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NUMERICAL MODEL OF THE SPHERICAL PARTICLE SALTATION IN CHANNEL WITH TRANSVERSE TILTED ROUGH BED

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Summary: *The paper deals with 3D numerical model of the spherical particle saltation in the rectangular channel with rough bed and the effect of lateral slope of the bed. The stochastic method based on the concept of contact zone is used for the calculation of particle-bed collision. Some examples of calculation are presented, in particular, the lateral dispersions of the particles, which can be used for the particles sorting. The beams of the trajectories of particles starting their saltation from one point were calculated. It was found that the centrelines of the beams can be approximated by straight lines for low and moderate values of the bed transverse slope and the angles between centrelines and the stream direction depend on the lateral slope of the bed.*

1. Introduction

The modeling of the saltation process in the case of transversely tilted bed (i.e. in the rectangular channel with laterally sloped bed) is important for understanding of such natural phenomena as river morphology, i.e. bank erosion, topography of the river bed in fords and bends, sorting of the sediment transport and stream braiding (*Sekine and Parker, 1992)*. Most numerical models of saltation bed load transport are 2D (e.g. *Wiberg and Smith, 1985; Nino and Garcia, 1994; Lee et al., 2000; Lukerchenko et al., 2006b)*. But description of the saltation process in the case of transversely tilted bed can be realised properly only if the 3D pattern of particle saltation is introduced.

Sekine and Kikkawa (1992) and *Lee et al. (2006)* developed 3D saltation model, which consists of two parts; it is deterministic for the particle motion in fluid, but stochastic for the particle – bed collision. The saltating particle is supposed to be spherical and of uniform size, the bed is formed by the same particles. The above mentioned models do not take into account the particle rotation, which is an important component of the saltation process.

Due to the collisions with the bed the particles gain angular velocity, which can reach a few tens of revolution per second (*Nino and Garcia, 1998*). Therefore, to be close to nature of the saltation process, the model of saltation must consider not only translational but also rotational particle motion. The 3D numerical model of particle saltation *(Lukerchenko et al., 2003, 2004, 2006a),* in which the particle rotation is taken into account and the problem of particle-bed collision is solved using the concept of contact zone, is used in the present paper.

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This model of particle saltation in channel with laterally sloped bed allows computation of the particle trajectory and determining of the relationships between the deviation angle of the particle trajectory and value of the lateral slope of the bed and the other saltation parameters. For the purpose of particle movement simulation we shall use the values of the fluid density ρ = 1000 kg/m^3, fluid kinematical viscosity ν = 10^{-6} m^2/s (corresponding to water) and the particle density corresponding to sand ρ_p = 2650 kg/m^3.

2. The mathematical model of the spherical particle saltation

Since the saltation process is very complicated some simplifications were used to develop the model. The main assumptions of the model are:

The particle translational and rotational movement is taken into account.
The effect of turbulence on the particle is neglected.
The concentration of the conveyed particles is sufficiently low and the collisions between particles and the effect of the particles on fluid flow can be neglected.

Therefore the motion of a set of particles can be represented by the motion of a single particle. The effect of the drag force, submerged gravitational force, Basset history force, force due to added mass, Magnus force and drag moment acting on the particle are taken into account in the model.

Let us define the channel bed as a plane spaced over the top of the bed roughness. It is inclined to the horizontal plane in the downstream direction (longitudinal slope θ_s) and also in the transverse direction (transversal or lateral slope θ_l). The fluid flow is steady and uniform in the planes tangential to the bed level. The vertical velocity profile can be described by the logarithmic law. The lateral slope, the angle θ_l of the bed level to a horizontal plane can be defined in the following way, see Fig. 1. The coordinate axis *Oy''* is vertical, coordinate axis *Ox''* is in the downstream direction, i.e. in the direction of the fluid velocity vector and it is parallel to the plane *Ox''y''*.

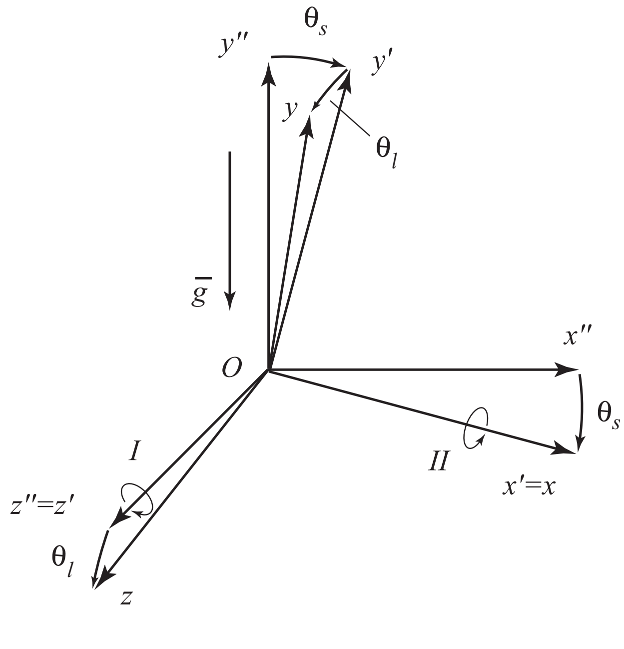

Fig.1. Definition of the coordinate system and its transformation

The plane *Oxz* associated with the bed level can be obtained from the coordinate's plane *Ox''z''* using two transformation of the coordinate system. The first is rotation of the coordinate system *Ox''y''z''* by the angle θ_s around the coordinate axis *Oz''*. The new coordinate system is *Ox'y'z'*. The rotation of the coordinate system *Ox'y'z'* by the angle θ_l around the coordinate axis *Ox'* is the second transformation. The system of governing equations of the particle motion is written in the resulting coordinate system *Oxyz*. The angle θ_s is the angle of the channel bed slope in the streamwise direction and the angle θ_l is the angle of the channel bed slope in the lateral direction or the so called lateral angle of the bed.

The gravitational acceleration vector in the coordinate system *Oxyz* can be written as

$$\bar{g} = \left(g \sin\theta_s; \qquad -g\cos\theta_s \cos\theta_l; \qquad g\cos\theta_s \sin\theta_l\right). \tag{1}$$

Let us consider a spherical homogeneous particle of diameter d_p and density ρ_p moving in the fluid of density ρ and kinematic viscosity ν. The system of the governing equations for the particle saltation motion can be written as

$$\frac{d\bar{r}_{Op}}{dt} = \bar{v}, \tag{2}$$

$$\rho_p \frac{d\bar{v}}{dt} = \bar{F}_D + \bar{F}_m + \bar{F}_g + \bar{F}_B + \bar{F}_M, \tag{3}$$

$$J\frac{d\bar{\varpi}}{dt} = \bar{M}, \tag{4}$$

where t is time, $\bar{r}_{Op}\left(x_p, y_p, z_p\right)$ is the radius-vector of the particle centre of mass, $\bar{v}\left(v_x, v_y, v_z\right)$ is the vector of the velocity of the particle centre of mass, $\bar{\varpi}\left(\omega_x, \omega_y, \omega_z\right)$ is the vector of angular velocity of the particle rotation about its centre of mass, J is the particle moment of inertia, $\bar{F}_D$, $\bar{F}_m$, $\bar{F}_g$, $\bar{F}_B$, $\bar{F}_M$ are the drag force, force due to added mass, submerged gravitational force, Basset history force, and Magnus force per unit volume, respectively, $\bar{M}$ is the drag moment of viscous forces acting on the particle.

The particle motion is determined by two dimensionless parameters: translational Reynolds number (or Reynolds number) $\mathrm{Re} = \left|\bar{v}_R\right| d_p / \nu$ and rotational Reynolds number $\mathrm{Re}_\omega = \left|\bar{\varpi}_R\right| r_p^2 / \nu$, where $\bar{v}_R = \bar{v} - \bar{v}_f$ is the vector of particle-fluid relative velocity, $\bar{v}_f\left(v_{fx}, v_{fy}, v_{fz}\right)$ is the vector of the fluid velocity, $\bar{\varpi}_R = \bar{\varpi} - \frac{1}{2} rot\, \bar{v}_f$ is the particle relative angular velocity, and $r_p = 0.5\, d_p$ is the particle radius.

The flow velocity distribution can be described by the logarithmic law

$$v_{fx}\left(y\right) = \frac{u_*}{k} \ln\left(\frac{y}{y_0}\right), \tag{5}$$

where $k = 0.4$ is the Karman constant; $y_0 = 0.11\left(\nu / u_*\right) + 0.033\, k_s$ *(Nikuradze, 1933)*, where u_* is the fluid shear velocity, k_s is the bed roughness and $v_{fy} = v_{fz} = 0$. The numerical model of the particle saltation including particle-bed collision is described in *Lukerchenko et al., 2004, 2005 and 2006a* in details.

3. Results of the calculations

The effect of the transversely tilted bed results mainly in the lateral deviation of the saltating particles trajectories from the downstream direction in agreement with the lateral slope of the bed. A single particle trajectory has a random character therefore the beam or fascicle of the trajectories of particles that started their movement from one point in the channel were studied.

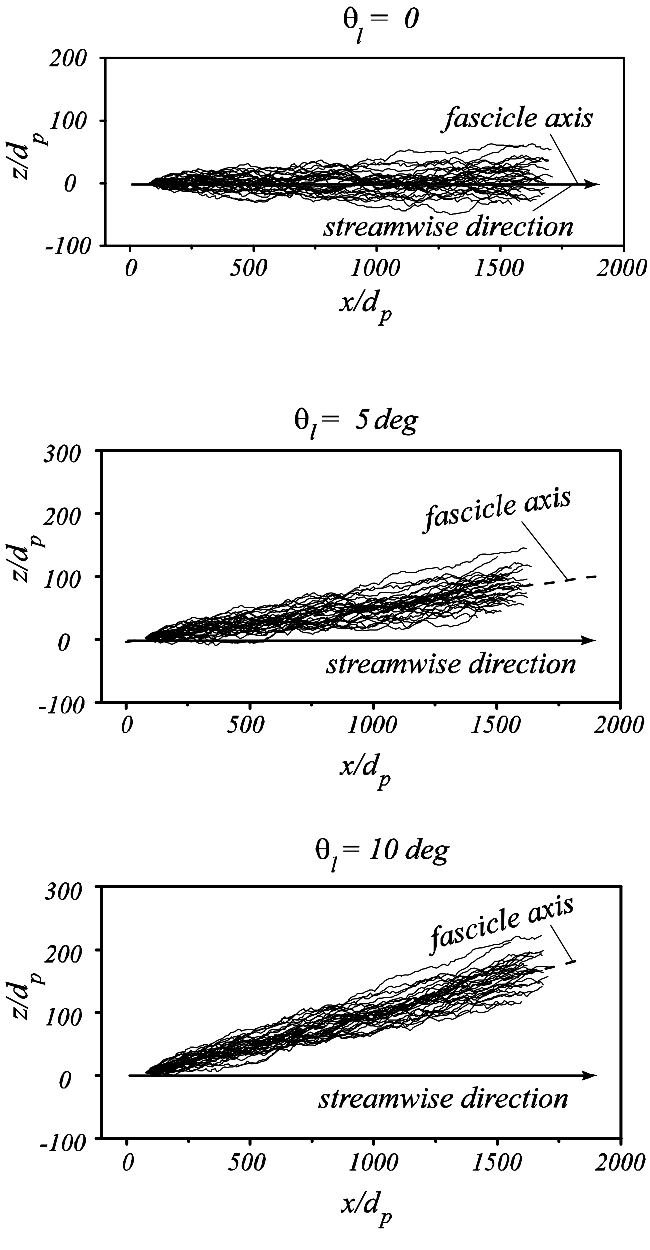

Fig. 2. Effect of the bed lateral slope on the saltating particle trajectories (d_p = 1 mm, ρ_p / ρ = 2.65, k_s = 1 mm, u_* = 0.025 m/s).

3.1 The beams of the saltating particle trajectories

Examples of the beams of the saltating particle trajectories are shown in Fig. 2 for the case of the bed without the lateral slope and for the cases of the bed with two different lateral slopes. The values of the angle of the bed lateral slope θ_l are 5 and 10 degrees, respectively.

Let us define the origin of the trajectories and the centreline of the beam, which can be approximated by a nearly straight line. Let us call it a beam axis and let us define also the beam boundaries as the envelope of the computed particle trajectories and the region between the beam boundaries as the beam region.

We call the angle between the beam axis and the downstream direction the deviation angle α_p and the angle between the beam boundary and the beam axis the disperse angle $\Delta\alpha_p$. Both angles α_p and $\Delta\alpha_p$ are functions of the saltation parameters:

$$\alpha_p = \alpha_p(\theta_l, d_p, u_*, k_s, \rho_p, \rho, \nu, g), \tag{6}$$

$$\Delta\alpha_p = \alpha_p(\theta_l, d_p, u_*, k_s, \rho_p, \rho, \nu, g). \tag{7}$$

The value of the deviation angle α_p grows when the value of the bed lateral slope θ_l increases. Effect of the shear velocity is shown in Fig. 3. In the studied range of θ_l , i.e. from zero to 20 degrees, the dependence is close to the linear. With growing shear velocity the deviation angle decreases. Effect of the particle diameter on the deviation angle is shown in Fig. 4 for three values of the particle diameter. The deviation angle increases with growing particle diameter.

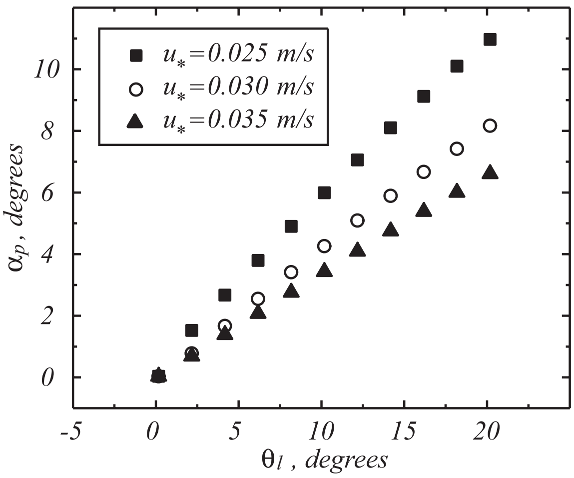

Fig. 3. The deviation angle α_p vs. the bed lateral slope θ_l (d_p = 1 mm, ρ_p/ρ = 2.65, k_s = 1 mm)

3.2. The particles sorting in the channel with the transverse tilted rough bed

The dependence of the deviation angle α_p on the particle diameter d_p is shown in the Fig. 5 for $\theta_l = 20^0$ and the bed roughness k_s = 1 mm. Since it is a monotonously increasing function, this suggests that the saltation of particles in the channel with laterally sloped bed can result in the selection of particles according to the size of particle diameter. This process can be called the particle sorting. However, the particle sorting is relatively complex process, which depends not only on particle diameter, but also on lateral slope of the channel bed and on liquid velocity.

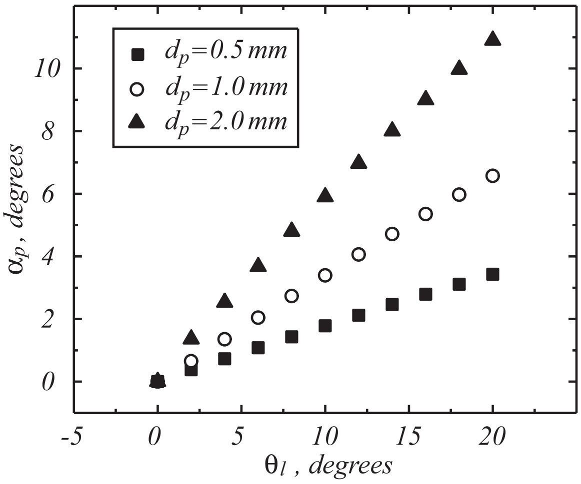

Fig. 4. The deviation angle α_p vs. the bed lateral slope θ_l
($u_* = 0.35$ m/s, $\rho_p / \rho = 2.65$, $k_s = 1$ mm)

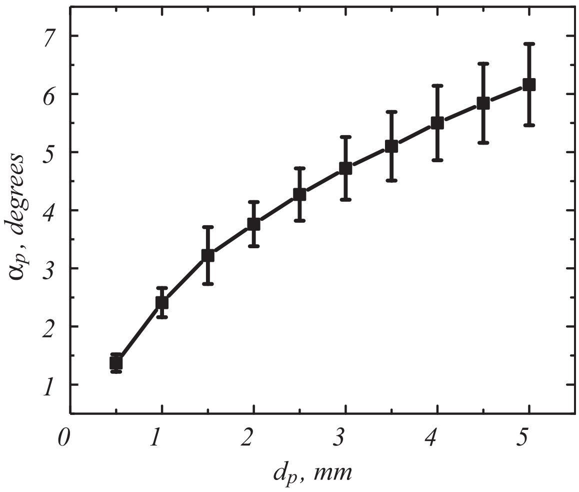

Fig. 5. The effect of the particle diameter d_p on the deviation angle α_p
($\theta_l = 20^0$, $\rho_p / \rho = 2.65$, $k_s = 1$ mm, $u_* = 0{,}07$ m/s)

4. Conclusions

The 3D numerical model, which describes the saltation of the spherical particles in rectangular channel with the transversely tilted rough bed, was developed.

The effect of the transversely tilted bed results mainly in the lateral deviation of the particles trajectories from the downstream direction to the direction of the lateral slope of the bed. The trajectories of the saltating particles starting from one point were computed and as it was shown they create a beam of trajectories.

The deviation angle of the beam axis from the downstream direction α_p is nearly linearly dependent on the lateral slope of the bed θ_l in the region $0 < \theta_l < 20$ degrees.

The deviation angle depends on the lateral bed slope, the particle diameter and shear velocity. It was found that the saltation of particles in the channel with transverse tilted bed can result in the selection of particles according to the size of the particle diameter.

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Notation

d_p - diameter of the moving particle;
$\overline{F}_B$ - Basset history force per unit volume;
$\overline{F}_D$ - drag force per unit volume;
$\overline{F}_g$ - submerged gravitational force per unit volume;
$\overline{F}_m$ - force due to added mass per unit volume;
$\overline{F}_M$ - Magnus force per unit volume;
$\overline{g}$ - vector of gravitational acceleration;
J - particle moment of inertia;
k - Karman constant;
k_s - bed roughness;
$\overline{M}$ - drag moment of viscous forces acting on a rotating particle in fluid;
r_p - radius of the moving particle;
$\bar{r}_{Op}(x_p, y_p, z_p)$ - radius-vector of the particle centre of mass;
Re - translational Reynolds number;
Re_ω - rotational Reynolds number;
t - time;
u_* - fluid shear velocity;
$\bar{v}_f(v_{fx}, v_{fy}, v_{fz})$ - vector of the fluid velocity;
$\bar{v}(v_x, v_y, v_z)$ - vector of velocity of the particle centre of mass;
$\bar{v}_R$ - vector of the particle relative velocity;
y_0 - elevation of zero fluid velocity;
α_p - angle of the fascicle axis deviation from the downstream direction;
$\Delta\alpha_p$ - angle between the fascicle boundary and the fascicle axis;
ν - kinematic viscosity;
θ_l - angle of the bed level lateral slope;
θ_s - angle of the bed level streamwise slope;
ρ - fluid density;
ρ_p - density of the moving particle;

$\overline{\omega}\left(\omega_x, \omega_y, \omega_z\right)$- vector of angular velocity of the particle rotation about its centre of mass;

$\overline{\omega}_R$ - particle relative angular velocity;

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