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MODAL PROPERTIES OF A CRACKED BEAM

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Summary: *The existence of a crack in a structural member introduces local modification of flexibility and consequently affects the dynamic behaviour of the cracked member. In this paper, the modal properties such as the natural frequencies and mode shapes for bending vibrations of a cracked beam are studied. The focus of this paper is the modal analysis of a beam with a transverse single-edge open crack. These modal properties are obtained using the finite element method. The influence of the crack parameters, especially the position and depth of the crack, on the modal properties is examined. The results of the change in modal parameters due to the crack appearance are presented.*

1. Introduction

In the present time, the effort to produce the machines and constructions with higher performance, materially and economically effective is graduated. The increasing of performance is mostly achieved by increasing the operating velocities and cycles. The modern constructions are often loaded by external dynamical effects, which produce undesirable dynamic difficulties and overloading of structural elements. Consequently, these structures can be subjected to degenerative effects that may cause initiation of structural defects such as cracks. The presence of crack in the structure could not only cause a local variation in the stiffness but it could affect the mechanical behaviour of the entire structure to a considerable extent. Cracks present in vibrating structures, as time progresses, could lead to catastrophic failure and breakdown of the structure. In order to improve the safety, reliability and operational life, it is important to ensure the integrity of structural elements. The cracks or other defects in a structural element influence its dynamical behaviour and change its stiffness and damping properties. Cracks in a vibrating structure reduce its natural frequencies because it becomes more flexible (Loya & other, 2006). The natural frequencies and mode shapes of the structure contain information about the location and dimensions of the damage.

The vibration characteristics of structures can be useful for an on-line detection of defects such as cracks, what offers an effective, inexpensive and fast means of "non-destructive testing“ without actually dismantling the structure (Musil, 2006). For these reasons, there is a need to understand the dynamics of cracked structures.

It is well-known that the beam is one of the most important structural elements, which is very often used in machines and constructions. The effect of cracks on the dynamic properties of beams has received much attention because of its importance in mechanical and civil engineering applications. In particular, the natural frequencies and mode shapes of cracked beams

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can provide an insight into the extent of damage. As a representative model of a cracked structure a clamped-free and clamped-clamped beams with a transverse edge crack are studied in this paper. The modal analysis approach is used to formulate the corresponding eigenvalue problem (Nánási, 2001). The effect of two of the crack parameters on the dynamic behaviour of the cracked beam under consideration is investigated. These two parameters are the depth and position of the crack.

2. Problem formulation

The problem considered here is a simple beam with crack as it is shown schematically in Fig. 1. The beam of length l_b has a uniform rectangular cross-section (width - b_b, height - h_b). It is supposed that the beam has the crack of depth h_t located at a distance l_t from the right-side of the beam. It is assumed that the crack has a uniform depth across the width of the beam. Only fully open crack is considered.

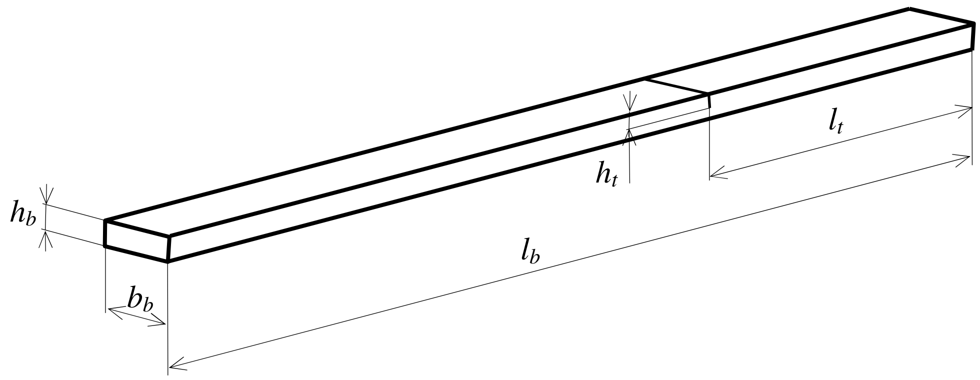

Fig. 1 *Model of a beam with transverse crack*

The mathematical model used for modal analysis of this beam is based on the finite element method. The three-dimensional finite element model of the beam with transverse non-propagating crack to analyse the crack effect on the modal properties is used.

Generally, the equation of motion for a free vibration of beam without crack has the form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0}, \qquad (1)$$

where $\mathbf{M}$ is a mass matrix, $\mathbf{K}$ is a stiffness matrix of the beam without crack, vectors $\mathbf{q}$ and $\ddot{\mathbf{q}}$ are displacements and accelerations, respectively.

The basic dynamical properties of a beam - natural angular frequencies and mode shapes, can be obtained from the following eigenvalue problem

$$(\mathbf{K} - \omega^2\mathbf{M})\boldsymbol{\phi} = \mathbf{0}, \qquad (2)$$

where ω is the natural angular frequency and $\boldsymbol{\phi}$ is the mode shape vector.

Both the birth and growth of the crack are affecting natural frequencies and mode shapes of cracked beam. It must be noted that the mass distribution of cracked beam is not changed, but stiffness of this beam is considerably changed. When the crack is considered, the stiffness matrix of the beam has to be changed.

The equation of motion for a free vibration of cracked beam has the form

$$\mathbf{M}\ddot{\mathbf{q}}_t + \mathbf{K}_t \mathbf{q}_t = \mathbf{0}. \quad (3)$$

Then, the eigenvalue problem can be written

$$(\mathbf{K}_t - \omega_t^2 \mathbf{M})\boldsymbol{\phi}_t = \mathbf{0}, \quad (4)$$

where $\mathbf{K}_t$ is a stiffness matrix of the cracked beam, ω_t is a natural angular frequency and $\boldsymbol{\phi}_t$ is a mode shape vector of the cracked beam.

The modal analysis of a beam with crack was realised using FEM code ANSYS.

3. Numerical example

The effect of the geometrical parameters of transverse crack on the basic dynamical properties of cracked beam is investigated. The dynamical properties of clamped-free (C-F) beam and clamped-clamped (C-C) beam are studied.

The beams under analysis have the following properties: uniform rectangular cross-section with width b_b =12 mm, height h_b = 6 mm and length l_b = 450 mm for clamped-free beam, l_b = 1000 mm for clamped-clamped beam. Material properties of the beams are: Young's modulus of elasticity E = 210 GPa, Poisson number μ = 0.3 and material density ρ = 7800 kgm^3. The value of the crack depth is considered in range $h_t = 1 \div 5$ mm.

The geometrical parameters of the crack are specified by using of the following non-dimensional crack parameters:

- non-dimensional crack position $$\xi_t = \frac{l_t}{l_b} \in \langle 0.0; 1.0 \rangle, \quad (5)$$
- non-dimensional crack depth $$\delta_t = \frac{h_t}{h_b} \in \langle 0.0; 0.83 \rangle. \quad (6)$$

The non-dimensional relative natural frequency for *i*-th mode shape is introduced and it is defined as a frequency ratio

$$\chi_i = \frac{f_{ti}}{f_i}, \quad (7)$$

where f_i is *i*-th natural frequency of beam without crack and f_{ti} is *i*-th natural frequency of beam with crack.

The dependencies of the first three natural frequencies of C-F beam on crack position ξ_t for the different crack depths h_t are shown on Fig. 2. The curves for the first natural frequency have minimum for value ξ_t = 0.1 and maximum of this frequency is for ξ_t tend to the free end of beam. Similar situation we can see for higher natural frequencies of bending vibration of C-F cracked beam. These curves have more local minima and maxima, respectively. Number of local extremes (maximum, minimum) of the curve depends on the order of the particular natural frequency.

Next, the dependencies of the first three natural frequencies of C-C beam on crack position ξ_t and for the different crack depths h_t are shown on Fig. 3.

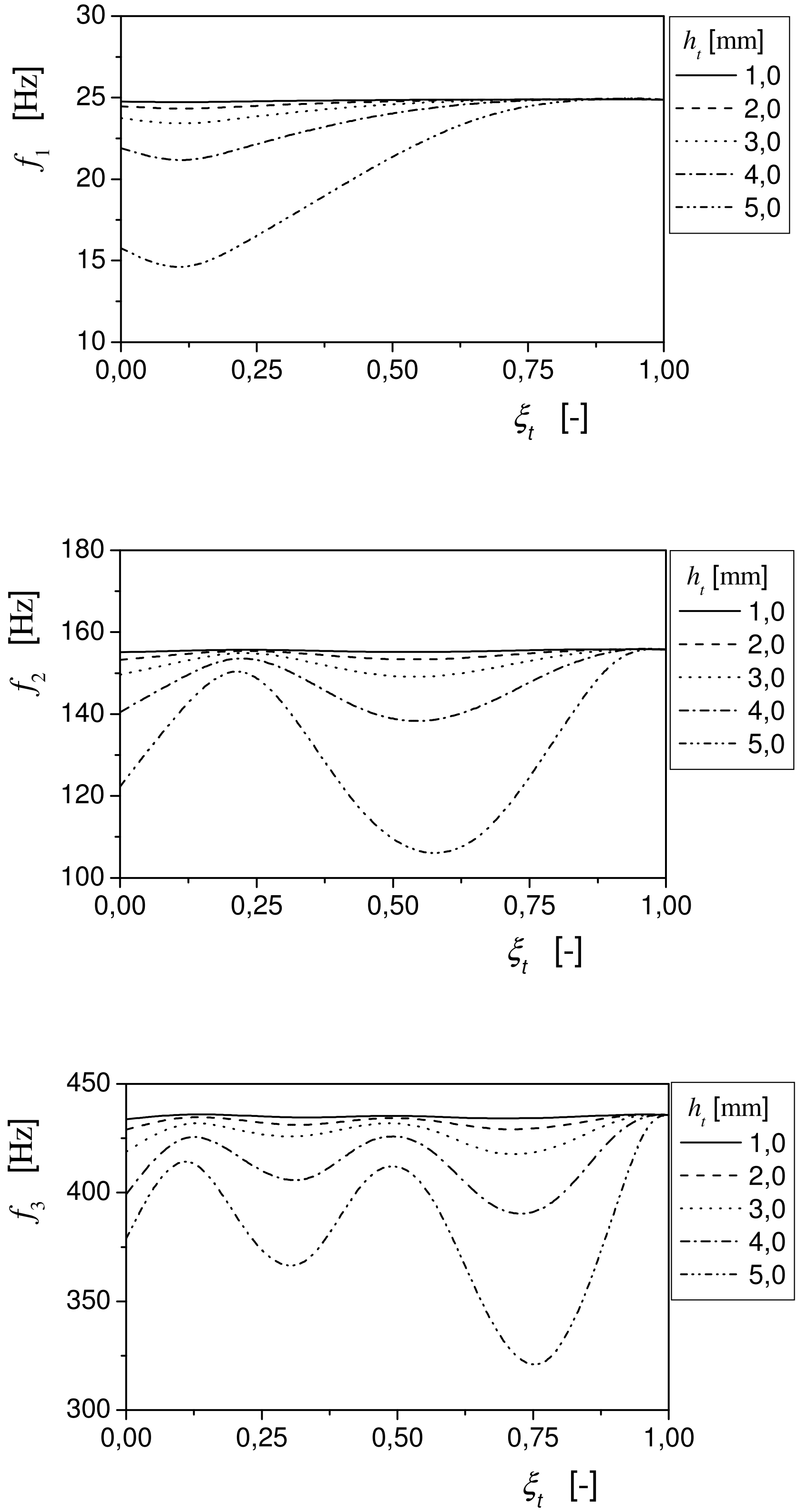

Fig. 2 *Dependency of the first three natural frequencies C-F beam on non-dimensional crack positi*on ξ_t for different h_t.

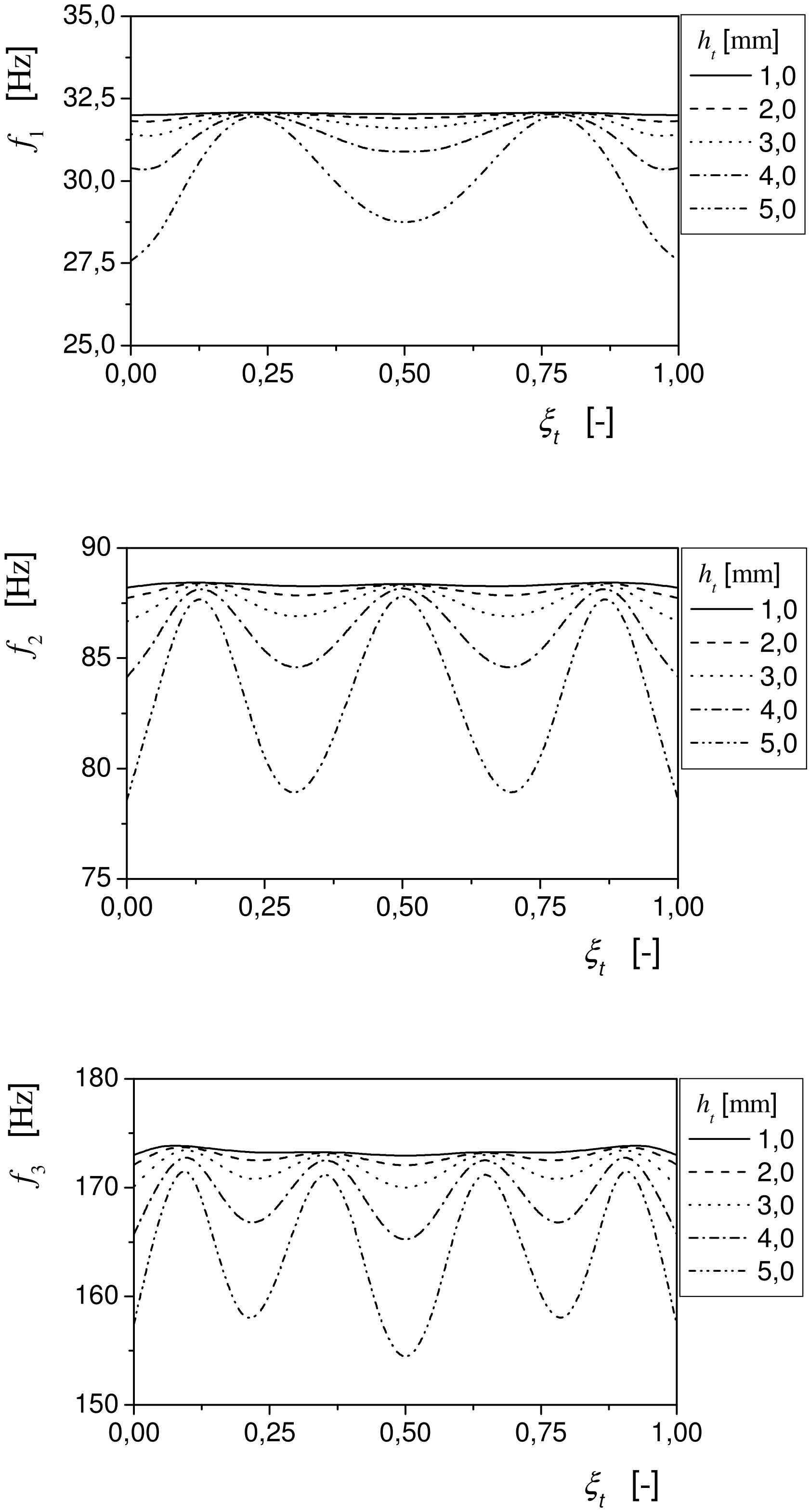

Fig. 3 *Dependency of the first three natural frequencies C-C beam on non-dimensional crack positi*on ξ_t for different h_t.

The dependencies of the first three non-dimensional relative natural frequencies χ_i on non-dimensional crack position ξ_t of the cracked beam (C-F beam - Fig.4; C-C beam - Fig.5) for crack depth $h_t = 5$ mm are compared.

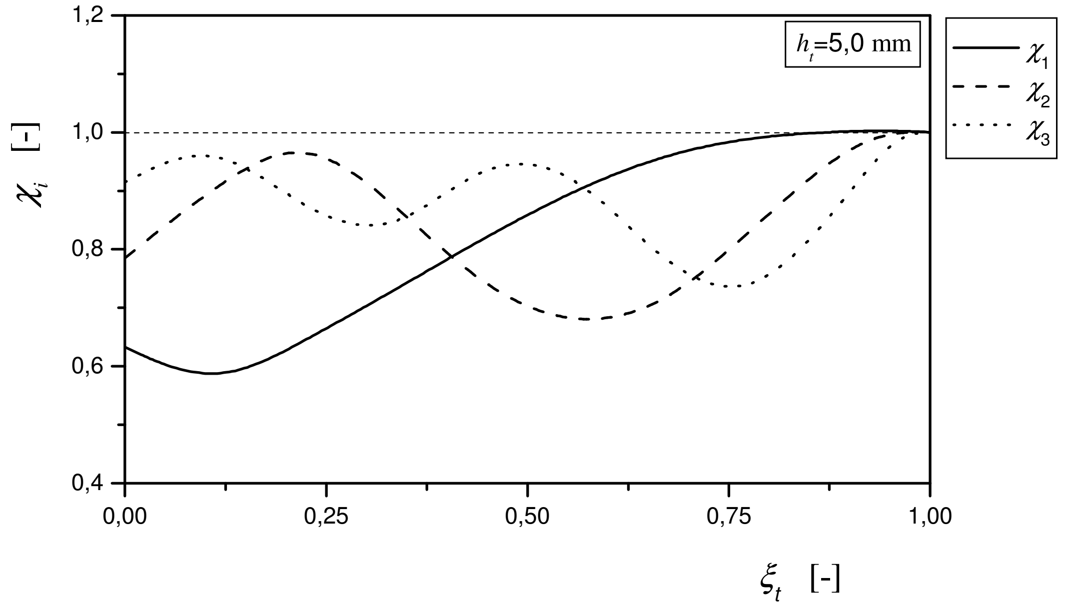

Fig. 4 *Dependency of the first three non-dimensional relative natural frequencies* C-F beam *on non-dimensional crack position for h_t* = 1.0 mm.

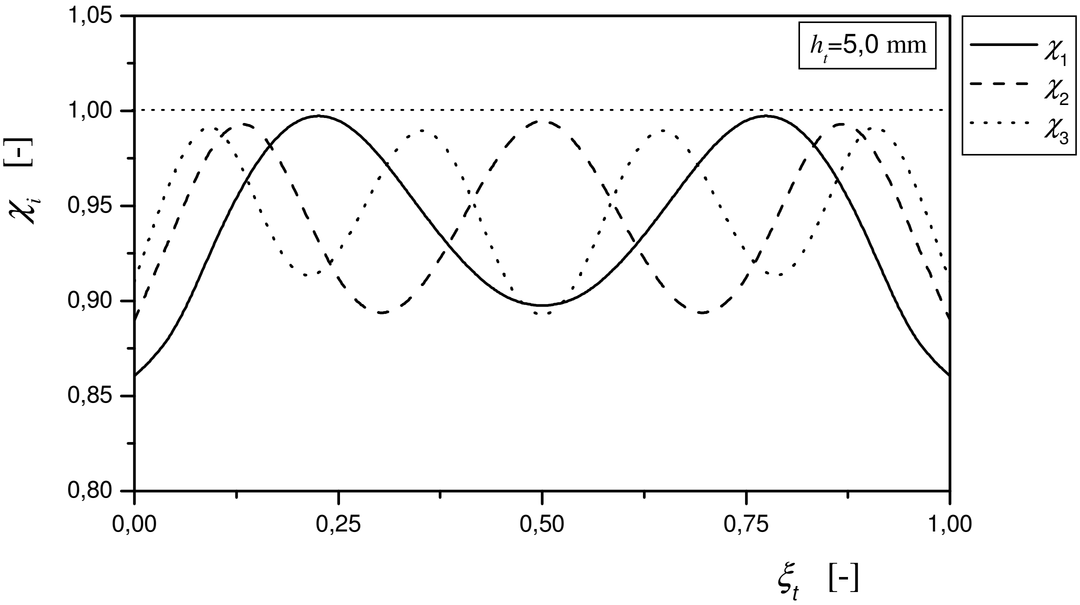

Fig. 5 *Dependency of the first three non-dimensional relative natural frequencies* C-C beam *on non-dimensional crack position for h_t* = 1.0 mm

4. Conclusion

In the present paper, the effect of location and dimensions of crack on natural frequencies and mode shapes of C-F beam and C-C beam are analysed. The crack depth and crack position cause notable modification of natural frequencies of bending vibration of beams. Maximal decreasing of natural frequencies occurs for the first natural frequencies. For the C-F beam is decreasing of the first relative frequency approximately 40% (crack depth h_t = 5 mm, crack position ξ_t = 0.1). The decrease of the first relative frequency of C-C beam is up to 13% (crack depth h_t = 5 mm, crack position ξ_t = 0.0 or ξ_t = 1.0)

Finally, we can say that the crack is a damage that often occurs in structural members and may cause serious failure of the structures. From this follows, that the crack must be detected in the early state. It can be said that the obtained results are applicable for detection and localization of cracks in structures.

5. Acknowledgements

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