, after the main blood flow flows off, elastic forces of the widened wall induce a return of the wall to its initial sizes and the blood collected in the extension zone is ejected in the direction from the heart (which is a direction of the smallest resistance). This way potential elastic energy of the wall is backward turned to kinetic energy of blood. For details see Křen et al., (2001).

3. Interaction Problem for a Blood Vessel

We investigate blood flow in the aorta. The flow is caused by the pressure gradient along the tube. Inlet pressure is larger than outlet pressure. The tube is elastic, therefore its deformations are dependent on the pressure and velocity distribution in the considered fluid. Hence the radius of the tube is constant only in the passive state. The current work deals with solution of interaction problem of two continua from the point of view of uncoupled method. The method is based on alternating, relatively independent solutions of both of the continua, interacting and affecting each other through actual boundary conditions on their common bound. This way the solution disintegrates to two basic parts, which represent solutions of two basic problems – fluid flow in the rigid body and the solution of deformation of elastic tube subjected to internal pressure obtained from the solved fluid.

At the first step, we solve blood flow in the ideal stiff tube, the form of which corresponds with the initial undeformed configuration. This way we obtain pressure and velocity distribution for the considered shape of tube. Later, we load elastic tube with the pressure, obtained from the above described solution for fluid and observe deformation of the tube. The form of tube changes, hence changes the area filled with fluid. The above performed solution should be corrected. Again we suppose deformed tube to be a "stiff" body. These steps of the interaction are repeated until the conditions on the common bound do not satisfy to the desired accuracy. Details could have been found in the work Křen at al., (1999). Let's symbolically say the fluid to be continuum 1 and the aortic wall continuum 2. On their common bound Γ_{12} the following conditions of contact of both continua and a condition of strength equilibrium satisfy for $x \in \Gamma_{12}$ and $t \in (0,T)$ the following relationships

$$ {}^{1}u_i(x,t) = {}^{2}u_i(x,t)\,, \tag{1} $$

$$ {}^{1}\dot{u}_i(x,t) = {}^{2}v_i(x,t), \tag{2} $$

$$ {}^{1}\tau_{ij} n_j = -{}^{2}\tau_{ij} n_j . \tag{3} $$

We should notice, however, that the above described algorithm of the interaction problem solution implies remeshing on the each step of iteration. It is, of course, a disadvantage of the method, which is, however, covered by the comparative simplicity of the independent algorithms of hydromechanics and elastostatics problems solutions.

The blood in the largest artery of the body has a Newtonian character. It represents homogeneous incompressible fluid. Boundary problem of the hydromechanics is described by the system of equations - Navier-Stokes equation and continuity equation

$$ \rho \frac{\partial v_i}{\partial t} + \rho v_j \frac{\partial v_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial}{\partial x_j}\left(\frac{\partial v_i}{\partial x_j}\right) + \rho f_i , \tag{4} $$

$$ \frac{\partial v_i}{\partial x_i} = 0 , \tag{5} $$

for $x \in \Omega$, $t \in (0,T)$ and standard boundary conditions for hydromechanics problem. The Navier-Stokes equation in the current work was considered without convexity and simplified to stationary case.

The unknown in this way described problem of hydromechanics are components of velocity v_i and pressure p. Which are being evaluated simultaneously as a velocity and pressure distributions.

We use Finite Element Method in order to solve this problem. The method allowed transformation of the system of the differential equations into a system of algebraic equations. The elements are standard 20-noded hexahedrons with linear approximation functions defining the pressures and polynomial functions of second order defining the velocities. The final algebraic equation after the discretization can be symbolically written in the simplified matrix form as:

$$ \begin{bmatrix} \mathbf{A} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{v}} \\ \dot{\mathbf{w}} \\ \mathbf{p} \end{bmatrix} + \begin{bmatrix} \mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{C}_1 \\ \mathbf{0} & \mathbf{B} & \mathbf{0} & \mathbf{C}_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{B} & \mathbf{C}_3 \\ \mathbf{K}_1 & \mathbf{K}_2 & \mathbf{K}_3 & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{w} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \\ \mathbf{E}_3 \\ \mathbf{0} \end{bmatrix}, \tag{6} $$

where $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ are unknown components of velocity and $\mathbf{p}$ is unknown pressure.

The aorta is considered to be a thick-walled circular cylindrical tube subjected to internal pressure. In order to discover the complex microstructure of the aortic wall, the fiber-reinforced composite macroscopic 3D linear model is assumed. The model is homogenized and reflects at any point the behavior of the microstructure. The fibers of collagen, which are assumed to be parallel give the aortic wall tissue its strength and mechanical integrity. The model is structural in the sense that it requests the information about the orientations of the collagen fibers to aortic axis, for a detailed view see Shakirova, (2006). The blood vessel wall is considered to be pseudo-isotropic, homogeneous and incompressible material with linear response to stretch.

The problem of elasticity is fully described by three equations (cinematic relation, constitutive law and equilibrium equation) and two standard boundary conditions for the elasticity problem. In order to solve the problem the Finite Element Method was chosen. The elements are standard 8-noded hexahedrons with linear approximation functions defining the unknown displacements.

4. Composite Material and Failure

Composite materials are materials, composed of two or more constituent components, which remain separate and distinct on a macroscopic level. At least one component, called reinforcement is often a strong fiber that gives the material its tensile strength, while another component, called matrix binds the fibers together and maintains their relative positions. The reinforcement is a discontinuous phase of the composite, it is stiff, strong and of a specific direction. The properties of the composite material depend on individual properties of its different components, geometry, volume fraction and distribution of the phases.

The constitutive relation for the linear elastic material is the relationship between stress and strain. For orthotropic material it can be written in the principal material axes coordinate system $\{L,T,W\}$ - longitudinal, in-plane transverse and out-of-plane transverse as:

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon}, \qquad \text{or} \qquad \boldsymbol{\sigma} = \mathbf{S}\,\boldsymbol{\varepsilon}, \tag{7}$$

$$\boldsymbol{\sigma} = [\sigma_L, \sigma_T, \sigma_W, \tau_{TW}, \tau_{LW}, \tau_{LT}], \tag{8}$$

$$\boldsymbol{\varepsilon} = [\varepsilon_L, \varepsilon_T, \varepsilon_W, \gamma_{TW}, \gamma_{LW}, \gamma_{LT}], \tag{9}$$

where $\mathbf{C}$ and $\mathbf{S}$ are elastic stiffness and compliance matrices, respectively, and satisfy: $\mathbf{S} = \mathbf{C}^{-1}$. The compliance matrix $\mathbf{S}$ in case of orthotrophy depends on elastic moduli E_L, E_T, E_W, shear moduli G_{TW}, G_{WL}, G_{LT} and Poisson's ratios $\nu_{TW}, \nu_{WL}, \nu_{LT}$.

Most of failure criteria for orthotropic materials suppose the homogenity of the material (no distinction between fibers and matrix). Therefore, the theories like Maximum Stress Theory, Maximum Strain Theory, Tsai-Hill or Tsai-Wu criteria are sort of predictions from the macroscopic approach, for details see work of Decolon, (2002).

The prediction of the tissue failure, introduced in this work, has been computed using the Tsai-Hill failure criterion, which is for an orthotropic material written as:

$$\left(\frac{\sigma_L}{F_L}\right)^2 + \left(\frac{\sigma_T}{F_T}\right)^2 + \left(\frac{\sigma_{LT}}{F_{LT}}\right)^2 - \frac{\sigma_L \sigma_T}{F_L^2} < 1. \tag{10}$$

The function, situated on the left-hand side of the relation (10) is so-called failure index. It represents the interest in further computations. The basic ultimate parameters for the criterion are longitudinal and transverse failure stresses in compression, longitudinal and transverse stresses in tension and shear failure stress.

5. Numerical Simulations

We consider blood to be a Newtonian fluid of dynamic viscosity $\mu = 3.3\times10^{-3}\ [Pa \cdot s]$ to fill space inside of the aortic tube of internal radius 13 $[mm]$, thickness $2\,[mm]$, and length $100\,[mm]$. We assume inlet pressure equal to systolic pressure $p_{inlet} = 120\,[mmHg] = 16\,[kPa]$. The obtained distribution of pressure from the solution of hydrodynamics problem, given my the fluid with above described parameters, represents the internal pressure to which the aortic elastic tube is subjected.

The media layer of the aorta has 4 sublayers with $\pi/6$ angle of the collagen fibers orientation, laying in opposite directions. The fibers in the adventitia, that has 2 sublayers have orientation $\pi/3$. The other parameters - Young's moduli, Poisson ratio, volume fractions for single layers and ultimate stresses for all constituents of the aortic layers material are given in tab. 1.

Tab. 1 Input data for single constituents of the aortic wall

	matrix	collagen	elastin	sm.musc
$E\ [Mpa]$	0.01	350	0.4	0.03
ν	0.45	0.3	0.39	0.42
V_m [%]	2	23	36.8	38.2
V_a [%]	2	36.2	36.6	25.2
ε^* [%]	60	4	130	70

Restricted displacements for the tube are in two planes - the top and the bottom of the tube. Both of the boundaries consist the displacements restricted against movement along x and y axis in all nodes situated on the outer circle of these planes and all nodes within these planes are restricted against movement along z-axes. The load forces represent the part of internal pressure, known from the pressure distribution obtained from the blood flow solution, with which the nodes situated on the very inner surface of the tube, are loaded

The solution of blood – aorta interaction problem is realised as a programming module, written within the Matlab environment. The obtained results are graphically worked up and are demonstrated on the following group of figures.

The velocity profile in the middle of the aorta and pressure distribution for the fluid are schematically given in the fig. 1. We can see that the velocity profile is paraboloid (fig. 1, left), which corresponds with fact, that for laminar flow the velocity profile is paraboloid in case of rotational tube. The pressure distribution is drawn on the deformed state of the internal part of the tube, i.e. on the space, filled with blood (fig. 1, right). A colour in the figure changes from red to blue with decrease of pressure magnitude. As we can see the pressure in the region close to the inlet is higher and decreases towards outlet. The velocity profile and pressure distribution are given for the last iteration when the solution had converged.

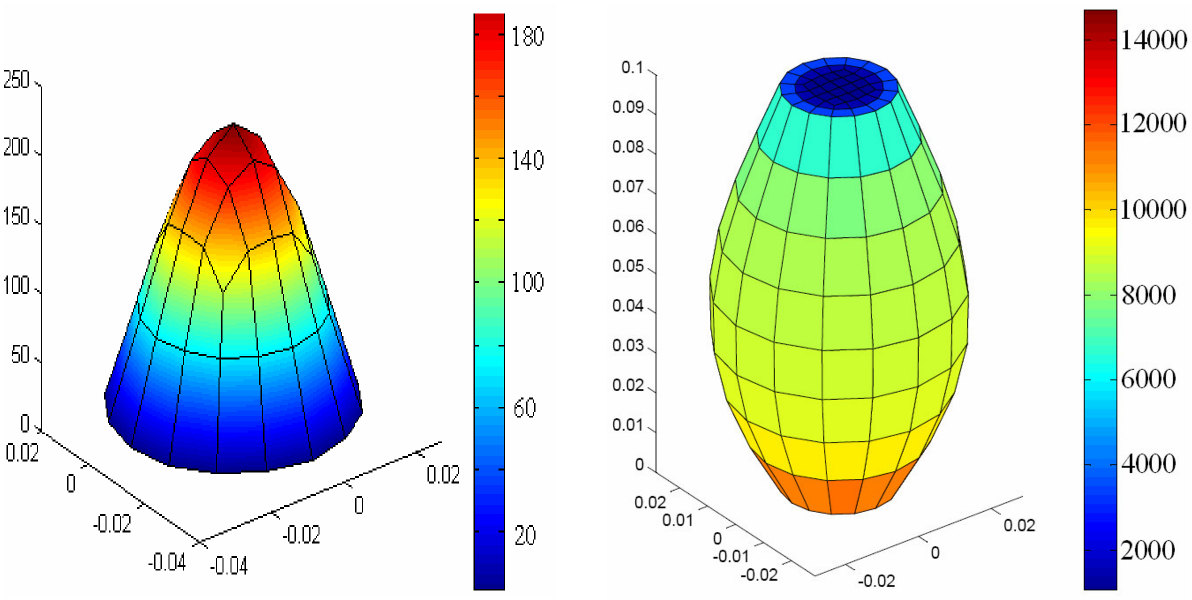

Fig. 1 Velocity profile (left), pressure distribution along the aorta (right)

In the fig. 2, the Tsai-Hill failure index is drawn on the deformed – active state of the aortic wall for media and adventitia layers separately. From the figures we can see how actually large the deformations of the aortic wall are. The colour change from blue to red corresponds with probability of damage from smallest to largest.

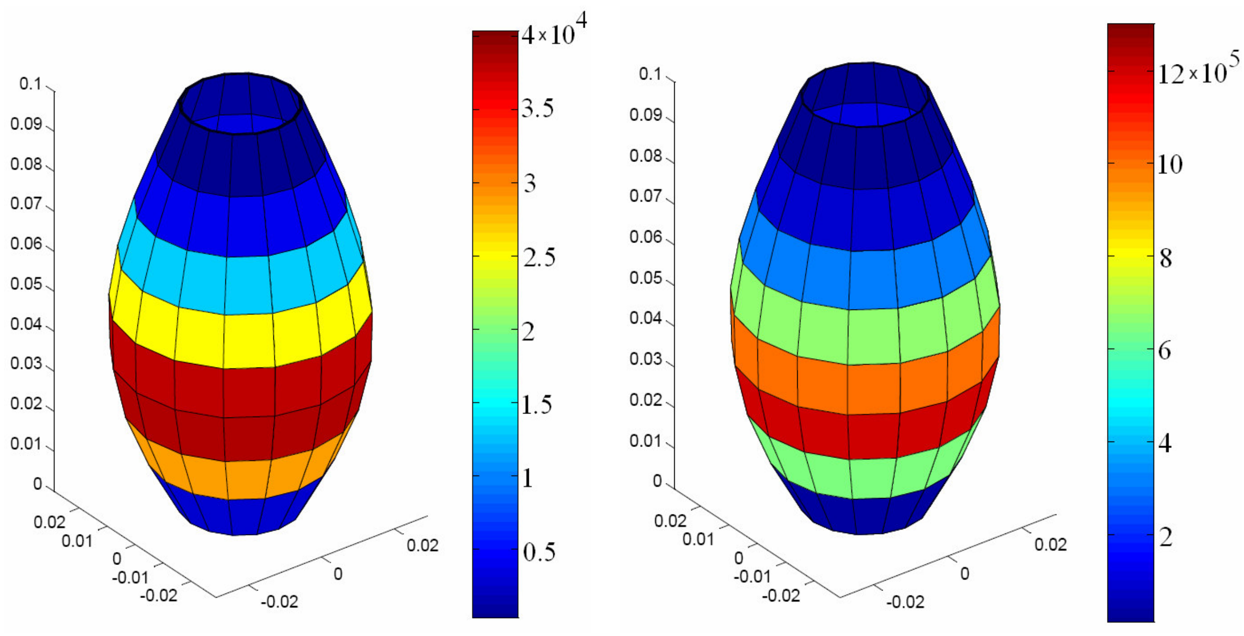

Fig. 2 The Tsai-Hill failure index for the media (left) and the adventitia (right)

As we can see, tissue damage probability increases from the region close to inlet to the most widened part of the wall and decreases further towards outlet region. Besides, the failure index for the media layer belongs to interval $[2.3204, 4.0663\times10^4]$, whilst for the adventitia layer it is in the interval $[57.8766, 1.3116\times10^6]$. We notice, that the failure index for the adventitia is an order of magnitude higher than the one for the media, from which we can conclude that it is the adventitia layer in the aorta, which damages with load first, which reflects the reality.

6. Conclusion

The paper is focused on the solution of blood flow – blood vessel interaction problem, which was solved with usage of uncoupled method. The requested application in presented as an own programming module, written within the Matlab environment. The authors' own implementation was motivated by the opportunity to further extend and enhance the model making it more realistic. The presented model, is yet simplified, it doesn't cover issues like material and geometric nonlinearities of the blood vessel, respects only small deformations and uses a failure criterion which requests only three out of six components of stress. Besides the blood flow is simplified to stationary case. The consideration of these and other parameters in this model in future could lead to a more realistic model.

7. Acknowledgement

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