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AEROELASTIC MODEL WITH AND WITHOUT THE FROUDE NUMBER

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Summary: *Stress-ribbon footbridge is the pre-stressed concrete deck, which has the shape of a catenary. Four types of pre-stressed footbridges were investigated in the wind tunnels with modelled atmospheric boundary layer in the Aeronautical Research and Test Institute in Prague. The types differ in both torsion and bending stiffnesses. The last part of the research contributed to the discussion, whether or not the Froude number should be complied with.*

1. Introduction

The problem of aeroelastic safety of stress-ribbon footbridges was investigated experimentally on the model in a boundary layer wind-tunnel, with particular reference to the influence of the torsional stiffness of the bridge-deck and of the viewing platform at the middle of the bridge span. The results obtained, together with previous studies of the same authors and with the results of full-scale measurements.

Stress-ribbon in catenary form, stress-ribbon supported at its mid-span by two parallel arches, stress-ribbon suspended on a pair of load-bearing ropes and stress-ribbon suspended on a pair of ropes and pre-stressed by the pair of tendons.

2. Reynolds and the Froude numbers

When modelling certain physical phenomena using a major number of dimensionless quantities Π, it is not always possible to comply with all of them – it is impossible to achieve perfect physical similitude. The use of approximate similitude is possible only, if the non-observed dimensionless quantity Π or the model law λ are of insignificant importance for the examined phenomenon. The assessment, whether the results of the approximate model test are not unduly loaded with errors or the quantitative and qualitative assessment of these errors, depends on the experimentator´s experience. For instance, when modelling the phenomena influenced also by the velocity of the flowing liquid, its viscosity and the forces of inertia, it is necessary to observe also two dimensionless quantities (Koloušek et al., 1984).

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$$\Pi_1 = \frac{V^2}{l \cdot g} \quad \text{and} \quad \Pi_2 = \frac{V \cdot l}{\upsilon}, \tag{1}$$

which are the Froude number and the Reynolds´ number, in which υ denotes the kinematic viscosity of the liquid. The model laws for Eq. (1) can be determined as

$$\frac{\lambda_V^2}{\lambda_l \cdot \lambda_g} = 1 \quad \text{and} \quad \frac{\lambda_V \cdot \lambda_l}{\lambda_\nu} = 1. \tag{2}$$

It is obvious that the simultaneous compliance with both laws (2), when $\lambda_g = 1$ and $\lambda_\upsilon = 1$, is possible only if $l_M = l_{\text{real}}$, which is impossible in the modelling. The experimentator must decide which of the two numbers of Eq. (1) is more important for the examined phenomenon.

NOTE
According to experiments with vessel models in a water canal Dr. William Froude (+1879) came to the conclusion that the length of the vessel model *(L_M)* and its velocity *(V)*, and gravity *(g)* determined the quality of results and the shape of the wave accompanying the vessel. According to theory the wave length *(l)* is proportionate with the square of velocity *(V)*

$$l = \frac{2\pi}{g} \cdot V^2 \quad \text{or} \quad V = \sqrt{\frac{g \cdot l}{2\pi}}. \tag{3}$$

Its application to the vessel yields

$$V_M = \sqrt{\frac{g.L_M}{2\pi}} \tag{4}$$

and after the introduction of the ratios λ we obtain the first expression in Eq.(2).

The difficulties arising from this discrepancy have been and are discussed by numerous authors (ASCE, 1999). The sharp-edged exterior geometry models require that $Re \geq 4{\cdot}10^5$ the Froude number requires lower air flow model velocities that the Reynolds number. However the compliance with the model velocity according to the Froude law may make it impossible to produce aeroelastic instabilities. Therefore, for instance in case of the structures for which the gravity loads are important, such as suspension and cable-stayed bridges or guyed masts, the air flow velocity must be modelled according to the Froude law.

In "A wind tunnel investigation of a retractable factory roof" Irwin and Wardlaw (1979) write:

„The Reynold´s number, $\rho Vb/\upsilon$, was as with most tests ignored. After some analysis it was also considered unnecessary to achieve similarity of the Froude number, V^2/bg, because the gravitational forces at full scale tend to be much smaller, owing to the light weight and high tension of the roof system, than the aerodynamic forces in high winds or those due to cable tensioning. An exceptional situation, where gravitational forces would play a role, would be when a cable went slack at high wind speeds due to the tendency of the wind to pull the roof upwards in some areas. The gravitational force would then be of importance for determining the excitation of the slack cable itself but not for the loads at the perimeter of the roof which were under study here. If a cable were sufficiently slack for gravity effects to influence its stiffness, it would no longer contribute significantly to the loads at the roof edge."

3. Demonstration of the observation or non-observation of the Froude law on a model

Even on the world scale we have not found any case in which the examiner has compared the results of an experimental analysis of two models (of the same real structure) the first of which (A) would and the second (B) would not satisfy the Froude law.

For this reason the laboratory of the Institute of Theoretical and Applied Mechanics of the Academy of Sciences of the Czech Republic produced two aeroelastic models of a single-span footbridge of catenary form representing a real structure of the dimensions according to Fig. 1. The linear scale of both models is 1 : 20, i.e. λ_1 = 0.05. The deck of the actual footbridge is a strip of R.C. slabs supported by two dia. 62.7 mm circular sections.

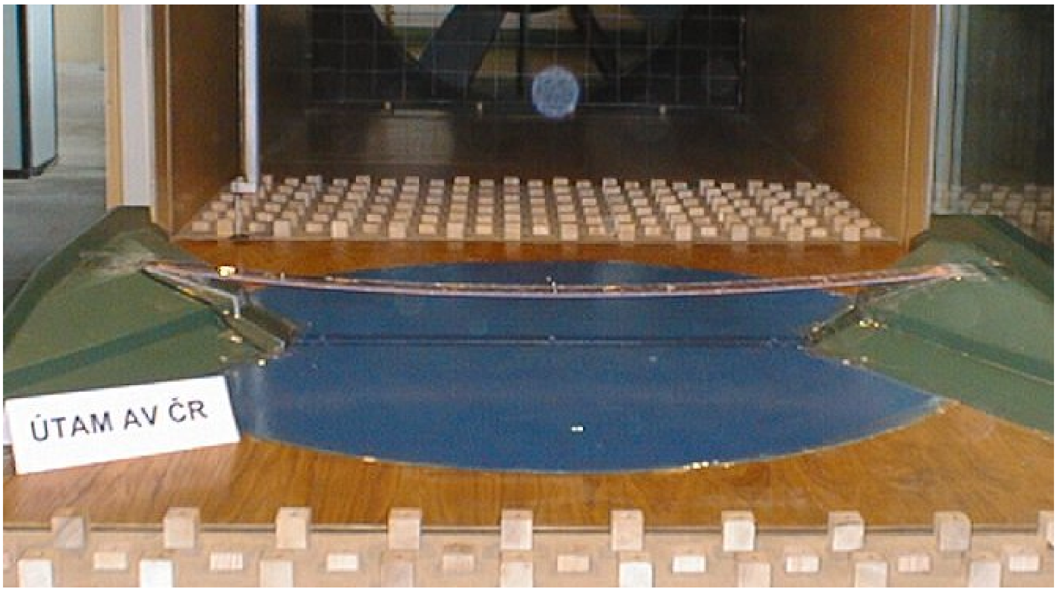


Figure 1. Model of a footbridge 1:20

Model scales of Model A (satisfying the Froude law):

scale of air flow velocity $\lambda_V = \sqrt{0.05} = 0.22$, scale of frequencies $\lambda_f = \sqrt{\dfrac{1}{0.05}} = 4.47$

(the lowest model natural frequency $f_{(2)Real}$ = 0.831 Hz, $f_{(2)M}$ = 3.516 Hz);

scale of tensile stiffness $\lambda_{EA} = 1.21 \cdot 10^{-4}$,

scale of concentrated mass and concentrated loads $\lambda_m = \lambda_F = 4.23 \cdot 10^{-5}$,

scale of wind load $\lambda_{Fw} = \lambda_V^2 \cdot \lambda_l^2 = 1.21 \cdot 10^{-4}$.

From the scale of tensile stiffness we shall derive the model of the lateral load-bearing sections: dia. 1 mm wire.

Model scales of Model B (not satisfying the Froude law):
scale of air flow velocity $\lambda_v = 0.45$ (chosen);

scale of frequencies $\lambda_f = \frac{\lambda_V}{\lambda_l} = \frac{0.45}{0.05} = 9$,

scale of tensile stiffness $\lambda_{EA} = 5.06 \cdot 10^{-4}$,
scale of concentrated mass and load $\lambda_m = 4.34 \cdot 10^{-5}$,
scale of wind load $\lambda_{Fw} = 5.1 \cdot 10^{-4}$.

From the tensile stiffness scale we shall derive the model of lateral load-bearing sections: 4 dia 1 mm wires.

4. San Diego footbridge model

The model on the linear scale of 1 : 70, tested in the wind tunnel in aerodynamic research institute in Prague, had to be made of plexiglass of minimum thickness of 2 mm. This limitation was due to un-guaranteed mechanical properties of the material less than 2 mm thick. The limitation manifested itself in the model similitudes of masses, flexural and torsional stiffnesses and air flow velocity. Therefore the Froude law could not be complied with, although its compliance is recommended for this type of structures. The ratio determined by the Froude number was approximately ≈ 5. Fig. 2 shows the model in the wind tunnel (view in the air flow direction). The pier consists of a steel section coated with balsa wood). Fig. 3 shows the model ready for dynamic analysis (vertical vibration exciter is installed at a quarter-span). Fig. 4a shows a lateral view and the plan of vibration modes, Fig. 4b the vibration modes corresponding with theoretical analysis.

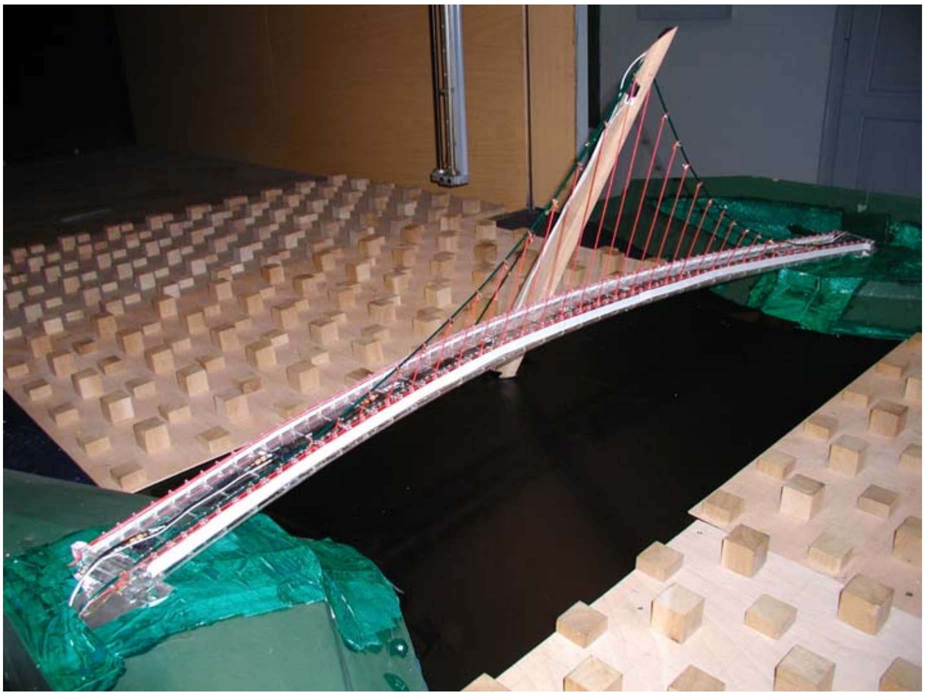
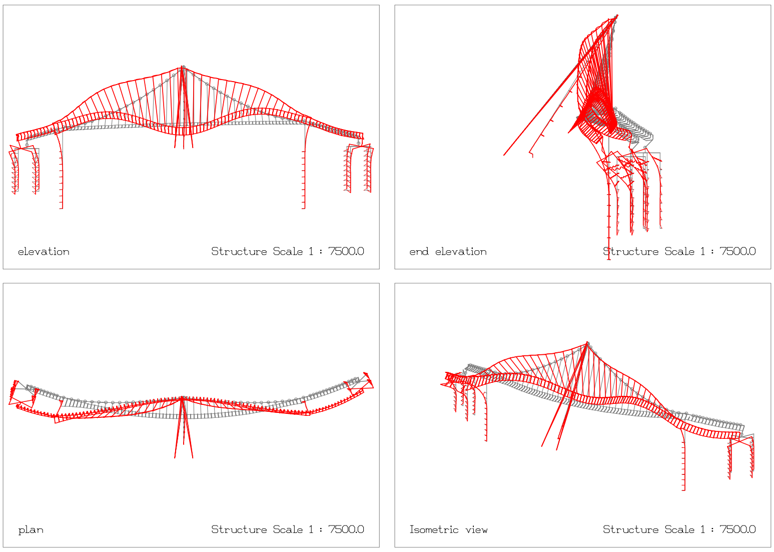


Figure 2. Model of the footbridge in San Diego (1:70) in wind tunnel.

Figure 3. Model at experimental dynamic analysis.

a)



b)

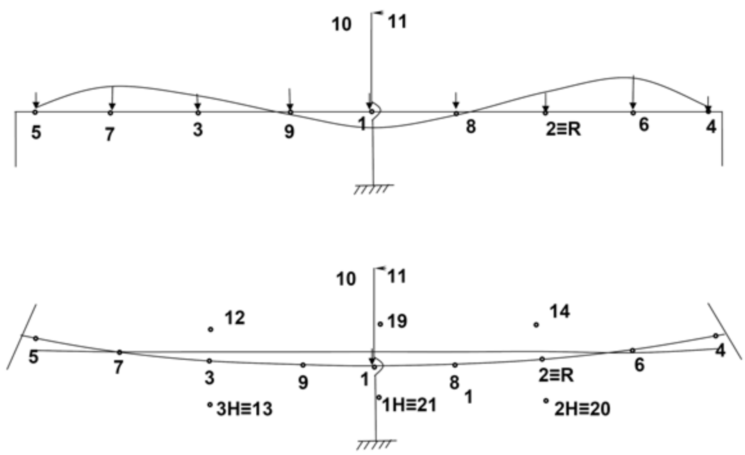


Figure 4a, 4b. Natural modes, a–theory, b–experiment (side view and in groundplan).

The model has shown an agreement of natural vibration frequencies and their modes with the theoretical analysis. It has also proved the aerodynamic stability of the footbridge and the fact that its dynamic response to wind load does not exceed the limit of unpleasant feeling of pedestrians up to the velocity of V_{real} = 22 m/s..

5. Conclusions

The article presents some problems related to experimental verification of footbridge vibrations excited by wind and pedestrian loads on models with and without the Frude number. It has come to light that the satisfaction of requirements imposed on pedestrian comfort is an important aspect which should be assessed in footbridge design and that with current footbridge spans these requirements are difficult to satisfy. In such cases it is necessary to assure sufficient damping of the footbridge. In this respect the stress-ribbon footbridges have proved particularly well which are mentioned in the article. Another possibility is the application of dynamic vibration absorbers based on different principles.

6. Acknowledgements

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